Full Name	Class Index No	Class

Anglo-Chinese School
(Barker Road)

PRELIMINARY EXAMINATION 2023
SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)

ADDITIONAL MATHEMATICS
4049
PAPER 1

2 HOURS 15 MINUTES

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

For Examiner's Use

This question paper consists of **18** printed pages and **2** blank pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 A cuboid has a square area $\left(6-\sqrt{35}\right)\text{m}^2$. The capacity of the cuboid is $\left(45\sqrt{5}-38\sqrt{7}\right)\text{m}^3$. Find the height of the cuboid, in m, in the form $\left(a\sqrt{5}-b\sqrt{7}\right)$, where a and b are integers. [3]

2 **(a)** Show that $(x-1)$ is a factor of $\mathrm{f}(x)=x^3-(2h+1)x^2-(k-2h)x+k$,where h and k are constants. [1]

(b) Explain with working why the equation $\mathrm{f}(x)=0$ will have 3 real roots if $h^2+k\geq 0$. [4]

3 The temperature, $T°\mathrm{C}$, of glass being heated in a kiln is given by the formula $T = 2030 - 2000e^{-kt}$, where t is the time in hours since the glass was placed in the kiln. The temperature of the glass after 90 minutes is $1000\,°\mathrm{C}$.

(a) Find the temperature of the glass before it was heated. [1]

(b) The glass can be molded into different shapes when its temperature reaches at least $1300\,°\mathrm{C}$. What is the earliest time that the glass can be removed from the kiln and molded? [4]

(c) The instruction manual of the kiln gives the range of the temperature of the kiln as $T < y°\mathrm{C}$. State the value of y. [1]

4 The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}''(x) = \dfrac{18}{(1-2x)^3}$ and $2\mathrm{f}'(-1) = 1$. Find the equation of the curve given that it passes through the point $(1, 0)$. [6]

5 It is given that $\mathrm{f}(x) = \dfrac{(x+2)^2}{p-x}$, $x \neq p$, where p is an integer.

(a) Obtain and simplify an expression for $\mathrm{f}'(x)$ in terms of p. [3]

(b) The range of values of x for which $\mathrm{f}(x)$ is an increasing function is $-2 < x < 8$. Show that the value of p is 3. [3]

(c) Hence find the equation of the normal to the curve at the point where the curve crosses the x-axis. [3]

6 Given that $5\sin^2 A - 3\cos^2 A = 7\sin 2A$ where $0 \le A \le 90^\circ$.

(a) Show that $\tan A = 3$. [3]

(b) Hence find the value of $\cos\left(60^\circ + A\right)$, leaving your answer in the form $\dfrac{a + b\sqrt{3}}{2\sqrt{10}}$, where a and b are integers. [3]

(c) Without finding the value of A, explain whether $60^\circ + A$ is acute or obtuse. [1]

7 **(a)** Given that $y=(3x-1)\left(\sqrt{6x+1}\right)$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{27x}{\sqrt{6x+1}}$. [3]

(b) Hence find the value of $\displaystyle\int_0^4 \frac{9x-3}{\sqrt{6x+1}}\,\mathrm{d}x$. [4]

8 **(a)** In the binomial expansion of $\left(2x-\frac{3}{x}\right)^n$, where n is a positive integer, the coefficient of the third term is $\frac{270}{8}\left(2^n\right)$. Show that $n = 6$. [4]

(b) Using the value of n in **part (a)**, find the coefficient of x^4 in the expansion of $\left(1+x^2\right)\left(2x-\frac{3}{x}\right)^n$. [3]

9 It is given that $y = a\cos 2x + b$ for $-180^\circ \le x \le 90^\circ$ where $a < 0$ and $b > 0$. Given that the amplitude of y is 3 and the maximum value of y is 7.

(a) State the value of a and of b. [2]

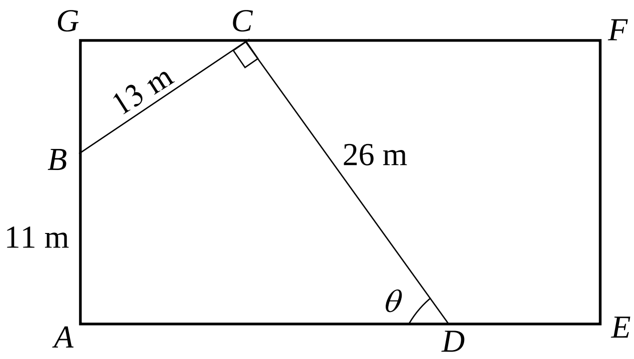
(b) Sketch the graph of $y = a\cos 2x + b$ for $-180^\circ \le x \le 90^\circ$. [3]

(c) Hence state the number of solution(s) of the equation $1 - \frac{1}{2}\cos 2x = 0$ for $-180^\circ \le x \le 90^\circ$. [2]

10 **(a)** Solve $49^x - 7^{x+1} = 18$, leaving your answer in the form $\log_a b$, where a and b are integers to be determined. [5]

(b) Find the largest value of the integer q such that $-3x^2 + qx - 8$ is negative for all real values of x. [3]

11



The diagram shows a rectangular futsal court, *AEFG*.
From a point *A* on the court, players are to run along the straight paths *AB*, *BC*, *CD* and *DA*. The lengths of *AB*, *BC* and *CD* are 11 m, 13 m and 26 m respectively. Angle *ADC* is θ, where $0^0 < \theta < 90°$. The total distance covered by each player is *T* metres.

(a) Show that *T* can be expressed as $50 + 26\cos\theta + 13\sin\theta$. [3]

(b) Express T in the form $50+R\cos(\theta-\alpha)$, where $R>0$ and $0^{\circ}<\alpha<90^{\circ}$. [3]

(c) Bryan claims that he ran a total distance of 85 m. Explain with calculations if this is possible. [2]

12 The line $y = 4$ and the positive y-axis are tangents to a circle C. It is given that the x-coordinate of the centre of C is a, where $a > 0$.

(a) Write down, in terms of a, the largest possible y-coordinate of the centre of C. [1]

The line L is a tangent to C at the point (8, 13) on the circle. The centre of C lies below and to the left of (8, 13).

(b) Find the equation of C. [4]

(c) Find the equation of L. [2]

13 **(a)** Express $\dfrac{4x^3+2x^2-5}{x^2(2x-1)}$ in partial fractions. [7]

(b) Hence find $\int \frac{4x^3+2x^2-5}{x^2\left(2x-1\right)}\,\mathrm{d}x$. [3]

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