

# H2 Topic 8 – Temperature and Ideal Gases



**Infrared Fever Screening Systems** are a common sight in 2020 as the world battles the COVID-19 pandemic. It was a world-first, made-in-Singapore system back in 2003 to respond to the need for fast, mass measurements of body temperatures when battling the SARS epidemic. The prototype was produced within a week of the Ministry of Health approaching the Defence Science and Technology Agency (DSTA) for assistance.

## Content

- Thermal equilibrium
- Temperature scales
- Equation of state
- Kinetic theory of gases
- Kinetic energy of a molecule

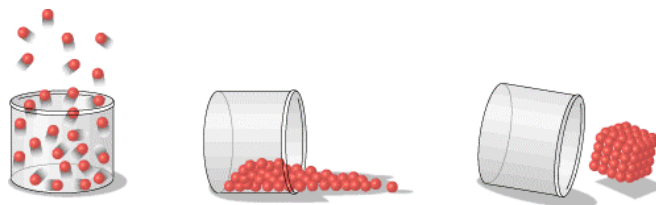
## Learning Outcomes

Candidates should be able to:

- show an understanding that regions of equal temperature are in thermal equilibrium
- explain how empirical evidence leads to the gas laws and to the idea of an absolute scale of temperature (i.e. the thermodynamic scale that is independent of the property of any particular substance and has an absolute zero)
- convert temperatures measured in degrees Celsius to kelvin:  $T / K = T / ^\circ C + 273.15$
- recall and use the equation of state for an ideal gas expressed as  $pV = nRT$ , where  $n$  is the amount of gas in moles
- state that one mole of any substance contains  $6.02 \times 10^{23}$  particles and use the Avogadro number  $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$
- state the basic assumptions of the kinetic theory of gases
- explain how molecular movement causes the pressure exerted by a gas and hence derive the relationship  $pV = \frac{1}{3} Nm\langle c^2 \rangle$ , where  $N$  is the number of gas molecules (a simple model considering one-dimensional collisions and then extending to three dimensions using  $\frac{1}{3} \langle c^2 \rangle = \langle c_x^2 \rangle$  is sufficient)
- recall and apply the relationship that the mean kinetic energy of a molecule of an ideal gas is proportional to the thermodynamic temperature (i.e.  $\frac{1}{2} m\langle c^2 \rangle = \frac{3}{2} kT$ ) to new situations or to solve related problems.

## 8.0 Introduction

Temperature, pressure, and volume are quantities that describe *macroscopic* properties of a system. Typically, when we discuss them, we refer to a bulk body of gas, liquid or solid. Together, the quantities ( $T$ ,  $p$  and  $V$ ) define a *state* of a system; in the field of thermal physics we refer to gas/liquid/solid as *phases*.



**Microscopic model.** Describes the extent of interactions between particles that lead to the bulk (microscopic) behavior – whether it is in the gaseous phase, liquid phase or solid phase.

In this field of study of these *macroscopic* properties, physicists hypothesize simple *microscopic* models to explain the bulk behaviour - that the system consists a large number of objects having complex and random interactions.

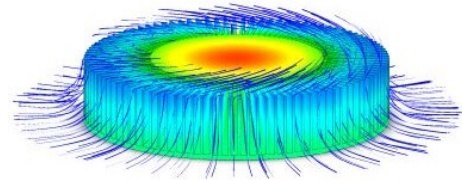
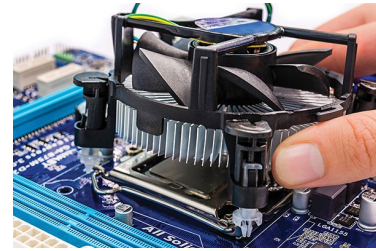
We often describe the *collective* behaviour of a large distribution of interacting objects by their *statistical averages*, which may be more meaningful than the chaos exhibited by individual objects. For example, rather than the individual changes in momentum of gas particles against the inner walls of the container, we describe the statistical average via the bulk property of *pressure* for a gas.

The fact that most real gases generally behave similarly to each other with only slight deviations suggests that the ideal gas model is a good approximation worth studying. Newtonian mechanics and its concepts of kinematics and dynamics is used to model the *microscopic* motion of gas particles.

## 8.1 Temperature and Thermal Equilibrium

When two bodies are in **thermal equilibrium**, there is

no net flow of thermal energy between the bodies that are in thermal contact because they are at equal temperature.



*Computer chips run hot. There needs to be very good thermal contact between the chip and the heatsink to dissipate the thermal energy built up. Thermal-conducting pastes are placed between the chip and heatsinks to fill in and squeeze out small pockets of air because air is an insulator of heat.*

To be in thermal contact means the 2 bodies are able to exchange thermal energy. There will be net flow of thermal energy (heat) *if* the bodies are at different temperatures.

**Heat** is thermal energy that flows  
  
from a region of higher temperature  
to a region of lower temperature.

Don't say "*heat* energy" as it means "(thermal energy) energy".

Temperature *difference* between 2 bodies provides 2 pieces of information

- direction of thermal energy flow  
(from higher to lower temperature)
- rate (speed) of transfer of thermal energy  
(the bigger the temperature difference, the faster the rate)

*also see Annex 1: The Zeroth Law of Thermodynamics*

## 8.2 Measuring Temperature

### Thermometric property

is a property of a substance that changes with temperature.

Some thermometric properties used in thermometers:

- volume of fixed mass of liquid
- resistance of thermistor or resistor
- e.m.f. between junctions of dissimilar metals exposed to different temperatures
- pressure of fixed mass of gas at constant volume

A suitable thermometric property should

- vary continuously and uniquely with temperature
  - different measurements of the thermometric property matches different temperatures in a 1-to-1 mapping
- change sufficiently noticeably
  - a more *sensitive* thermometer means a larger change in thermometric property for the same temperature change
  - reduces percentage uncertainty in that temperature measurement
- be reproducible

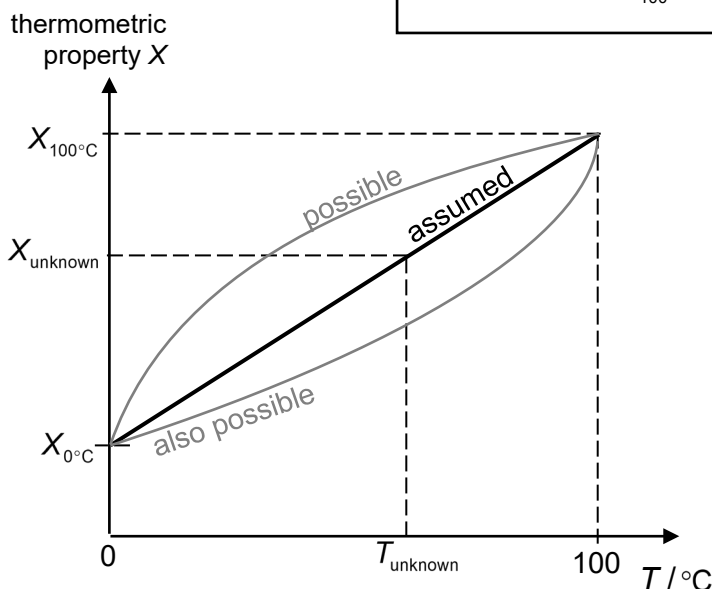
### 8.2.1 Empirical Centigrade Scales

Empirical means “verifiable from observation”. Centigrade means “to divide into 100 steps”. Therefore, an empirical centigrade scale is set up by the following:

- measure the thermometric property at fixed point of 0 °C (ice point),  $T_0$ 
  - pure melting ice in equilibrium with water at standard atmospheric pressure
- measure the thermometric property at fixed point of 100 °C (steam point),  $T_{100}$ 
  - pure boiling water in equilibrium with steam at standard atmospheric pressure
- assume that the thermometric property varies linearly with temperature

With this calibration, we measure an unknown temperature  $T_{\text{unknown}}$  by measuring its thermometric property  $X_{\text{unknown}}$  at the unknown temperature, and using the known values of  $T_0$  and  $T_{100}$  in the following equation:

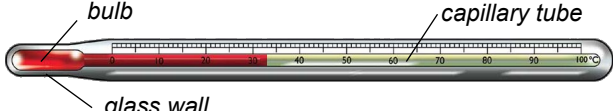
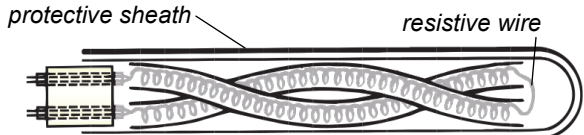
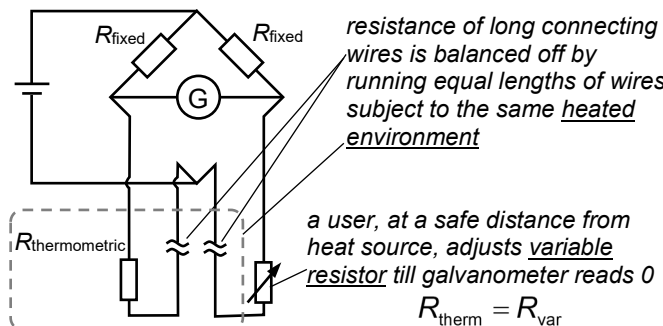
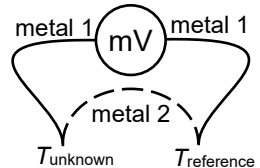

$$T_{\text{unknown}} = \frac{X_{\text{unknown}} - X_0}{X_{100} - X_0} \times 100 \text{ } ^\circ\text{C}$$



Temperature measurements from empirical centigrade scales across different types of thermometers may not agree because

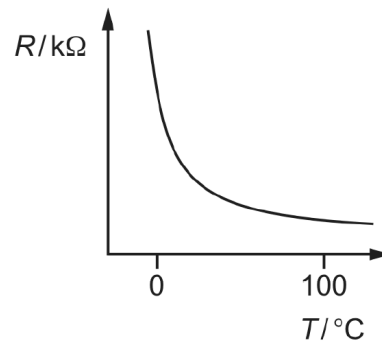
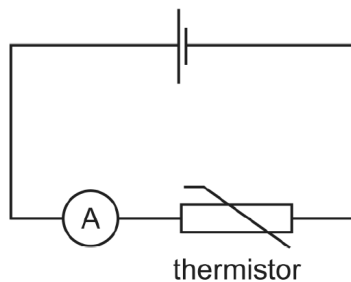
- thermometric properties are *assumed* to vary linearly with temperature, which is not the case in reality
- temperature measurements agree only at fixed points of ice point and steam points

Also see Annex 2: Different Types of Thermometer

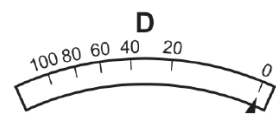
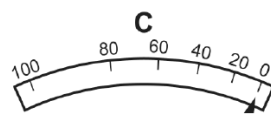
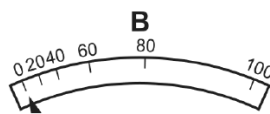
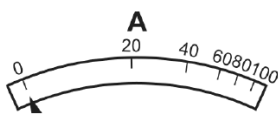
thermometer and thermometric property	advantages	disadvantages
<b>liquid-in-glass thermometer</b> <i>measures temperature via changes in volume of a fixed mass of liquid</i> 	<ul style="list-style-type: none"> <li>• simple construction</li> <li>• easy to use</li> <li>• portable</li> </ul>	<ul style="list-style-type: none"> <li>• fragile</li> <li>• need to surround bulb, so cannot measure temperature at a point/surface</li> <li>• slow response; bulb and glass wall at thermal equilibrium with measured substance</li> </ul>
mercury-in-glass	range of approx. -30 °C to 300 °C higher maximum range than alcohol-in-glass	cannot measure temperatures as low as alcohol-in-glass
alcohol-in-glass	<ul style="list-style-type: none"> <li>• cheaper than mercury-in-glass</li> <li>• safer when liquid spills during breakage</li> </ul>	range of approx. -100 °C to 70 °C smaller range than mercury-in-glass
<b>resistance thermometer</b> <i>measures temperature via changes in electrical resistance</i> 	<ul style="list-style-type: none"> <li>• very accurate when balanced electrically:</li> </ul> 	cannot register rapidly-changing temperature as time is needed to find balance condition of zero galvanometer reading (trade-off between accuracy and responsiveness)
using resistor	<ul style="list-style-type: none"> <li>• large range due to high melting point of resistance wire (usually metallic)</li> <li>• chemically inert</li> <li>• sensitive</li> </ul>	cannot register rapidly-changing temperature due to poor thermal contact (insulated by protective sheath)
using thermistor	higher sensitivity than resistance wire	narrower range than resistance wire
<b>thermocouple</b> <i>measures temperature via changes in e.m.f. between junctions of dissimilar metals at different temperatures</i> 	<ul style="list-style-type: none"> <li>• responsive, can measure rapidly changing temperature</li> <li>• wide range, can measure high temperatures</li> <li>• can measure temperatures at a point</li> <li>• relatively inexpensive due to simple build</li> </ul>	change in e.m.f. non-linear, so less accurate measurements 

### Example 1

In the circuit shown, an analogue ammeter is to be recalibrated as a thermometer. The ammeter is connected in series with a thermistor. The graph shows how the resistance  $R$  of the thermistor changes with temperature  $T$ .



Which of the diagram could represent the temperature scale on the ammeter?

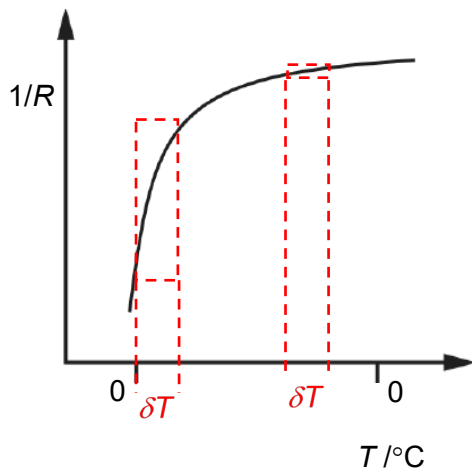


**Solution:**

The thermometric property used here is the resistance but what is measured is the current when the thermistor is subjected to a constant e.m.f.:  $I = \frac{V}{R}$ .

When temperature is low,  $R$  is high and so current is low  $\rightarrow$  Eliminate Options C and D because on analogue ammeter,  $I = 0$  starts from the left.

Looking at the  $\frac{1}{R}$  trend:



For the same increase in temperature  $\delta T$ , there is a larger change in  $1/R$  at the lower temperatures than higher temperatures.

The thermometer is *more sensitive* to changes in temperature at the lower temperatures.

**Option A**



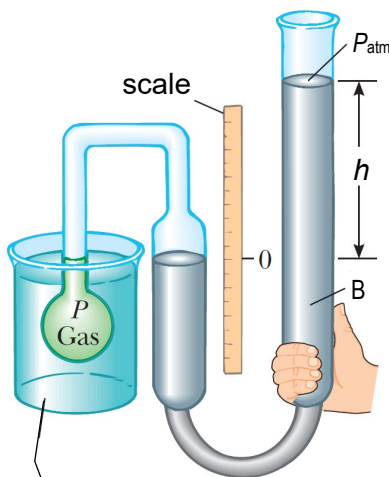
### 8.2.2 Absolute Temperature Scale

Since temperature readings from an empirical centigrade scale depend on the type of thermometer, the accuracy is unsatisfactory for scientific work. The *thermodynamic temperature*, measured on the *absolute temperature scale*, is the result of looking for a more accurate measurement.

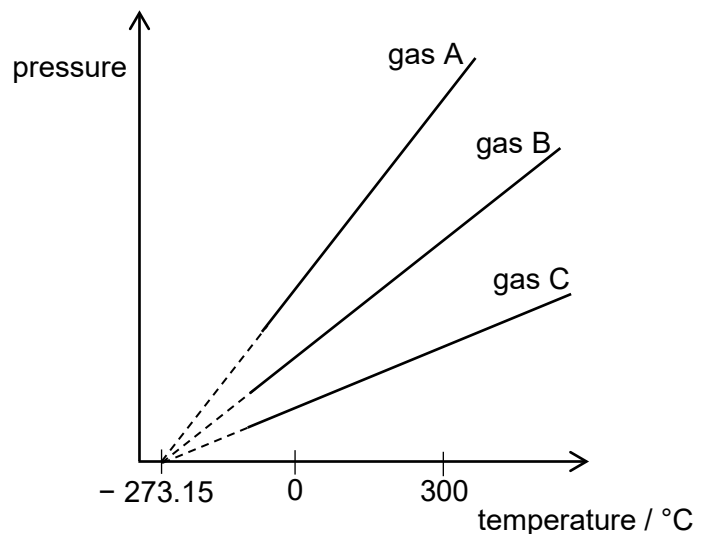
Thermodynamic temperature does not depend on the thermometric property of any substance.

[unit: kelvin, K]

Scientists experimented with *empirical centigrade* scaled constant-volume gas thermometers using pressure as the thermometric property. They found that the thermometer readings of all real gases tend towards a common temperature value when extrapolated to zero pressure.



liquid bath that is in thermal equilibrium with the fixed mass of gas



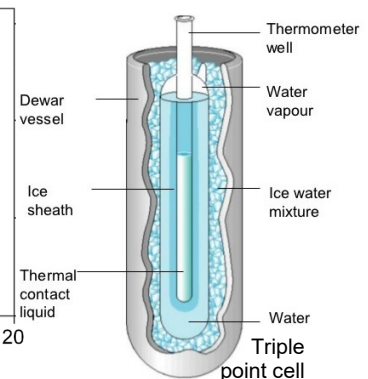
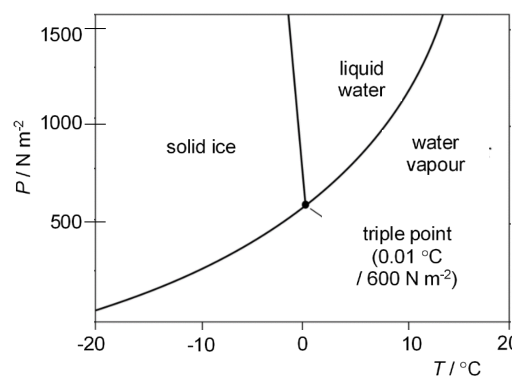
**Constant-volume gas thermometer** measures pressure of fixed mass of gas contained in the flask immersed in liquid bath. The gas volume is kept constant by raising or lowering reservoir B to keep the mercury level constant.

The results suggested that the lowest bound temperature of  $-273.15^{\circ}\text{C}$  is fundamentally important and does not depend on the thermometric property of any substance or gas. It is defined as absolute zero.

**Absolute zero** is defined as the zero point (0 K). It is a fixed point on the absolute temperature scale.

A substance at absolute zero has minimum internal energy.

2 fixed points are needed to establish a temperature scale. Absolute zero is taken as the lower fixed point. The triple point of water ( $0.01^{\circ}\text{C}$  /  $273.16\text{ K}$ ) is the upper fixed point. It is the single set of temperature and pressure at which ice, liquid water and water vapour can co-exist in equilibrium in a compact cell.



also see Annex 3: 2019 redefining of kelvin

### 8.2.3 The Celsius Scale

Since there are two identified fixed points the absolute temperature scale has, we can assign the value of what 1 K exactly means: the kelvin is  $\frac{1}{273.16}$  of the thermodynamic temperature of the triple point of water.

$$T_{\text{unknown}} = \frac{X_{\text{unknown}} - X_{\text{lower fixed pt}}}{X_{\text{upper fixed pt}} - X_{\text{lower fixed pt}}} \times \Delta T$$

for the constant volume thermometer  $T_{\text{unknown}} = \frac{P_{\text{unknown}}}{P_{\text{triple point}}} \times 273.16 \text{ K}$

The Celsius Scale (not to be confused with the *centigrade* scale) respects that  $1^\circ\text{C} = 1 \text{ K}$ , but shifts the absolute scale such that ice point is at  $0^\circ\text{C}$  and boiling point is at  $100^\circ\text{C}$ .

The conversion is:

$$T / \text{K} = T / ^\circ\text{C} + 273.15$$

### 8.3 The Ideal Gas

In 8.2.2 where we discussed fixed masses of real gases all approaching a common lower bound, there is an implied idea that their behaviours approached a theoretical *ideal*.

#### 8.3.1 Equation of State (a.k.a. Ideal Gas Law)

An **ideal gas** is a gas that

obeys the *equation of state*  $pV = nRT$   
at all pressures, volumes and temperature,

where  $p$  is pressure of the gas in Pascals  
 $V$  is volume of the gas in cubic metre  
 $n$  is amount of gas in moles  
 $R$  is molar gas constant  $8.31 \text{ J mol}^{-1} \text{ K}^{-1}$   
 $T$  is the thermodynamic temperature

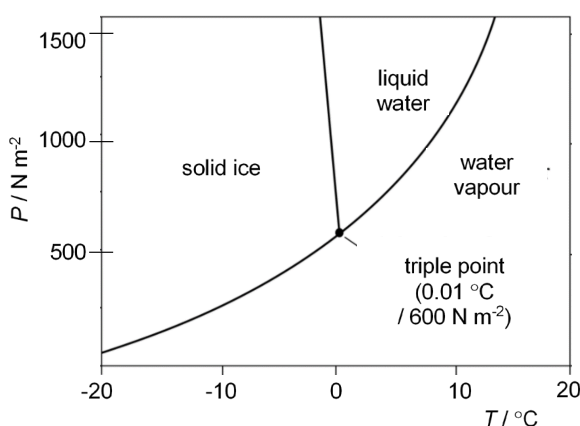
There is an equivalent equation of state which uses actual number of gas particles  $N$  rather than the amount of gas in moles.

It reads as  $pV = NkT$

where  $k$  is the Boltzmann's constant  $1.38 \times 10^{-23} \text{ J K}^{-1}$ .

Rather than “states of matter” such as solid, liquid or gas, the “state” in “equation of state” refers to a quantity that describes the condition of the system.

Think of it as for e.g. “this gas is in a highly pressurized state”, “this gas is in a low temperature state”. When describing systems, we “phase” to describe regions where all physical properties are uniform.



Referring to the diagram on the left, we can say that water typically exist in solid *phase* at lower temperatures and higher pressures.

We say that that there is a solid-liquid *phase boundary* near  $T = 0^\circ\text{C}$  if pressure is greater than  $500 \text{ N m}^{-2}$ . Using the same lingo, when water boils, we say that there is a *phase transition* from liquid to gas.



### Example 2

The volume of an oxygen tank is 50.0 l. As oxygen gas exits the tank, the tank pressure drops from 21.4 atm to 7.8 atm and the temperature decreases from 30.0°C to 10.0°C. Calculate the number of oxygen molecules withdrawn. (1 atm =  $1.01 \times 10^5$  Pa)

### Solution

Assume oxygen gas molecules behave like an ideal gas.

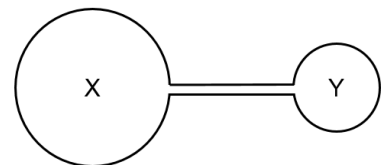
$$pV = NkT \quad \Rightarrow \quad N = \frac{pV}{kT}$$

Number of particles removed =  $N_i - N_f$

$$\begin{aligned} &= \frac{V}{k} \left( \frac{p_i}{T_i} - \frac{p_f}{T_f} \right) \\ &= \frac{50 \times 10^{-3}}{1.38 \times 10^{-23}} \left( \frac{21.4 \times (1.01 \times 10^5)}{30.0 + 273.15} - \frac{7.8 \times (1.01 \times 10^5)}{10.0 + 273.15} \right) \\ &= 1.58 \times 10^{25} \end{aligned}$$

### Example 3

Two vessels X and Y of volumes  $10 \times 10^{-4} \text{ m}^3$  and  $5 \times 10^{-4} \text{ m}^3$  respectively, are connected by a tube of negligible volume and kept at temperatures 400 K and 200 K respectively. Both contain the same ideal gas.



What is the ratio  $\frac{\text{number of molecules in X}}{\text{number of molecules in Y}}$  ?

### Solution

At equilibrium condition, there is uniform pressure throughout the vessel.

$$pV = NkT \quad \Rightarrow \quad p = \frac{NkT}{V}$$

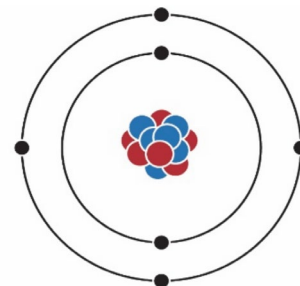
$$\begin{aligned} p_X &= p_Y \\ \frac{N_X k T_X}{V_X} &= \frac{N_Y k T_Y}{V_Y} \\ \frac{N_X k (400)}{10 \times 10^{-4}} &= \frac{N_Y k (200)}{5 \times 10^{-4}} \\ \frac{N_X}{N_Y} &= 1 \end{aligned}$$

### 8.3.2 The Avogadro's Constant

The actual number of gas particles in a typical volume of gas considered is very big. To aid in the counting, we refer to the “amount in moles” rather than the actual number:

One **mole** of any substance  
contains  $6.02 \times 10^{23}$  particles

**Avogadro's number**,  $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$   
is the number of atoms in 12 grams (0.012 kg) of carbon-12



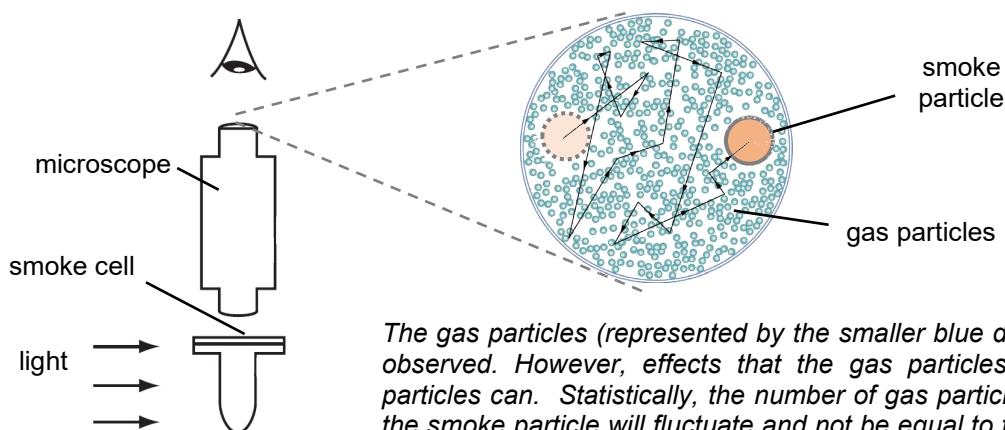
A carbon-12 atom consists  
6 protons, 6 neutrons and  
6 electrons

The *molar mass* of hydrogen gas molecules ( $\text{H}_2$ ) is  $2 \text{ g mol}^{-1}$ , i.e.  $6.02 \times 10^{23}$  hydrogen gas molecules will have a total mass of 0.002 kg.

### 8.4 The Kinetic Theory of Gases

Kinetic theory explains the properties of gases in terms of the motions of the particles that make up the gas. It is an attempt to explain *macroscopic behaviour* in terms of *microscopic motion*. The number of particles involved is so large that keeping track of each particle's individual motion is impossible. The theory relies on the average behaviour of all the particles being similar.

#### 8.4.1 Macroscopic Demonstration of Microscopic Motion of Gas Molecules



The gas particles (represented by the smaller blue dots) cannot be directly observed. However, effects that the gas particles have on the smoke particles can. Statistically, the number of gas particles striking one side of the smoke particle will fluctuate and not be equal to the other side, causing the smoke particles to have unpredictable paths.

Under a microscope, the motion of small particles such as smoke or pollen enclosed in an illuminated closed cell is seen as specks of reflected light in random motion. Interpreting the results:

- cannot see what is causing smoke particles to move  $\Rightarrow$  gas particles are much smaller than smoke particles
- smoke particles are in continuous (not *constant*) motion  $\Rightarrow$  gas particles are in continuous motion
- smoke particles are in random motion  $\Rightarrow$  gas particles are in random motion

The motion of gas particles

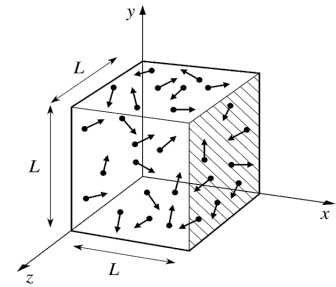
- are random and haphazard
- has a constant speed in a straight line between collisions
- shows a distribution of speeds

Note: If the smoke particles used are bigger, the randomness of collisions with gas particles will be ‘averaged out’ and so the movement of the smoke particles will be *less haphazard*.

### 8.4.2 Assumptions behind the Kinetic Theory of Gases

The model is built on the following assumptions:

- a) Gas molecules are hard, elastic identical spheres
- b) Large numbers of gas molecules are in continuous random motion
- c) No intermolecular forces except during collisions
- d) Total volume of molecules negligible compared to volume of containing vessel
- e) Time of collisions negligible compared to time between collisions



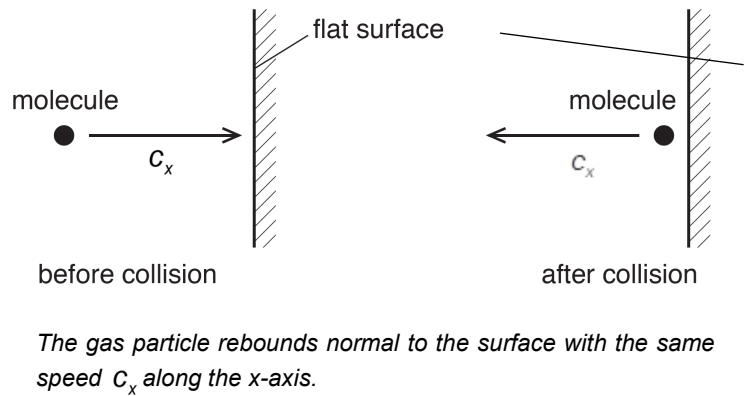
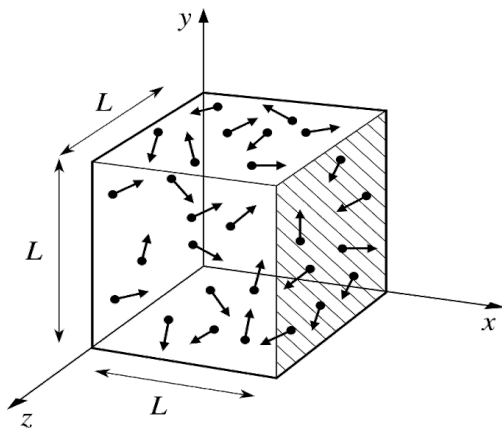
*A typical visual representation of the model of gas particles under the Kinetic Theory of Gases*

### 8.4.3 Macroscopic Property of Pressure Demonstrates Microscopic Collisions of Gas Molecules Against Inner Walls of Container

The gas particles in the kinetic theory model are hard, elastic identical spheres which obey Newton's Laws of motion whenever they collide i.e. forces are only exerted during collisions. As pressure is force / (area normal to the force exerted), we can use the kinetic model to explain the pressure exerted by a gas on a container.

In essence, gas particles exert pressure on the wall of its container because:

- **Gas particles** are in **continuous random motion** and **experience changes in momentum when they collide with inner walls of container**.
- **By Newton's 2<sup>nd</sup> Law**, a **gas particle experiences a force** from the rate of change in momentum during the collision.
- **By Newton's 3<sup>rd</sup> Law**, there is a **force exerted on the wall** that is equal in magnitude and opposite in direction to the force exerted on the gas particle.
- Hence, an **average force** is exerted over the area of the wall due to many collisions from many gas particles in a random distribution of velocities



Let us **derive the relationship**  $pV = \frac{1}{3}Nm\langle c^2 \rangle$ , where  $\langle c^2 \rangle$  is the mean square speed of the gas molecules within a cube container of sides  $L$  containing  $N$  particles of ideal gas each of mass  $m$ .

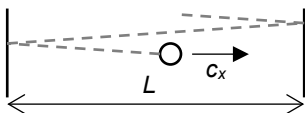
**Consider 1 gas particle moving only in the positive x-direction:**

Change in momentum of one particle during collision

$$\begin{aligned}\Delta P &= P_{\text{final}} - P_{\text{initial}} \\ &= (-mc_x) - mc_x \\ &= -2mc_x\end{aligned}$$

**Use Newton's 2<sup>nd</sup> Law to find average force on each gas particle:**

Distance covered for "1 round trip" =  $2L$



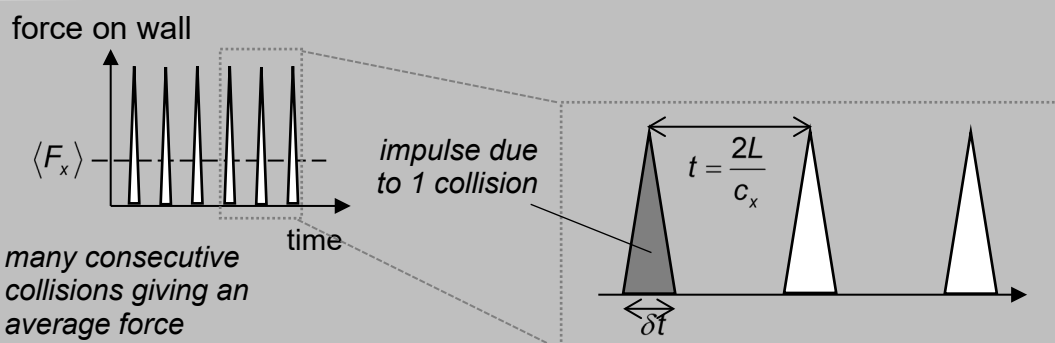
$$\text{time taken} = \frac{\text{distance}}{\text{speed}} = \frac{2L}{c_x}$$

$$\begin{aligned}\text{By N2L, } F_{\text{on 1 particle}} &= \frac{dP}{dt} \approx \frac{\Delta P}{\Delta t} \\ &= \frac{-2mc_x}{\left(\frac{2L}{c_x}\right)} = -\frac{mc_x^2}{L}\end{aligned}$$

We usually apply N2L for a collision using the time interval  $\delta t$  over which collision takes place.

In those cases, we are trying to find the force in that 1 collision.

Here instead, we find the **average force over many consecutive collisions**, so we use the time taken **between** successive collisions (time duration between 2 consecutive collisions with same wall).



### By Newton's 3rd Law,

$$F_{\text{on wall by 1 particle}} = -F_{\text{on 1 particle}} \\ = \frac{mc_x^2}{L}$$

### Total force on container due to $N$ particles moving in the x-direction:

$$F_{\text{total}} = \frac{mc_{x,1}^2}{L} + \frac{mc_{x,2}^2}{L} + \frac{mc_{x,3}^2}{L} + \dots + \frac{mc_{x,N}^2}{L} \quad \text{where } c_{x,N}^2 \text{ is square of x-velocity of } N^{\text{th}} \text{ particle}$$

$$= \frac{Nm}{L} \frac{(c_{x,1}^2 + c_{x,2}^2 + c_{x,3}^2 + \dots + c_{x,N}^2)}{N}$$

$$= \frac{Nm}{L} \langle c_x^2 \rangle$$

$$\langle c_x^2 \rangle = \frac{(c_{x,1}^2 + c_{x,2}^2 + c_{x,3}^2 + \dots + c_{x,N}^2)}{N}$$

is **mean-square** speed in x-direction,  
a useful statistical quantity!

### Pressure on the container due to $N$ particles moving in the x-direction:

$$p = \frac{F_{\perp}}{\text{area}} = \frac{F_{\text{total}}}{L^2}$$

$$= \frac{Nm}{L^3} \langle c_x^2 \rangle = \frac{Nm}{V} \langle c_x^2 \rangle \quad \text{where } V \text{ is volume of cube container}$$

Hence  $pV = Nm \langle c_x^2 \rangle$

Since there is no preference for occurrence of velocity in any direction:

Hence  $pV = \frac{1}{3} Nm \langle c^2 \rangle$

Alternatively  $p = \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle$

$$= \frac{1}{3} \frac{m_{\text{total}}}{V} \langle c^2 \rangle$$

$$p = \frac{1}{3} \rho \langle c^2 \rangle$$

By Pythagoras Theorem,

$$c^2 = c_x^2 + c_y^2 + c_z^2$$

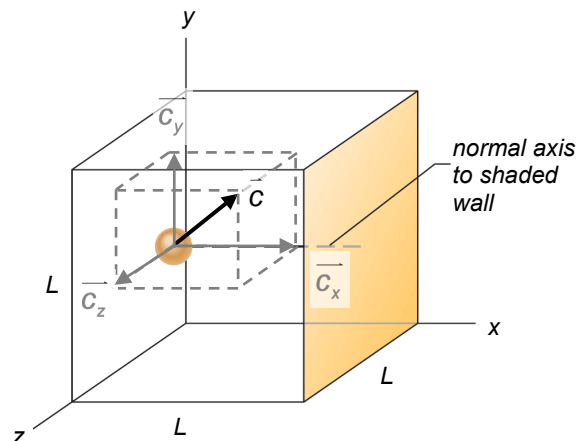
Considering mean square value

$$\langle c^2 \rangle = \langle c_x^2 \rangle + \langle c_y^2 \rangle + \langle c_z^2 \rangle$$

Since there is no preference for occurrence of velocity in any direction:

$$\langle c_x^2 \rangle = \langle c_y^2 \rangle = \langle c_z^2 \rangle$$

Hence  $\langle c^2 \rangle = 3 \langle c_x^2 \rangle \Rightarrow \frac{1}{3} \langle c^2 \rangle = \langle c_x^2 \rangle$



### 8.4.4 Macroscopic Property of Temperature Reflects Kinetic Energy of Gas Particles

In this section, we discover the quantitative relationship between a gas' internal energy and its temperature. From section 8.3.1, we have the equation of state:

$$pV = NkT$$

In section 8.4.3, we proved

$$pV = \frac{1}{3}Nm\langle c^2 \rangle$$

Combining these 2 equations, we get

$$\frac{1}{3}Nm\langle c^2 \rangle = NkT$$

$$\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT$$

Hence

$$\langle E_k \rangle = \frac{3}{2}kT \quad \text{i.e.}$$

The mean translational kinetic energy of molecules of an ideal gas is directly proportional to the thermodynamic temperature of the gas.

### 8.4.5 The Root-Mean-Square speed

The r.m.s. speed is an indicative speed that represents the average translational kinetic energy of gas particles:

$$\frac{3}{2}kT = \frac{1}{2}m\langle c^2 \rangle$$

$$\langle c^2 \rangle = \frac{3kT}{m}$$

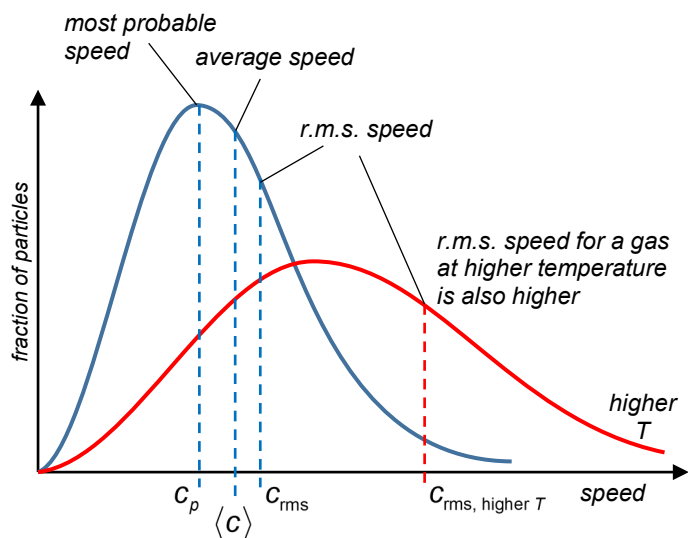
$$c_{\text{rms}} = \sqrt{\langle c^2 \rangle}$$

$$= \sqrt{\frac{(c_{x,1}^2 + c_{x,2}^2 + c_{x,3}^2 + \dots + c_{x,N}^2)}{N}}$$

$$c_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

$$\text{(also)} = \sqrt{\frac{3RT}{M_r}}$$

where  $M_r$  is molar mass



The Maxwell-Boltzmann Distribution shows how the speed of particles vary with the proportion of all particles in random motion. As all particles are in continuous random motion, there are no particles at rest. There is no upper limit – there will always be a small proportion of particles with very high speeds.



**Example 4**

A vessel contains 20 gas particles and their velocities are given in the table below. Find the (i) most probable speed, (ii) average speed and (iii) root-mean-square speed of the particles.

velocities / m s <sup>-1</sup>	- 3.0	- 2.0	- 1.0	0.0	1.0	2.0	3.0
number of particles	1	7	4	0	2	5	1
$v^2$	9.0	4.0	1.0	0.0	1.0	4.0	9.0

**Solution**

- (i) there are 2 particles of speed 3.0 m s<sup>-1</sup>,  
 12 particles of speed 2.0 m s<sup>-1</sup>, and  
 6 particles of speed 1.0 m s<sup>-1</sup>. The most probable speed is 2.0 m s<sup>-1</sup>.

- (ii) average speed,

$$\langle v \rangle = \frac{2(3.0) + 12(2.0) + 6(1.0)}{20}$$

$$= 1.8 \text{ m s}^{-1}$$

- (iii) root-mean-square speed

$$v_{\text{rms}} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{v_1^2 + \dots + v_N^2}{N}}$$

$$= \sqrt{\frac{2(9.0) + 12(4.0) + 6(1.0)}{20}}$$

**Example 5**

In order for an atom to escape completely from the Earth's gravitational field, it must have a speed of approximately  $1.1 \times 10^4 \text{ m s}^{-1}$  at the top of Earth's atmosphere.

- (a) Estimate the temperature at the top of the atmosphere such that helium, assumed to be an ideal gas, could escape from the Earth. The molar mass of helium is  $4.0 \text{ g mol}^{-1}$ .  
 (b) Suggest why some helium atoms will escape at temperatures below that of (a).

**Solution**

- (a)

$$\frac{1}{2} m \langle c^2 \rangle = \frac{3}{2} kT$$

$$T = \frac{m \langle c^2 \rangle}{3k}$$

$$= \frac{\left( \frac{4.0 \times 10^{-3}}{6.02 \times 10^{23}} \right) (1.1 \times 10^4)^2}{3(1.38 \times 10^{-23})}$$

$$= 19\,400 \text{ K}$$

- (b) In a gas, the particles are in a random distribution and not all have the same speed. There is a wide range of speeds and a small proportion will have speeds higher than the escape speed.

## 8.5 Summary

When two bodies are in **thermal equilibrium**, they are at equal temperature and so there is no net flow of thermal energy between these bodies that are in thermal contact.

Where there is a temperature difference, heat flows from higher to lower temperature.

The bigger the temperature difference, the faster the rate of flow of thermal energy.

To measure temperature, we use **thermometric properties**, properties of substances which change with temperature.

An empirical centigrade scale assumes that a thermometric property varies linearly between 2 fixed points of temperature. Temperatures measured this way may not be very accurate.

The search for an accurate scale resulted in the **absolute temperature scale**. On this scale, the **thermodynamic temperature does not depend on the thermometric property of any substance**.

The conversion for Celsius scale is  $T / K = T / ^\circ\text{C} + 273.15$ .

Constant-volume gas thermometers tended towards a common ideal behaviour as absolute zero temperature was approached.

An ideal gas obeys the *equation of state*  $pV = nRT$  at all pressures, volumes and temperature.

The **Kinetic Theory of Gases** uses a simplified, microscopic model to describe an ideal gas. The assumptions are:

- a) Gas molecules are hard, elastic identical spheres
- b) Large numbers of gas molecules are in continuous random motion
- c) No intermolecular forces except during collisions
- d) Total volume of molecules negligible compared to volume of containing vessel
- e) Time of collisions negligible compared to time between collisions

Brownian motion is a macroscopic demonstration of the **continuous random motion** of microscopic gas particles.

The pressure of a gas is a macroscopic demonstration of the collisions of gas particles against the inner walls of containers.

The temperature of a gas is a macroscopic demonstration of the speeds of the gas particles. Specifically, the mean translational kinetic energy of particles is directly proportional to thermodynamic temperature of gas,  $\langle E_k \rangle = \frac{3}{2}kT$ .

To further understand the model behind the Kinetic Theory of Gases, we need the next topic where we discuss Internal Energy and the First Law of Thermodynamics.