VICTORIA JUNIOR COLLEGE

2023 PROMO PRACTICE PAPER A

(Modified from 2017 VJC H2 Math PROMO)

- Exam conditions (one sitting, 3 hours)
- Manage your time well
- Check against solutions and learn from your mistakes before the next practice

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1 **TJC Promo 9758/2020/Q1**

The sum, S_n, of the first n terms of a sequence $u_1, u_2, u_3, \ldots$ is given by

$$S_n = \ln(n+1).$$

(i) Find u_n in terms of n. [2]

(ii) Describe the behaviour of the sequence. [1]

2 By using an algebraic approach, solve $\dfrac{10}{3-x} \geq x$. [4]

3 **2017 A-level Paper 1 Q5 (Modified)**

When the polynomial $x^3 + ax^2 + bx + c$ is divided by $(x-1)$, $(x-2)$ and $(x-3)$, the remainders are 8, 12 and 25 respectively.

(i) Find the values of a, b and c. [3]

A curve has has equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = x^3 + ax^2 + bx + c$, with values of a, b and c found in part **(i)**.

(ii) Show that the gradient of the curve is always positive. Hence, explain why the equation $\mathrm{f}(x) = 0$ has only one real root. [2]

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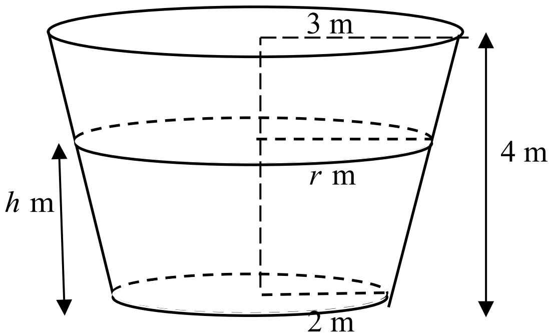


Water is poured at a rate of 9 m^3 per minute into an open container in the form of a frustum of a right circular cone as shown in the diagram above. The container has bottom radius of 2 m, top radius of 3 m and height of 4 m. After t minutes, the radius of the water surface is r m and the depth of water is h m, where $0 \leq h < 4$.

(i) Show that $r = \dfrac{h}{4} + 2$. [2]

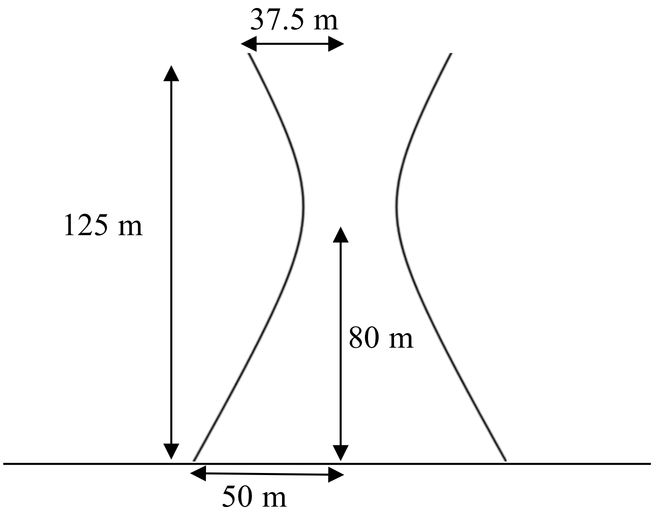
(ii) Find, in exact form, the rate of increase of the depth of water when $h = 1$. [4]

[The volume of a frustum of a right circular cone of bottom radius r_1, top radius r_2 and height h is given by $V = \dfrac{1}{3}\pi\left(r_1^2 + r_2^2 + r_1 r_2\right)h$.]

5 **(i)** Find the expansion of $\dfrac{1-2x}{\sqrt{4-x}}$ in ascending powers of x, up to and including the term in x^2. [3]

(ii) By substituting $x=\dfrac{3}{4}$ into the expansion in part **(i)**, find an approximation for $\sqrt{13}$ in the form $\dfrac{a}{b}$, where a and b are integers to be determined. [2]

6 The diagram below shows the cross section of a cooling tower of height 125 m. The radii of its circular base and its circular top are 50 m and 37.5 m respectively. The narrowest part of the tower occurs at a height of 80 m above the base of the tower.



Take the origin to be at the center of the base, and 1 m to be 1 unit on both axes. The walls of the tower is part of the curve C with equation $\dfrac{x^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$.

(i) Find the values of a^2, b^2 and k. [3]

For the rest of the question, use 80, 900 and 3600 as the values of k, a^2 and b^2 respectively.

(ii) Sketch C, indicating the equations of any asymptotes and coordinates of the points where C crosses the axes, where appropriate. [3]

(iii) Given that r is a positive constant and C intersects the curve with equation $x^2+(y-80)^2=r^2$ at exactly two distinct points, state the value of r. [1]

7 A curve C has parametric equations

$$x=3t^2+2t-\frac{5}{3}, \qquad y=t^3+t^2.$$

(i) The tangent to C at the point P where $t=p$ is denoted by l. It is given that l is parallel to the y-axis. Find the equation of l. [3]

(ii) Find the least value of x. [1]

(iii) Sketch C, indicating the coordinates of the points where C crosses the x-axis. [3]

8 Given that $y = \tan\left(\frac{1}{2}x\right)$, $-\pi < x < \pi$, show that

$$\frac{d^2y}{dx^2} = y\frac{dy}{dx}.$$ [2]

(i) By further differentiation of this result, find the Maclaurin series for y up to and including the term in x^3. [4]

(ii) Find $\int \tan\left(\frac{1}{2}x\right) dx$. [1]

(iii) Find the Maclaurin series for $\ln\left(\sec\left(\frac{1}{2}x\right)\right)$ up to and including the term in x^4. [3]

9 **ASRJC Prelim 9758/2020/01/Q8**

Do not use a calculator in answering this question.

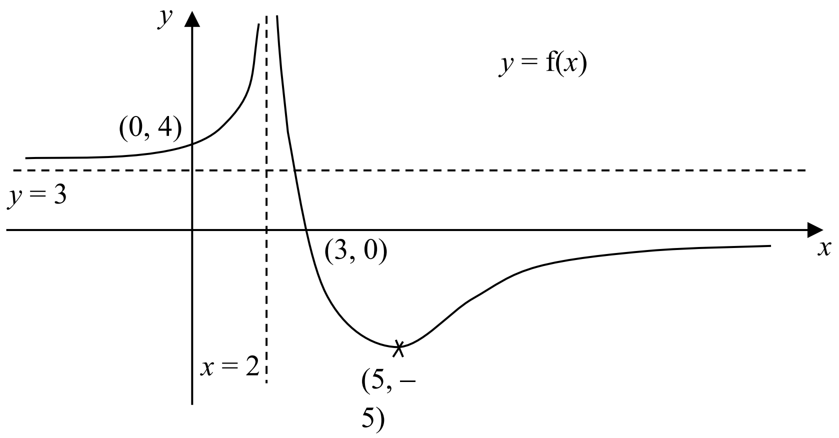
(a) (i) Solve the equation $w^2 = 3 - 4\mathrm{i}$, giving your answers in cartesian form $a + \mathrm{i}b$. [4]

(ii) Hence find the roots of the equation $z^2 - 4\mathrm{i}z + 4\mathrm{i} - 7 = 0$, giving your answers in cartesian form $p + q\mathrm{i}$. [2]

(b) For positive integer n, a complex number z is such that $|z|^n = \frac{1}{\sqrt{2}}$. The complex conjugate of z is z^*. State the conjugate of $1 + 2z^{2n}$ in terms of z^*.

Hence show that $\dfrac{2z^n}{1+2z^{2n}}$ is a real number. [5]

10 **(a)** The diagram below shows the graph of $y = \mathrm{f}(x)$. The graph cuts the x- and y-axes at the points $(3,0)$ and $(0,4)$ respectively. It has a turning point at $(5,-5)$. The equations of the asymptotes are $x = 2$, $y = 0$ and $y = 3$.



On separate diagrams, sketch the graph of

(i) $y = \mathrm{f}(2x+1)$, [3]

(ii) $y = 2\mathrm{f}(x) - 3$, [3]

(b) **2018/DHS/Prelim/2/5**

A closed cylinder is designed to contain a fixed volume of p cm^3 of liquid such that its external surface area is a minimum. Find the radius of the cylinder in terms of p in cm. [5]

11 TMJC Promo 9758/2020/Q5(a); CJC Prelim 9758/2018/01/Q6

(a) Find $\int \frac{\sin 3x}{2-\cos 3x}\,dx$. [2]

(b) Find $\int \frac{e^x}{\left(1+2e^x\right)^3}\,dx$. [2]

(c) Find $\int \frac{2-x}{4+x^2}\,dx$. [3]

(d) Use the substitution $x = \tan y$ to find the exact value of $\int_0^1 \frac{1}{\sqrt{1+x^2}}\,dx$. [4]

(e) Write down $\int x^2 e^{x^3}\,dx$. Hence find $\int x^5 e^{x^3}\,dx$. [4]

12 (a)(i) A particular bond is issued at \$100 per unit with a 5% *annual coupon rate*. The bondholder will receive a fixed amount of $\$(100\times 5\%)$ as coupons for 1 bond that he holds, at the end of **every year** until the bond matures after 30 years.

At the start of 2016, Mr Reech purchased 10 units of this particular bond. At the start of each subsequent year, he purchases another 10 units of the same bond. Assuming that Mr Reech receives the coupons as cash at the end of each year, show that the total amount of cash he receives over a period of 20 years is $\$10,500$. [2]

(ii) Mr Puwer, on the other hand, puts his money in a savings account that pays him k% compound interest at the end of every year. At the start of 2016, Mr Puwer deposits \$800 in the savings account. At the start of each subsequent year, he deposits \$800 into the same account. Mr Puwer does not withdraw any money from this account. Show that the total amount of money Mr Puwer has in his savings account at the end of n years is $\$\left(\frac{800(100+k)}{k}\left[\left(\frac{100+k}{100}\right)^n-1\right]\right)$. [3]

(iii) Hence, find the range of values of k such that at the end of 20 years, Mr Reech receives more cash in coupons than what Mr Puwer receives in **total interest** paid. [4]

(b) Mr Soo has \$100,000 in his savings account, and the prevailing interest rate is 2% per annum. Mr Soo does not deposit any more money in his account. Interest paid at the end of every year is withdrawn by him from the account, but the annual interest rate is halved every year. Find the minimum number of years it takes for the total interest withdrawn by Mr Soo to be at most \$20 less than the theoretical maximum total interest payable by the bank. [4]

[End of Paper]