Solutions
Systems of Linear Equations

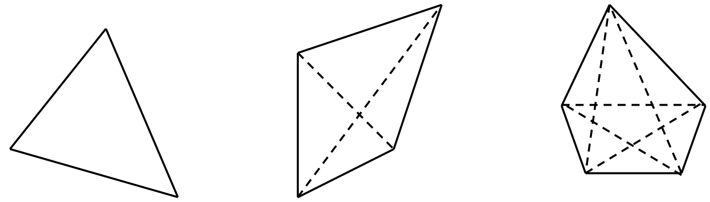
1	Let $y = ax^3 + bx^2 + cx + d$, where a, b, c and d are real constants. When $x = 0$, $y = 0$, $\therefore\ d = 0$ Given $x = 2$, $y = 0$, $a(2^3) + b(2^2) + c(2) = 0$ $8a + 4b + 2c = 0$ ------ (1) Given $x = 2.55$, $y = -0.0631$, $a(2.55^3) + b(2.55^2) + c(2.55) = -0.0631$ ------ (2) Differentiating y with respect to x, $\frac{dy}{dx} = 3ax^2 + 2bx + c$ Given $x = 0.785$, $\frac{dy}{dx} = 0$, $3(0.785^2)a + 2(0.785)b + c = 0$ ------ (3) Solving (1), (2) and (3) using GC, $a = 0.0993$, $b = -0.497$, $c = 0.596$ (3 s.f.) Hence, $y = 0.1x^3 - 0.5x^2 + 0.6x$.

2	Let \$$x$, \$$y$ and \$$z$ be the cost of a ticket for a senior citizen, adult and child respectively. $2x + 19y + 9z = 1982$ $10y + 3z = 908$ $x + 7y + 4z = 778$ Using GC, $x = 36$ $y = 74$ $z = 56$ Thus, the cost of a ticket for a senior citizen is \$36, for an adult is \$74 and for a child is \$56. 4(36) + 5(74) + 1(56) = 570 Therefore, the total cost for Group D = \$570

3	$420+6.4-17.5a-25b-100c=102.4$ $17.5a+25b+100c=324$ ----- (1) $420+3.8-18.5a-19b-120c=92.6$ or $18.5a+19b+120c=331.2$ ---- (2) $420+8.6-17a-23b-90c=121.2$ $17a+23b+90c=307.4$ ----- (3) Using GC, $a=\frac{58}{5}$, $b=\frac{13}{5}$, $c=\frac{14}{25}$

4	Let S, B and W be the number of strawberry, blueberry and walnut muffins purchased respectively. $S+B+W=30$ $1.6S+1.75B+2.2W=53.40$ $S=2W \Rightarrow S-2W=0$ From GC, $S=12$, $B=12$, $W=6$

5	Let x, y, z be the price rate of electricity, gas and water respectively. $23x+12y+16z=103$ $33x+16y+21z=142$ $49x+22y+33z=209$ Solving $x=2.2$, $y=2.5$, $z=1.4$ Her monthly utility bill for the month of July is $35(2.2)(1.07)+17(2.5)(0.95)+26(1.4)=159.165\approx 159.17$

6	Let the number of diagonals be $d=An^2+Bn+C$  Triangle (3 sides) Quadrilateral (4 sides) Pentagon (5 sides) No of diagonals in a triangle = 0 No of diagonals in a quadrilateral = 2

	No of diagonals in a pentagon = 5 Therefore, $9A+3B+C=0$ $16A+4B+C=2$ $25A+5B+C=5$ Solving using GC, $A=\frac{1}{2},\quad B=-\frac{3}{2},\quad C=0$ Thus, $d=\frac{1}{2}n^2-\frac{3}{2}n$ For a polygon of 200 sides, The number of diagonals = $\frac{1}{2}(200)^2-\frac{3}{2}(200)=19700$

7(i)	m : weight of mackerel in kg s : weight of salmon in kg t : weight of tuna in kg $m + s + t = 800$ $7m + 21s + 39t = 20300$ $5m + 23s + 49t = 23900$ Using GC, $m=200,\ s=250,\ t=350.$ Therefore the fisherman has 250 kg of salmon.
(ii)	m : weight of mackerel in kg s : weight of salmon in kg t : weight of tuna in kg $m + s + t = 600$ $7m + 21s + 39t = 20300$ $5m + 23s + 49t = 23900$ Using GC, $m=-460,\ s=990,\ t=70.$ Since the weight of all fishes must be non-negative, the fisherman’s claim is not possible. Or Since the weight of salmon and tuna is more than 600kg, the fisherman’s claim is not possible.

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2) Let the price (per kg) for dab, lobster and bamboo clam for his first visit be c, ℓ, b.

$$3.20c + 1.50\ell + 7b = 277.50$$
$$5.60c + 1.20(1.1\ell) + 6.50b = 347$$
$$4.50c + 2(1.1^2\ell) + 6.50b = 395.18$$

From GC, $c = 36.20$, $\ell = 79.9983$, $b = 5.95$

Required price: <u>\$36.20</u>, \$79.9983 × 1.1^2 = <u>\$96.80</u> and \$5.95 respectively.

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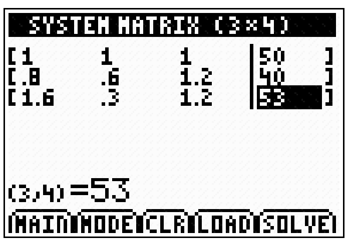
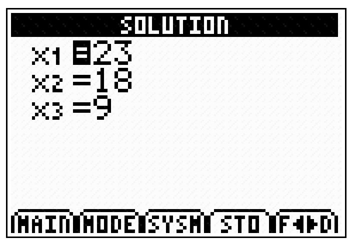
Let x, y, z be the number of apples, oranges and pineapples respectively.

$$x + y + z = 50$$
$$0.8x + 0.6y + 1.2z = 40$$
$$1.6x + 0.3y + 1.2z = 53$$





From GC, Adam bought 23 apples, 18 oranges and 9 pineapples.

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Let the price of a X-box console be \$$x$, a Kinect sensor be \$$y$ and a Game DVD be \$$z$.

$$x + y + z = 499$$
$$0.9x + 0.85y + 0.9z = 439.15$$
$$0.95x + 0.75y + 0.8z = 426.30$$

$$\text{Aug matrix} = \begin{pmatrix} 1 & 1 & 1 & 499 \\ 0.9 & 0.85 & 0.9 & 439.15 \\ 0.95 & 0.75 & 0.8 & 426.30 \end{pmatrix} \Rightarrow \text{rref} = \begin{pmatrix} 1 & 0 & 0 & 247 \\ 0 & 1 & 0 & 199 \\ 0 & 0 & 1 & 53 \end{pmatrix}$$

<u>$x = 247$, $y = 199$, $z = 53$</u>

Employees of Company P will pay \$[0.9(247)+0.8(199)+0.85(53)] = \$426.55 > \$426.30.
No, it will not be more attractive for employees to purchase all the 3 items from own company.

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Let x, y and z be the digits on the hundreds, tens and ones position respectively.

$$x + y + z = 8 \quad -----(1)$$
$$(100x + 10y + z) - (100z + 10y + x) = 297 \quad ----(2)$$

$$\Rightarrow \begin{aligned} x+y+z&=8 \\ x-z&=3 \end{aligned}$$

Using GC: $x=3+z,\ \ y=5-2z,\ \ z=z$

Since *x*, *y* and *z* are non-negative integer values,

$3+z\geq 0 \Rightarrow z\geq -3$

$5-2z\geq 0 \Rightarrow z\leq 2.5$

$z\geq 0$

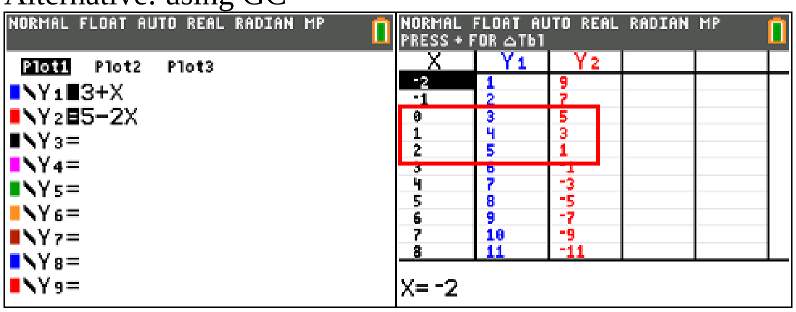
$\therefore 0\leq z\leq 2.5$

When $z=0, x=3, y=5$

When $z=1, x=4, y=3$

When $z=2, x=5, y=1$

Alternative: using GC



Hence, the possible original numbers are 350, 431 or 512.

12 $f'(x)=3ax^2+2bx+2$

$\Rightarrow f(x)=ax^3+bx^2+2x+c$

Curve passes through (1,2) $\Rightarrow a+b+c=0$ ----------(1)

Curve passes through (-1,3) $\Rightarrow -a+b+c=5$ ----------(2)

Curve passes through (2,2) $\Rightarrow 8a+4b+c=-2$ ----------(3)

$a=-\dfrac{5}{2}$, $b=\dfrac{31}{6}$, $c=-\dfrac{8}{3}$

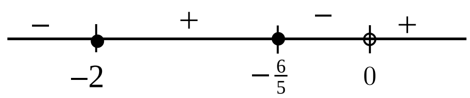
$\therefore f(x)=-\dfrac{5}{2}x^3+\dfrac{31}{6}x^2+2x-\dfrac{8}{3}$

13	Let x, y and z be the usual retail price of a packet of cashew nuts, macadamia nuts and almonds respectively. $4x+6y+7z=57.05$ $4(0.7)x+4y+7z-10=33.05 \Rightarrow 2.8x+4y+7z=43.05$ $4(0.8)x+3y+7(z-0.15)=33.05+5.45 \Rightarrow 3.2x+3y+7z=39.55$ By G.C., $x=3.5, y=4.9, z=1.95$ Thus, the usual retail price of a packet of cashew nuts, macadamia nuts and almonds is \$3.50, \$4.90 and \$1.95 respectively.

14	At the points of intersection, $\sqrt{a}\,x+bx^2=\ln(cx)$ When x =1, $\sqrt{a}+b-\ln c=0$ ------------ (1) When x =2, $2\sqrt{a}+4b-\ln c=\ln 2$ ------------ (2) When x =3, $3\sqrt{a}+9b-\ln c=\ln 3$ ------------ (3) Using GC, $\sqrt{a}=1.124670 \quad \Rightarrow \quad a=(1.124670)^2=1.265$ (to 3 d.p.) $b=-0.144$ (to 3 d.p.) $\ln c=0.98083 \quad \Rightarrow \quad c=\mathrm{e}^{0.98083}=2.667$ (to 3 d.p.)

15	$x+y+z=21$ **------ (1)** $3x-z=0$ **------- (2)** $0.05y+0.8z=3(0.9x) \Rightarrow 2.7x-0.05y-0.8z=0$ **----- (3)** Using GC, $x=2.1$, $y=12.6$ and $z=6.3$

16	Sub $(1,1)$ and $(2,2)$ into $y=\mathrm{h}(x)$. $a+b+c+d=1$ ----- (1) $8a+4b+2c+d=2$ -----(2) Since (2,2) is also the stationary point, $\mathrm{h}'(2)=0$. i.e. $12a+4b+c=0$ ----- (3) Using the GC, $a=-\frac{1}{2}-\frac{1}{4}d$ $b=\frac{3}{2}+\frac{5}{4}d$ $c=-2d$

	$\frac{ab}{c} \le 0$ $\frac{\left(-\frac{1}{2}-\frac{1}{4}d\right)\left(\frac{3}{2}+\frac{5}{4}d\right)}{-2d} \le 0$  $\{d \in \mathbb{R} : d \le -2 \text{ or } -\frac{6}{5} \le d < 0\}$

CJC Prelim 9758/2018/01/Q2	
17.	Marking Scheme: $\mathrm{f}(x) = x^3 \ln a + bx^2 + cx + d$ $\mathrm{f}'(x) = 3x^2 \ln a + 2xb + c$ At $\left(\frac{5}{3}, \frac{320}{27}\right)$, $\left(\frac{5}{3}\right)^3 \ln a + b\left(\frac{5}{3}\right)^2 + \frac{5}{3}c + d = \frac{320}{27}$ $125\ln a + 75b + 45c + 27d = 320 \qquad -(1)$ $3\left(\frac{5}{3}\right)^2 \ln a + 2\left(\frac{5}{3}\right)b + c = 0$ $25\ln a + 10b + 3c = 0 \qquad -(2)$ Let $\mathrm{g}(x) = \mathrm{f}(x+1) = (x+1)^3 \ln a + b(x+1)^2 + c(x+1) + d$ $\mathrm{g}'(x) = 3(x+1)^2 \ln a + 2(x+1)b + c$ At $(0, 12)$, $\ln a + b + c + d = 12 \qquad -(3)$ $3\ln a + 2b + c = 0 \qquad -(4)$ Using GC and solve, $\ln a = 1, b = -4, c = 5, d = 10$ $\therefore a = \mathrm{e}, b = -4, c = 5, d = 10$

18	$\mathrm{f}(x) = x^4 + ax^2 + bx + c$ $\mathrm{f}(2) = 0 \quad \Rightarrow 16 + 4a + 2b + c = 0 \quad \Rightarrow 4a + 2b + c = -16 \ldots(1)$ $\mathrm{f}(3) = 12 \quad \Rightarrow 81 + 9a + 3b + c = 12 \Rightarrow 9a + 3b + c = -69 \ldots(2)$ $\mathrm{f}(4) = 26 \quad \Rightarrow 256 + 16a + 4b + c = 26 \Rightarrow 16a + 4b + c = -230 \ldots(3)$ Solving (1), (2) and (3), $a = -54,\ b = 217,\ c = -234$

NYJC Prelim 9758/2022/01/Q1

Q19	**Suggested Answers**
(i)	$\text{f}(x) = ax^3 + bx^2 + cx + d$ At (–1, –1), $-a + b - c + d = -1$-----------------------------(1) $\text{f}'(x) = 3ax^2 + 2bx + c$ At minimum point (–1, –1), $3a - 2b + c = 0$ ---------------(2) $\int_0^1 \text{f}(x)\,\text{d}x = \frac{9}{4}$ $\int_0^1 \left(ax^3 + bx^2 + cx + d\right)\text{d}x = \frac{9}{4}$ $\left[\frac{ax^4}{4} + \frac{bx^3}{3} + \frac{cx^2}{2} + dx\right]_0^1 = \frac{9}{4}$ $\frac{a}{4} + \frac{b}{3} + \frac{c}{2} + d = \frac{9}{4}$ ---(3) When f(x) is divided by (x + 2), the remainder is 3 $\text{f}(-2) = a(-2)^3 + b(-2)^2 + c(-2) + d = 3$ $-8a + 4b + -2c + d = 3$--------------------------------------(4) Solving, $a = -1$, $b = 0$, $c = 3$ and $d = 1$