Anglo - Chinese School

(Independent)

FINAL EXAMINATION 2020

YEAR 3 INTEGRATED PROGRAMME

CORE MATHEMATICS
PAPER 2

MONDAY **xxth October 2020** **1 hour 30 minutes**

ADDITIONAL MATERIALS:

Answer Paper (7 sheets)
Graph Paper (1 sheet)

INSTRUCTIONS TO STUDENTS

Do not open this examination paper until instructed to do so.
A calculator is required for this paper.
Answer all the questions on the answer sheets provided.
At the end of the examination, fasten the answer sheets together.
Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures. Answers in degrees are to be given to one decimal place.

INFORMATION FOR STUDENTS

The maximum mark for this paper is 80.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. You are therefore advised to show all working.

Answer all the questions on the answer sheets provided. Begin each question on a new page.

1. *[Maximum mark: 6]*

(a) Evaluate $\dfrac{(0.3578)^2 \times \sqrt{7.647}}{\sqrt[4]{43.96}}$, leaving your answer correct to 3 significant figures.

[2]

$$\frac{0.354019}{2.57492}$$

$$= 0.137$$

Some student take square of the expression to get rid of the square root first. They need to know that this is an expression, hence they cannot do that!

(b) Express $\dfrac{1}{x+2} + \dfrac{1}{2-x} + \dfrac{10+3x}{x^2-4}$ as a single fraction in its simplest form.

[4]

$$\frac{1}{x+2} + \frac{1}{2-x} + \frac{10+3x}{x^2-4}$$

$$= \frac{1}{x+2} - \frac{1}{x-2} + \frac{10+3x}{(x-2)(x+2)}$$

$$= \frac{x-2-x-2+10+3x}{(x-2)(x+2)}$$

$$= \frac{6+3x}{(x-2)(x+2)}$$

$$= \frac{3}{(x-2)}$$

Very careless

$$\frac{1}{2-x} = -\frac{1}{x-2}$$

Some students did not leave it in the simplest form

2 *[Maximum mark: 7]*

(a) Solve the equation $2x - \frac{9}{2x} = 3$, giving your answers correct to two decimal places.

[4]

$2x(2x) - 9 = 3(2x)$

$4x^2 - 9 = 6x$

$4x^2 - 6x - 9 = 0$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(-9)}}{2(4)}$

$x = -0.93$ or $x = 2.43$

The question specifically asked for 2 dp. Some student left it as 3 sf.

(b) Expand and simplify $\left(p^{\frac{2}{3}} + q^{\frac{1}{3}}\right)\left(q^{\frac{2}{3}} - q^{\frac{1}{3}}p^{\frac{2}{3}} + p^{\frac{4}{3}}\right)$.

[3]

$p^{\frac{2}{3}}q^{\frac{2}{3}} - q^{\frac{1}{3}}p^{\frac{4}{3}} + p^2 + q - q^{\frac{2}{3}}p^{\frac{2}{3}} + q^{\frac{1}{3}}p^{\frac{4}{3}}$

$= p^2 + q$

Easy question, but surprising very poorly done. Many tried to factorize the expression instead of manually expanding it.

3 *[Maximum mark: 9]*

(a) Simplify $\frac{\left(2x^{-2}\right)^3 \times 4x^{12}}{(4x^{-4})^2\left(x^3\right)^4}$, expressing your answer in positive indices.

[3]

$\frac{8x^{-6} \times 4x^{12}}{16x^{-8} \times x^{12}}$

$= \frac{32x^6}{16x^4}$

$= 2x^2$

A lot of misconceptions such as:

$(2x^{-2})^3 = 2x^{-6} = \frac{1}{2x^6}$

$(4x^{-4})^2 = 4x^{-8}$

(b) Given that $\dfrac{16^{x+1}+2(2^{2x})^2}{2^{x-2}8^{x+1}}=k$, find the value of k.

[3]

$$\frac{16^x(16)+2(2^{4x})}{2^x2^{-2}2^{3x}2^3}$$

$$=\frac{2^{4x}(16)+2(2^{4x})}{2(2^{4x})}$$

$$=\frac{2^{4x}(16+2)}{2(2^{4x})}$$

$$=\frac{18}{2}$$

$$=9$$

Very careless. Mistakes such as the following are very common:

$16^{x+1}=2^{4x+1}$

$2(2^{2x})^2=2(4^{4x})=8^{4x}$

(c) Find the value of $\dfrac{1}{\log_p pqr}+\dfrac{1}{\log_q pqr}+\dfrac{1}{\log_r pqr}$.

[3]

$$\log_{pqr}p+\log_{pqr}q+\log_{pqr}r$$

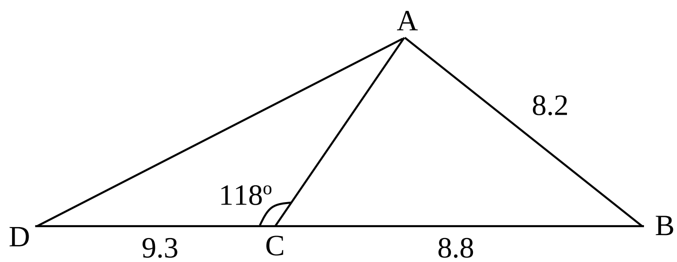
$$=\log_{pqr}(pqr)$$

$$=1$$

Poorly attempted for a very simple question. Many just split the denominator into 3 separate terms or just tried to combine the 3 fractions using common denominator.

4 *[Maximum mark: 15]*

The diagram shows four towns A, B, C and D. Given that Town C and Town D lie west of B, AB = 8.2 km, BC = 8.8 km, CD = 9.3 km and $\measuredangle ACD=118^o$, calculate



(a) $\measuredangle ABC$,

[4]

$\measuredangle ACB = 180 - 118$

$\measuredangle ACB = 62$

$\frac{\sin 62}{8.2} = \frac{\sin \measuredangle CAB}{8.8}$

$\sin \measuredangle CAB = 0.947554$

$\measuredangle CAB = 71.4$

$\measuredangle ABC = 180 - 71.4 - 62$

$\measuredangle ABC = 46.6$

Alternative methods such as finding the length of AC first, then use consine rule and then sine rule again is quite common.

Students should try to use the easiest method to solve a problem.

(b) the bearing of B from A,

[2]

$90 - 46.6 = 43.4$

$180 - 43.4 = 136.6$

Alternative:

90 + 46.6 = 136.6

(c) the distance of AD.

[3]

$AD^2 = 18.1^2 + 8.2^2 - 2 * 18.1 * 8.2 * \cos 46.6$

$AD^2 = 190.895$

$AD = 13.8$

Well attempted for students who has gotten part (a) and (b) correct. Very direct and straight forward.

A cyclist starts from D at 1030 and travels towards A at a constant speed of 15 km/h.

(d) Find the time, to the nearest minute, when he will be nearest to Town C.

[6]

$$\frac{AC}{\sin 46.6} = \frac{8.2}{\sin 62}$$

$AC = 6.74$

Area of triangle ACD = $\frac{1}{2} \times 9.3 \times 6.74 \times \sin 118$

Area of triangle ACD = 27.7

$$\frac{1}{2} \times 13.8 \times h = 27.7$$

$h = 4.02$

Distance cycled = $\sqrt{9.3^2 - 4.02^2}$

Distance cycled = 8.39

Time cycled = $\frac{8.39}{15} = 0.559$ hr = 34 minutes

Time nearest = 1104

- This is a 6 mark question, some students just used answer in part (c) to divide by speed.
- Many students failed to find the perpendicular distance.
- GOOD: Alternative method includes finding angle ADC and then use cosine to find the distance travelled.
- Question is asking for the time, not just the time taken.

5 *[Maximum mark: 13]*

(a) The equation of a curve is $y = 3qx^2 - 3px + 12q$ where p and q are positive integers. Show that the $x-$axis is the tangent to the curve if $\frac{p}{2q} = 2$.

[4]

$$(-3p)^2 - 4(3q)(12q) = 0$$

$$9p^2 - 144q^2 = 0$$

$$\frac{p^2}{q^2} = \frac{144}{9}$$

$$\frac{p}{q} = \frac{12}{3}$$

$$\frac{p}{2q} = 2$$

Some students are unable to explain why they need to let D = 0.

Alternative method includes attempting to complete the square and show that the turning point is at (2, 0), hence x-axis is a tangent.

(b) Explain why the line $y = kx - 2$ will always intersect the curve $y = \frac{3}{x-3}$.

[4]

$kx - 2 = \frac{3}{x-3}$

$kx^2 - 3kx - 2x + 6 = 3$

$kx^2 - 3kx - 2x + 3 = 0$

$\mathrm{D} = [-(3k+2)]^2 - 4(k)(3)$

$= 9k^2 + 12k + 4 - 12k$

$= 9k^2 + 4$

Since $D > 0$, hence the line always intersect the curve.

Very careless in manipulation.
Many students are not able to expand $(-3k-2)^2$ correctly! Hence, leading to the wrong conclusion or unable to explain why the two lines always intersect.
They expanded it as:

$(-3k-2)^2 = 9k^2 - 12k + 4$

(c) The roots of $(x-1)(3-x) = m$ are α and β.

(i) Find the value of $\alpha + \beta$ and $\alpha\beta$ in terms of m.

(ii) Given that $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{4}{(\alpha\beta)^2}$, find the value of m.

[5]

$3x - x^2 - 3 + x = m$

$-x^2 + 4x - 3 - m = 0$

$x^2 - 4x + 3 + m = 0$

$\alpha + \beta = 4$

$\alpha\beta = 3 + m$

Very poorly attempted.

Students must expand and ensure that the right hand side of the equation is 0 before reading off the a, b and c of the quadratic equation.

$$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{4}{(\alpha\beta)^2}$$

$$\frac{\alpha+\beta}{\alpha\beta}=\frac{4}{(\alpha\beta)^2}$$

$$\alpha+\beta=\frac{4}{\alpha\beta}$$

$$4=\frac{4}{3+m}$$

$$3+m=1$$

$$m=-2$$

6 *[Maximum mark: 18]*

(a) Evaluate $\log_3 2+\log_2 3$. [2]

$$\frac{\lg 2}{\lg 3}+\frac{\lg 3}{\lg 2}$$

$= 2.215$

Simple question but many failed to see that they can only obtain the value by changing to base 10 or base e. Hence, the question is poorly attempted.

(b) Find the value of k if $e^{e^{k+2}}=5$. [3]

$$e^{k+2}=\ln 5$$

$$k+2=\ln(\ln 5)$$

$$k=\ln(\ln 5)-2$$

$$k=-1.52$$

Surprisingly, this was much better than part (a).

(c) $2^{3x-5} = 7^{x-2}$ can be expressed as $\left(\frac{7}{a}\right)^x = \frac{49}{32}$ where a is a positive integer.

(i) Find the value of a.

[3]

$$\frac{2^{3x}}{2^5} = \frac{7^x}{7^2}$$

$$\frac{7^2}{2^5} = \frac{7^x}{8^x}$$

$$\left(\frac{7}{8}\right)^x = \frac{49}{32}$$

$$a = 8$$

A common error is to compare numerator and denominator of a fraction.

$$\left(\frac{7}{a}\right)^x = \frac{7^x}{a^x} = \frac{49}{32}$$

$$a^x = 32$$

And then students are stuck…

(ii) Hence, find the value of x.

[2]

$$\left(\frac{7}{8}\right)^x = \frac{49}{32}$$

$$x \lg\left(\frac{7}{8}\right) = \lg\frac{49}{32}$$

$$x = -3.19$$

Not difficult once students get part (i) correct.

(d) When a cup of hot water is left in a room, its temperature T °C at time t minutes is given by $T = 67e^{-0.2t} + 27$.

(i) Kenneth claimed that the initial temperature of the water is 100°C, explain clearly whether Kenneth is correct.

[2]

Let $t = 0$,

$$T = 67e^{-0.2(0)} + 27$$

$$T = 94$$

Kenneth is wrong.

Overall, this is a well attempted question. Most students are able to complete part (d) without any major problem.

(ii) Find the temperature when $T = 5$ minutes.

$$T = 67e^{-0.2(5)} + 27$$

$$T = 51.6$$

(iii) How long will it take for the temperature of the water to drop to 30°C. [3]

$30 = 67e^{-0.2t} + 27$

$3 = 67e^{-0.2t}$

$\frac{3}{67} = e^{-0.2t}$

$-0.2t = \ln\left(\frac{3}{67}\right)$

$t = 15.5$

7 *[Maximum mark: 13]*

Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = 2^{2-x}$. Some corresponding values of x and y are given in the following table.

x	–2	–1	0	1	2	3	4	5
y	16	a	4	2	1	0.50	b	0.125

(a) State the value of a. [2]

$y = 2^{-4+2}$

$y = 2^{-2}$

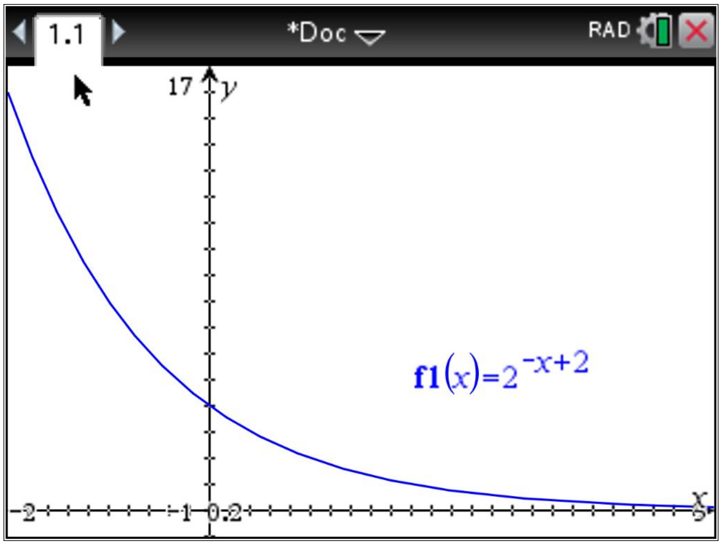
$y = 0.25$

$y = 2^{1+2}$

$y = 8$

(b) Taking 2 cm to represent 1 unit on the horizontal axis and 1 cm to represent 1 unit on the vertical axis, draw the graph of $y = 2^{-x+2}$ for $-2 \le x \le 5$.

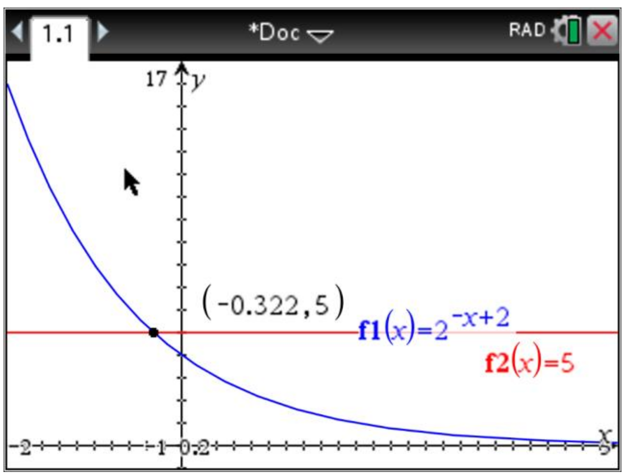
[4]



Students need to produce a smooth curve for a more accurate answer.

(c) Use your graph to solve the equation $2^{-x+2} = 5$

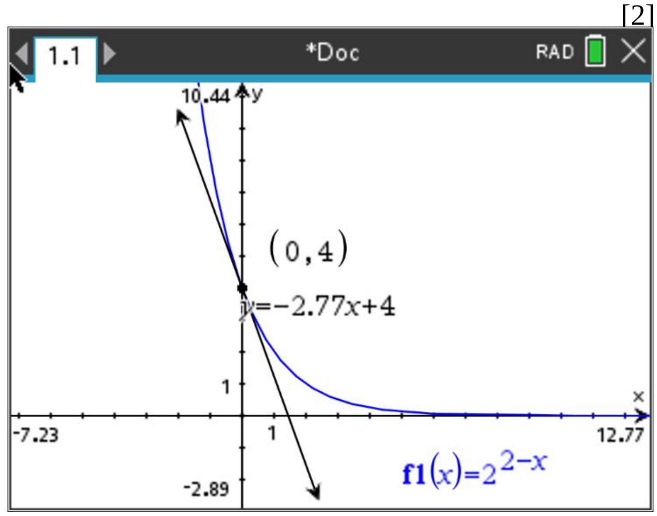
[2]



$x = -0.322$

(d) Find the gradient of the tangent at $x = 0$.

[2]



Gradient = -2.77

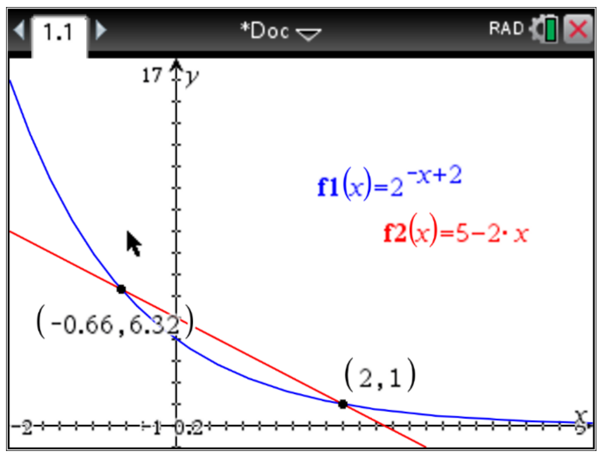
Accept -2.4 to -3

(e) By drawing a suitable straight line on the graph, find the solution of the equation $2^{x-2}(5-2x)=1$.

[3]

$x = -0.6$ to -0.7

or $x = 1.8$ to 2.2



Some students failed to manipulate the equation and cannot see that they need to draw the graph of $y = 5 - 2x$.

Also, there are two points of intersections. Some only gave one solution because they did not extrapolate the graph.

---END OF PAPER--