Core Maths Paper 1 2023 (Solutions)

1.

(a) $\dfrac{2\frac{3}{4}-\frac{1}{2}}{1-\frac{1}{4}}$

$=\dfrac{\frac{11}{4}-\frac{1}{2}}{\frac{3}{4}}$

$=\dfrac{11-2}{4}\div\dfrac{3}{4}$

$=\dfrac{9}{3}$

$=3$

(b) $\dfrac{\left(2a^4b^2\right)^3}{14b^{-2}c^6}\div\dfrac{1}{7\sqrt[4]{b^{-16}}}$

$=\dfrac{8a^{12}b^6}{14b^{-2}c^6}\times 7b^{-4}$

$=4a^{12-7}b^{6+(-4)-(-2)}c^{(-6)}$

$=\dfrac{4a^{12}b^4}{c^6}$

2.

(a) $\because y = 2(x-3)^2 + m - 4$, minimum point is $(3,\ -8)$,

$\therefore m - 4 = -8$

$m = -4$

(b) $y = 2(x-3)^2 - 8$

Let $y = 0$,

$2(x-3)^2 - 8 = 0$

$2(x^2 - 6x + 9) - 8 = 0$

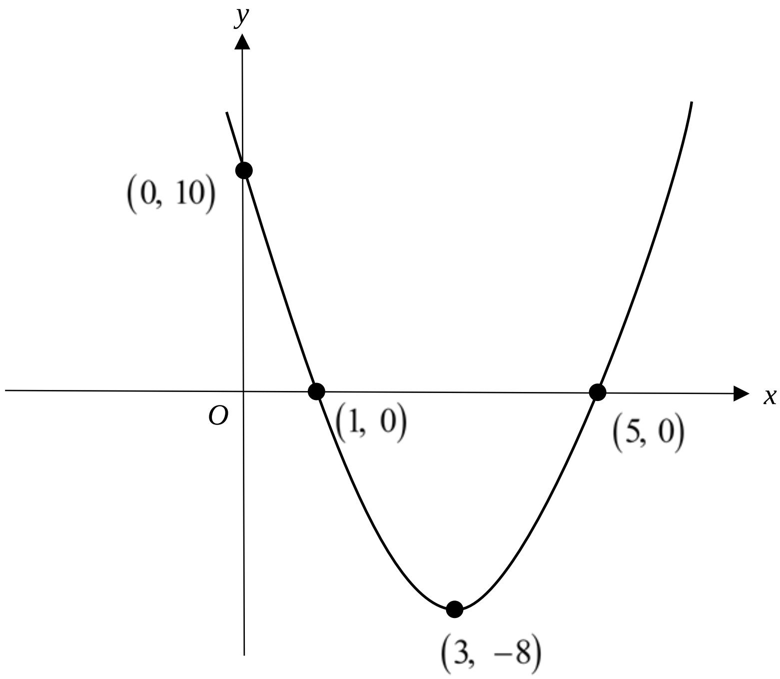
$x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

Let $x = 0$,

$y = 2(0-3)^2 - 8$

$y = 10$



3.

(a) $\dfrac{\sqrt{108}}{3}+18\sqrt{3}-\dfrac{4\sqrt{27}}{3}$

$=\dfrac{\sqrt{3\times36}}{3}+18\sqrt{3}-\dfrac{4\sqrt{3\times9}}{3}$

$=\dfrac{6\sqrt{3}}{3}+18\sqrt{3}-\dfrac{4\times3\sqrt{3}}{3}$

$=(2+18-4)\sqrt{3}$

$=16\sqrt{3}$

(b) **(i)** Since they are on the same straight line,

$m_{PQ}=m_{PR}$ OR $m_{PQ}=m_{QR}$

$\dfrac{1-\sqrt{5}}{\sqrt{5}-x}=\dfrac{3-1}{3\sqrt{5}-\sqrt{5}}$ $\quad$ $\dfrac{1-\sqrt{5}}{\sqrt{5}-x}=\dfrac{3-\sqrt{5}}{3\sqrt{5}-x}$

$(1-\sqrt{5})(2\sqrt{5})=2(\sqrt{5}-x)$ $\quad$ $(1-\sqrt{5})(3\sqrt{5}-x)=(3-\sqrt{5})(\sqrt{5}-x)$

$2\sqrt{5}-2(5)=2\sqrt{5}-2x$ $\quad$ $3\sqrt{5}-x-3(5)+\sqrt{5}x=3\sqrt{5}-3x-5+\sqrt{5}x$

$2x=10$ $\quad$ $2x=10$

$x=5$ $\quad$ $x=5$

OR

Equation of the line,

$y-1=\dfrac{1}{\sqrt{5}}(x-\sqrt{5})$

When $y=\sqrt{5}, x=5$

(ii) Length of *PR*

$=\sqrt{(3\sqrt{5}-\sqrt{5})^2+(3-1)^2}$

$=\sqrt{4(5)+4}$

$=\sqrt{24}$

$=2\sqrt{6}$ units

4.

$\left(\sqrt[3]{5}\right)^{3x} = 125^y$

$\left(5^{\frac{1}{3}}\right)^{3x} = \left(5^3\right)^y$

$5^x = 5^{3y}$

$\therefore x = 3y$.....(1)

$\log_3 x^2 + \log_3 y = 5$

$\log_3\left(x^2 \times y\right) = 5$

$\log_3\left(x^2 y\right) = 5$

$x^2 y = 3^5$......(2)

subs (1) into (2),

$(3y)^2 y = 243$

$9y^3 = 243$

$y^3 = 27$

$y = 3$

When $y = 3$,

$x = 9$

5.

(a) $m_{BC} = \frac{5-2}{0-3}$

$= -1$

Equation of the line L_1, which passes through *C* and is perpendicular to *BC* is

$y - 5 = 1(x - 0)$

$y = x + 5$

(b)

$L_1, y = x + 5$..............(1)

$5y - 10 = 2x$.................(2)

Subs (2) into (1),

$5(x+5) - 10 = 2x$

$5x + 25 - 10 = 2x$

$3x = -15$

$x = -5$

When *x* = - 5, *y* = 0.

Hence, the Coordinates of *D* are (-5, 0).

(c) Area of quadrilateral *ABCD*

$= \frac{1}{2}\begin{vmatrix} 0 & -5 & -4 & 3 & 0 \\ 5 & 0 & -2 & 2 & 5 \end{vmatrix}$

$= \frac{1}{2}\left|0 + 10 + (-8) + (15) - (-25) - 0 - (-6) - 0\right|$

$= \frac{1}{2}|48|$

$= 24 \text{ units}^2$

6.

(a) (i)

$$\frac{x}{x^2-1}+\frac{3}{x+1}$$

$$=\frac{x+3x-3}{x^2-1}$$

$$=\frac{4x-3}{x^2-1}$$

(ii)

$$\frac{x}{x^2-1}+\frac{3}{x+1}=0$$

$$\frac{4x-3}{x^2-1}=0$$

$$4x-3=0$$

$$x=\frac{3}{4}$$

(b) $2x-6<\frac{3(3-x)}{4}\leq\frac{-x+4}{2}$

$$2x-6<\frac{3(3-x)}{4}$$

$$8x-24<9-3x$$

$$11x<33$$

$$x<3$$

$$\frac{3(3-x)}{4}\leq\frac{-x+4}{2}$$

$$18-6x\leq-4x+16$$

$$2x\geq 2$$

$$x\geq 1$$

$$\therefore 1\leq x<3$$

Integers that satisfy the inequality are 1 and 2.

7.

(a) $25-4a^2+20ab-25b^2$

$= 25-4a^2+20ab-25b^2$

$= 25-\left(4a^2-20ab+25b^2\right)$

$= 25-\left(2a-5b\right)^2$

$= \left[5-\left(2a-5b\right)\right]\left[5+\left(2a-5b\right)\right]$

$= \left(5-2a+5b\right)\left(5+2a-5b\right)$

(b)

$\dfrac{32^{x^2}}{16} = 2^{8x}$

$\dfrac{2^{5x^2}}{2^4} = 2^{8x}$

$2^{5x^2-4} = 2^{8x}$

$\therefore 5x^2-8x-4=0$

$\left(5x+2\right)\left(x-2\right)=0$

$5x+2=0$ or $x-2=0$

$x=-\dfrac{2}{5}$ $\qquad x=2$

8.

(a) $mx^2 - 3x - 4 = 0$

$x^2 - \frac{3}{2}x - 2 = 0$

$\therefore \alpha + \beta = \frac{3}{2}$

$\alpha\beta = -2$

(b) $\alpha^2 + \beta^2$

$= (\alpha + \beta)^2 - 2\alpha\beta$

$= \left(\frac{3}{2}\right)^2 - 2(-2)$

$= \frac{9}{4} + 4$

$= \frac{25}{4}$

(c)

Sum of roots

$= \left(\alpha + \frac{\beta}{2}\right) + \left(\frac{\alpha}{2} + \beta\right)$

$= (\alpha + \beta) + \left(\frac{\alpha}{2} + \frac{\beta}{2}\right)$

$= \frac{3}{2} + \frac{3}{4}$

$= \frac{9}{4}$

Product of roots

$= \left(\alpha + \frac{\beta}{2}\right)\left(\frac{\alpha}{2} + \beta\right)$

$= \frac{\alpha^2}{2} + \alpha\beta + \frac{\alpha\beta}{4} + \frac{\beta^2}{2}$

$= \frac{\alpha^2 + \beta^2}{2} + \frac{5\alpha\beta}{4}$

$= \frac{25}{8} + \frac{5(-2)}{4}$

$= \frac{5}{8}$

$x^2 - \frac{9}{4}x + \frac{5}{8} = 0$

9.

(a) By using Pythagoras' Theorem, $AB^2 + BC^2$

$=11^2+\left(4\sqrt{3}\right)^2$

$=121+16(3)$

$=13^2$

$=AC^2$

Hence, angle *ABC* is a right angle.

(b) (i)

$\cos\angle DAC$

$=\cos\left(180-\angle BAC\right)$

$=-\cos\angle BAC$

$=-\dfrac{11}{13}$

(ii)

$\dfrac{\tan\angle ACB}{\cos\angle BAC}$.

$=\dfrac{\frac{11}{4\sqrt{3}}}{\frac{11}{13}}$

$=\dfrac{11}{4\sqrt{3}}\times\dfrac{13}{11}$

$=\dfrac{13}{4\sqrt{3}}$

$=\dfrac{13\sqrt{3}}{12}$

(c) Given that *AF* is the reflection of *AB* in the line of *AE*, *EF* = *BE*,

$EF^2+FC^2=EC^2$

$\left(4\sqrt{3}-x\right)^2+2^2=x^2$

$x^2-8\sqrt{3}x+16(3)+4=x^2$

$8\sqrt{3}x=52$

$x=\dfrac{13}{2\sqrt{3}}\,/\,\dfrac{13\sqrt{3}}{6}$

OR Using trigonometric ratio

$\cos\angle ACB=\cos\angle ECF$

$\dfrac{4\sqrt{3}}{13}=\dfrac{2}{x}$

$x=\dfrac{26}{4\sqrt{3}}$

10

(a) Let y = 0,

$(p-3)x^2-4x+p=0$

Discriminant >0

$(-4)^2-4(p-3)(p)>0$

$16-4p^2+12p>0$

$p^2-3p-4<0$

$(p-4)(p+1)<0$

$-1<p<4$

Given that the curve has a minimum point,

$a>0$

$p-3>0$

$p>3$

$\therefore 3<p<4$

(b) **(i)** x^2-4x+5

$=(x-2)^2-(-2)^2+5$

$=(x-2)^2+1$

(ii) $\dfrac{x^2-4x+5}{2x^2+2x-40}>0$

Since, $x^2-4x+5=(x-2)^2+1$ which is always positive for all real values of x,

$2x^2+2x-40>0$

$x^2+x-20>0$

$(x+5)(x-4)>0$

$x<-5$ or $x>4$

11.

(a) **(i)** $\log_5 xy^2$

$= \log_5 x + \log_5 y^2$

$= \log_5 x + 2\log_5 y$

$= m + 2n$

(ii) $\log_{\frac{x^2}{y^3}} 125$

$$= \frac{\log_5 125}{\log_5 \frac{x^2}{y^3}}$$

$$= \frac{\log_5 5^3}{\log_5 x^2 - \log_5 y^3}$$

$$= \frac{3}{2m - 3n}$$

(b)

$x^2 = (\log_3 p)x - \frac{1}{4}\log_{\sqrt{3}} p$

$x^2 - (\log_3 p)x + \frac{1}{4}\log_{\sqrt{3}} p = 0$

Discriminant $= 0$

$\left[-(\log_3 p)\right]^2 - 4(1)\left[\frac{1}{4}\log_{\sqrt{3}} p\right] = 0$

$(\log_3 p)^2 - \log_{\sqrt{3}} p = 0$

$(\log_3 p)^2 - \log_3 p^2 = 0$

$(\log_3 p)^2 - 2\log_3 p = 0$

$(\log_3 p)(\log_3 p - 2) = 0$

$\log_3 p = 0$ or $\log_3 p - 2 = 0$

$p = 3^0$ $\quad$ $p = 3^2$

$p = 1$ $\quad$ $p = 9$

End of Paper