# LOYANG VIEW SECONDARY SCHOOL
Preliminary Examination 2021
Secondary Four Express

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| CANDIDATE NAME | |

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| CLASS | 4 | E | | CLASS INDEX NUMBER | *CLASS INDEX NUMBER FOR THIS EXAMINATION ONLY* | 0 | 0 | | | |

## ADDITIONAL MATHEMATICS **4049/01**

Paper 1 **31 Aug 2021**

**2 hours 15 minutes**

Candidates answer on the Question Paper.

No Additional Materials are required.

| For examiner's use only | |
|---|---|
| Question number | Mark |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
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| 12 | |
| 13 | |
| Total | |

**READ THESE INSTRUCTIONS FIRST**

Write your Class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angle in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Setter: Mr Sim LE



## Mathematical Formulae

### 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial expansion*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

*Formulae for* $\Delta ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

[Turn Over

**1** A recycling company collects cans from a number of sites. It crushes them and sells the aluminium back to a manufacturer. The profit from processing *t* tonnes of cans each week is $*p*, where $p = 100t - \frac{1}{2}t^2 - 200$.

By completing the square, find the greatest profit the company can make each week, and how many tonnes of cans it has to collect and crush each week to achieve this profit. [4]

**2** **(a)** If $2^x 4^{-y} = \frac{1}{8}$, find a relation between *x* and *y*. [2]

**(b)** Solve the equation $x^{\frac{2}{3}} - 6x^{\frac{1}{3}} + 8 = 0$. [3]

3 Express $\dfrac{x^2 - 7x - 6}{x^3 - 3x^2}$ in partial fractions. [5]

**4** The function $g(x) = ax^4 + bx^3 - x^2 + 5$ has remainder $2x + 5$ when divided by $x^2 - 1$.

**(i)** Find the value of $a$ and of $b$. [3]

**(ii)** Hence, using your answer in **(i)**, find the polynomial h $(x)$ such that h $(x)$ is exactly divisible by $x^2 - 1$. [2]

**5** The curve $y - 8x^2 = k\left(1 + x^2\right) - 6x$ has a minimum value. Find the range of values of *k* if the curve cuts the *x*-axis at 2 distinct points. [5]

**6** **(i)** Prove that $\dfrac{\sin(A+B)-\sin(A-B)}{\cos(A+B)+\cos(A-B)}=\tan B$. [3]

**(ii)** It is given that $\tan(A+B) = -\frac{5}{12}$ and $\tan(A-B) = \frac{3}{4}$, where $0° < A+B < 180°$ and $0° < A-B < 90°$. Without evaluating for angle *A* and/or angle *B*, calculate the exact value of tan *B*. [4]

**7** **(i)** Show that $2\sin 2x(\operatorname{cosec} x + \cot x)$ can be expressed as a quadratic expression in the form $k\left(\cos x + \cos^2 x\right)$, where *k* is a positive integer. [2]

**(ii)** Hence, by first finding the exact solutions of the equation

$$2\sin 2y\left(\operatorname{cosec} y + \cot y\right) = 3,$$

and given that $y = x - \frac{4\pi}{5}$, solve for *x*, $0 \le x \le 2\pi$. Give your answers in the exact form. [5]

**8** It is given that $y = \frac{3x+4}{\sqrt{2x-1}}$.

**(i)** Express $\frac{dy}{dx}$ in the form $\frac{ax+b}{\sqrt{(2x-1)^3}}$, where a and b are integers. [2]

Hence, find the range of values of *x* for which *y* is

**(ii)** increasing, [3]

**(iii)** decreasing. [3]

**9** It is given that , $\mathrm{f}(x) = x^{2n} - 3x^3 + 5x^2 - 9x + 6$ where *n* is a positive integer.

**(i)** Show that $x - 1$ is a factor of f $(x)$. [1]

**(ii)** Find the value of *n* for which $x - 2$ is also a factor of f $(x)$. [2]

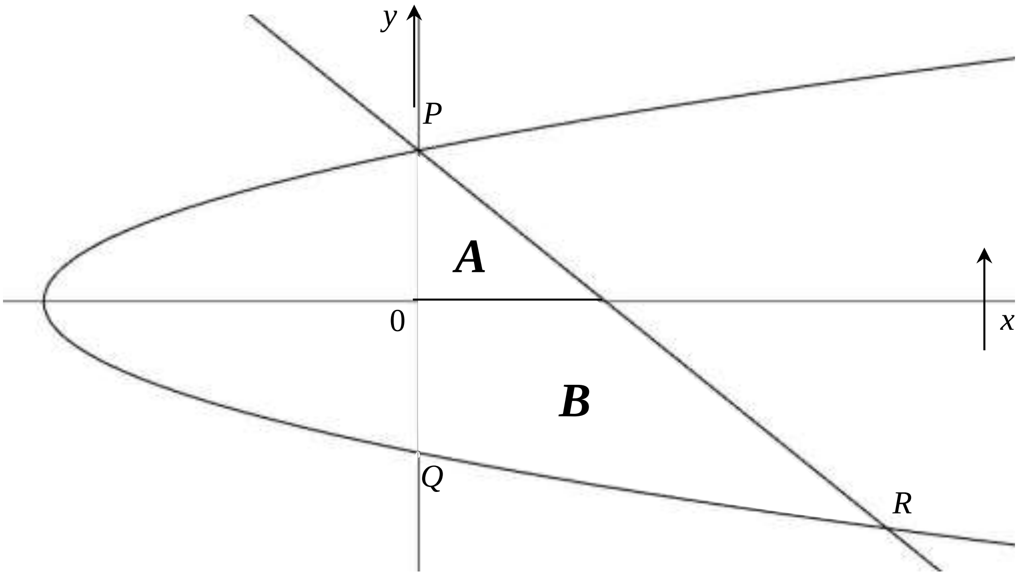
**(iii)** By finding the other factor(s), explain why the equation f ($x$) = 0 has only 2 real solutions. [3]

**(iv)** Hence, find the solutions to the equation $x^8 - 3x^6 + 5x^4 - 9x^2 + 6 = 0$, giving your answers in the exact form. [2]

**10** Find the stationary points of the curve $y = 3x^4 - 4x^3 + 5$ and determine the nature of each of the stationary points. [8]

**for continuation of working for Question 10:**

**11** The figure shows the curve $y^2 = x + 16$ and the line $y = -\frac{x}{2} + 4$. The curve intersects the *y*-axis at *P* and *Q* and the line intersects the curve at *P* and *R*.




**(i)** Find the coordinates of *P*, of *Q* and of *R*. [4]

**(ii)** Find the exact area of

[1]

**(a)** the shaded region ***A***, bounded by the axes and the line $y = -\frac{x}{2} + 4$.

**(b)** the shaded region ***B***, bounded by the axes, the line $y = -\frac{x}{2} + 4$ and the curve $y^2 = x + 16$. [3]

**(iii)** Suggest one other method of finding the total area (***A*** + ***B***) of the shaded region. [1]

**12** A particle travels at a velocity, $v = 2e^{2t} - 18$ m/s in a straight line, where *t* is the time in seconds after leaving *O*.

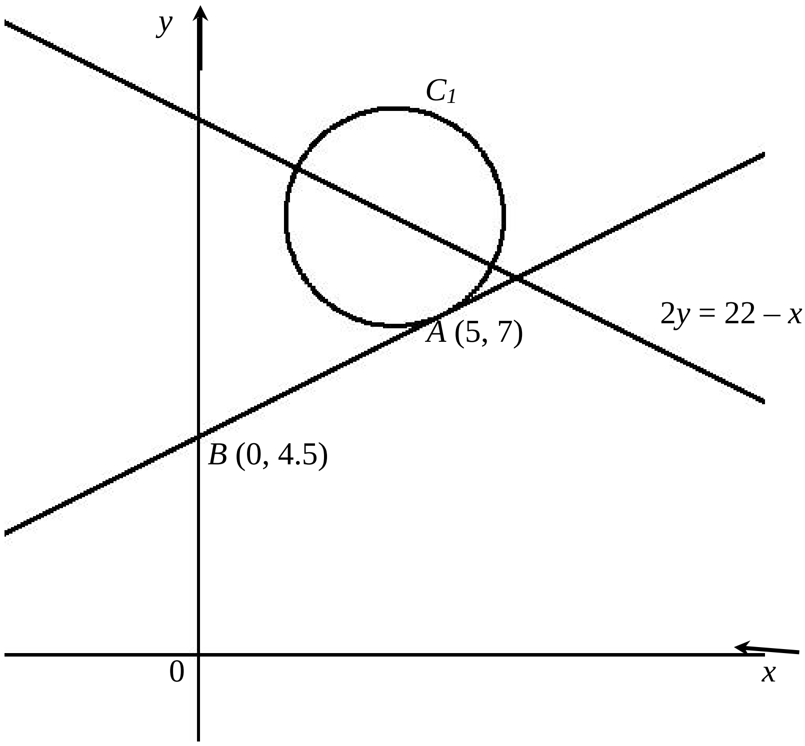
**(i)** Explain the significance of the value of *v* when *t* = 0. [1]

**(ii)** Find the acceleration of the particle when it is instantaneously at rest. [3]

**(iii)** Express, in terms of $t$, the displacement of the particle t seconds after leaving $O$. [2]

**(iv)** Find the total distance travelled by the particle in 2 seconds. [3]

**13** A tangent, to a circle $C_1$, at the point $A$ (5, 7) intersects the $y$-axis at the point $B$ (0, 4.5) as shown in the diagram. The line with equation $2y = 22 - x$ is a normal to the circle.




**(i)** The coordinates of the centre of the circle is $C(p, q)$. Show that $q = 11 - \frac{p}{2}$. [1]

**(ii)** Find the coordinates of the centre of the circle, and hence, its exact radius. [4]

**(iii)** The equation of the circle can be expressed in the form $x^2 + y^2 + 2gx + 2fy + c = 0$. State the value of *f* and of *g*, and find the value of *c*. [3]

A second circle $C_2$, has equation $(x+2)^2 + (y-6)^2 = 20$.

**(iv)** Determine, with justifications, if the two circles, $C_1$ and $C_2$ intersect, touch each other or do not intersect at all. [2]

**-END-**

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