Candidate Name: _____ **Index No:** ____ **Class:** _____

ZHENGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATIONS 2023
SECONDARY FOUR EXPRESS/NORMAL (ACADEMIC) (O LEVEL)
MATHEMATICS **4052/01**
Paper 1

22 AUGUST 2023

Candidates answer on the Question Paper. 2 hours 15 minutes

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

Name of Setter:

This document consists of 1 printed pages.

[Turn over]

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1. Calculate $\frac{\sqrt[3]{4\pi^2 - 15^2}}{0.63}$.

Leave your answer to 4 decimal places.

Answer ..... [1]

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2. Factorise $9x^2 + 12x - 12$.

Answer ..... [2]

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3. The sine of an angle is 0.427.

Give two possible values for the angle.

° °

Answer ..... or ..... [2]

---

4. 8 students took a test and their marks were recorded as follows.

8 10 9 13 10 9 x y

The modal score for the 8 students is 10 and the median is 9.5.

If the range of the scores is 5, and $x \leq y$, determine the values of x and y.

Answer $x =$ .....

$y =$ ..... [2]

---

5. A sum of money was shared among Allan, Ben and Chloe.

Allan receives $\frac{2}{5}$ of the money and the rest was shared among Ben and Chloe in the ratio 3:2.

Find the ratio of the money received in terms of Allan's : Ben's : Chloe's.

Answer ..... : ..... : ..... [2]

6. John is making two rectangular flower beds.

The dimensions of the larger rectangle will be four times the dimensions of the smaller rectangle.

There is going to be the same depth of soil in each flower bed.

John needs 120 kg of soil for the larger flower bed.

Work out how much soil John needs for the smaller flower bed.

Answer ..... kg [2]

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7. $T = \sqrt{\frac{w}{d^3}}$ for a particular value of w and d.

If w is increased by 20%, and d is decreased by 10%, determine the percentage change in T.

Answer ..... % [2]

---

8. Jamie is using the quadratic formula to solve a quadratic equation.

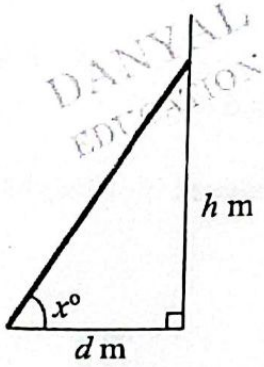
She substitutes values into the formula and correctly gets

$$x = \frac{-9 \pm \sqrt{81-48}}{6}$$

Find the quadratic equation that Jamie is solving, giving your answer in the form $ax^2 + bx + c = 0$, where a, b and c are integers.

Answer ..... [2]

9.



A ladder is leaning against a vertical wall as shown in the diagram.

The bottom of the ladder is d metres from the wall.

The top of the ladder is h metres above the ground.

The angle between the ladder and the ground is $x°$.

(a) Some safety instructions say it is safe to climb the ladder when $h = 3.5d$.

Determine the value of x when $h = 3.5d$.

Answer ..... [2]

(b) Some different safety instructions say the angle between the ladder and the ground should be $70°$. The ladder is then moved so that $x = 70°$.

Without further calculations, how does this change in angle affect the height, h metres, of the top of the ladder above the ground?

.....

..... [1]

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10. The table shows pairs of values of x and y.

x	4	5
y	400	625

(a) Which of the following statement is correct?

Statement A: $y \propto x$

Statement B: $y \propto x^2$

Statement C: $y \propto x^3$

Answer Statement ..... [1]

(b) Write a formula for y in terms of x.

Answer ..... [2]

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11. Simplify.

(a) $3+p(3-p)$

Answer ..... [1]

(b) $9x^{-4} \div \frac{1}{2}x^{6}$

Answer ..... [2]

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12. Solve the inequalities $x-3<\frac{3}{2}x-2\leq\frac{x+18}{6}$ and state the integer values of x that satisfy them.

Answer ..... [3]

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13. A bag contains 12 blue marbles and 8 red marbles.

(a) A marble is chosen at random and then replaced.

What is the probability that it is a red marble?

Answer ..... [1]

(b) How many red marbles must be placed in the bag so that the probability of choosing a red marble would be $\frac{3}{5}$?

Answer ..... [2]

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14. Express as $\frac{3}{x-3} - \frac{4}{x^2-9}$ a single fraction in its simplest form.

Answer ..... [3]

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15. A train 180 m long passes through a tunnel.

The average speed of the train is 42 km/h.

(a) Express 42 km/h in m/s.

Answer ..... m/s [1]

(b) The train passes through a 2.4 km tunnel.

Calculate the time for the train to completely pass through the tunnel.

Give your answer in minutes and seconds, to the nearest second.

Answer ..... min .....s [2]

---

16. $ab^2 + c = \dfrac{2c+b}{a}$

Rearrange the formula to make c the subject.

Answer ..... [3]

---

17. **(a)** Factorise completely $3ax+12by-9ay-4bx$.

Answer ..... [2]

(b) Show that $(3n+5)^2-7$ is a multiple of 3 for all integer values of n.

Answer

[2]

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18. The timings of 16 students running a 100 metre race is recorded below.

Time, x seconds	$12 \le x < 13$	$13 \le x < 14$	$14 \le x < 15$	$15 \le x < 16$
Frequency	3	5	6	2

(a) Calculate an estimate for

(i) the mean running time of the students,

Answer ..... s [1]

(ii) the standard deviation of the running time.

Answer ..... s [2]

(b) It was found that there was a mechanical error in the stopwatch used to time the run.

An actual 10 seconds is measured as 9 seconds by the stopwatch.

Explain how the mean will change after the adjustment.

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.....

.....

..... [1]

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19. Express as a product of prime factors, $168 = 2^3 \times 3 \times 7$ and $A = 2^p \times 3^3 \times 5$.

Using the above information, find

(a) the smallest integers m and n such that $\frac{168 \times m}{\sqrt{n}}$ is a perfect square and $m > n$,

Answer m = ...

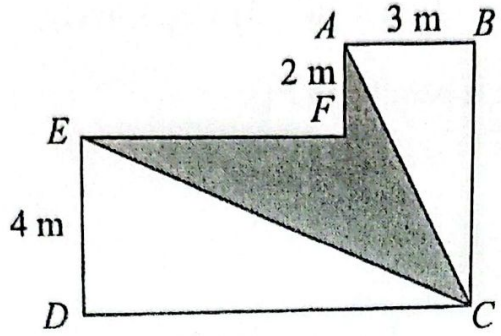
n = ... [2]

(b) the value of p and of A given that the highest common factor of 168 and A is 12.

Answer p = ...

A = ... [2]

20.



The diagram shows a shape *ABCDEF*.

All the corners of the shape are right angles.

$ED = 4$ m, $AF = 2$ m and $AB = 3$ m.

The perimeter of the shape is 38 m.

Find the area of the shaded part *ECAF* of the diagram.

Answer ..... m^2 [4]

21. The points P and Q have coordinates $(7, 2)$ and $(6, -3)$ respectively.

(a) Determine if the line $y + 5x = 4$ is parallel to PQ.

Answer

[2]

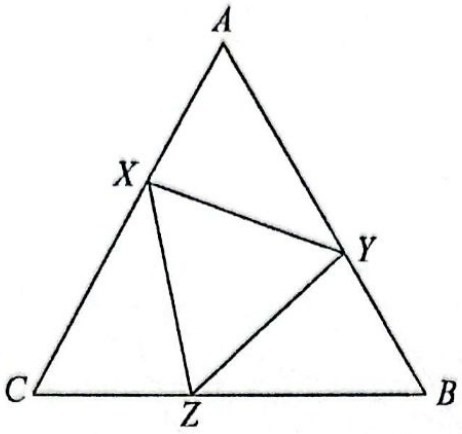
(b) **(i)** Find the column vector $\overrightarrow{PQ}$.

Answer ..... [1]

(ii) Find $\left|\overrightarrow{PQ}\right|$.

Answer ..... [2]

22.



In the diagram, triangle ABC is an equilateral triangle.

X, Y and Z lie on lines AC, AB and BC respectively and $AX = CZ = BY$.

(a) Prove that triangle AXY is congruent to triangle CZX.

Answer

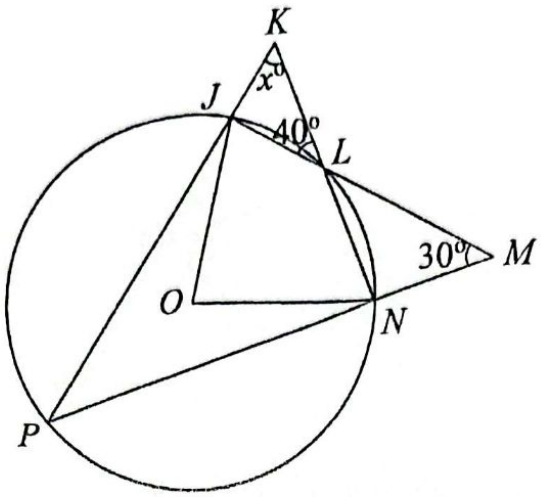
[3]

(b) Determine if triangle XYZ is an equilateral triangle.

Answer

[2]

23.



In the diagram, O is the centre of the circle. P, N, L and J lie on the circumference of the circle. The lines PJ and NL are extended to meet at K. The lines JL and PN are extended to meet at M. Angle JLK is 40°, angle LMN is 30° and angle JKL is x°.

Write down an expression, in terms of x, for

(a) **(i)** angle PJM,

Answer ..... [1]

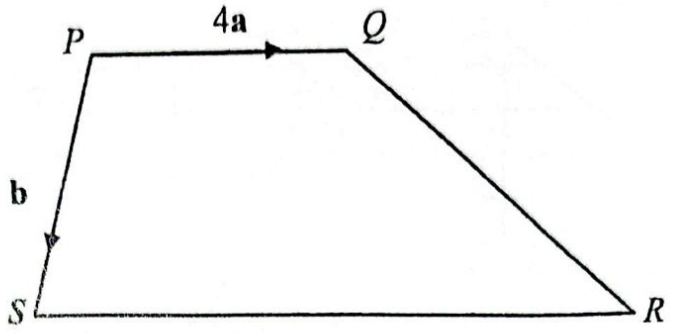
(ii) angle PNL.

Answer ..... [1]

(b) By considering triangle JMP, form an equation in x and solve it.

Answer $x =$ ..... [2]

24.



In the diagram, $PQRS$ is a trapezium where $\vec{PQ} = 4\,\mathbf{a}$ and $\vec{PS} = \mathbf{b}$.

A point T lies inside the trapezium where $QT : TS = 4 : 5$.

SR is twice the length of PQ.

(a) Express, as simply as possible, in terms of **a** and/or **b**,

(i) $\vec{QS}$,

Answer ..... [1]

(ii) $\vec{QT}$,

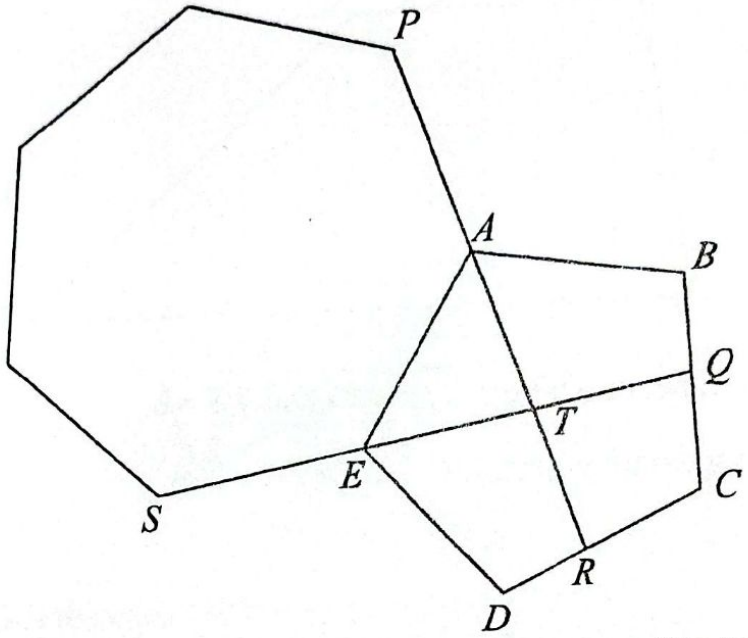
Answer ..... [1]

(b) Determine whether P, T and R lie on the same straight line.

Answer

[3]

25.



In the diagram, a regular heptagon and regular pentagon are fitted together at the edge *AE*.

PAR and *SEQ* are straight lines.

Find angle *TRD*.

Answer ..... ° [5]

26. The table below shows the distribution of three types of apartments in three blocks of flats.

	3-room	4-room	5-room
Block 1	10	20	50
Block 2	25	45	10
Block 3	30	50	20

The information can be represented by the matrix $\mathbf{T} = \begin{pmatrix} 10 & 20 & 50 \\ 25 & 45 & 10 \\ 30 & 50 & 20 \end{pmatrix}$.

The floor areas of the 3-room, 4-room and 5-room apartments are 60 m², 90 m² and 110 m² respectively and this information can be represented by the matrix $\mathbf{A} = \begin{pmatrix} 60 \\ 90 \\ 110 \end{pmatrix}$.

(a) Evaluate **TA**.

Answer ..... [2]

(b) Explain what the elements in **TA** represent.

.....

..... [1]

(c) Maintenance fees are charged based on the floor area of the apartments in the three blocks of flats. The maintenance fee rates ($/m²) for Block 1, 2 and 3 are are $3, $4, and $6 respectively. Write down a 1×3 matrix **C** to show the maintenance fee rates for the three blocks of flats.

Answer ..... [1]

(d) Hence, using matrix multiplication, find the total estate management fee for these three blocks of flats.

Answer ..... [2]

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27. **(a)** The first four terms of a sequence are 5, 9, 13 and 17.

The sum of the first n terms of this sequence is given by an^2+bn.

(i) When $n=1$, $a+b=5$.

Show that $4a+2b=14$.

Answer

[1]

(ii) Solve $a+b=5$

$4a+2b=14$.

Answer a = ...

b = ... [2]

(b) The nth term of another sequence is $n^2 + 4$.

Anthony says,

"The nth term of the sequence is always a prime number when n is an odd number."

Anthony is wrong.

Give an example to show that Anthony is wrong.

Answer ..... [2]

~ End of Paper ~