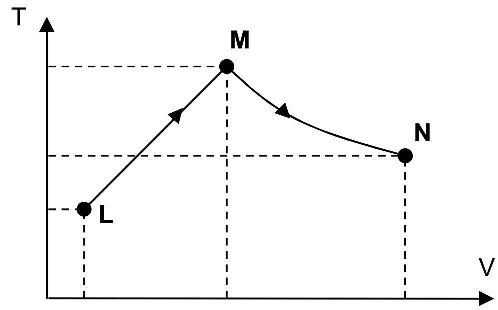
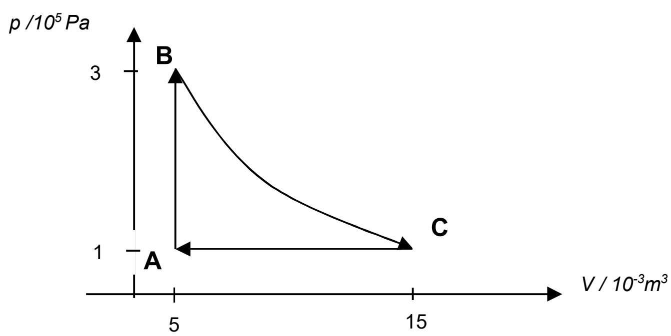
TUTORIAL 11: TEMPERATURE & IDEAL GAS SOLUTIONS

Level 1 Solutions

1 (a)(i)	There is no net transfer of heat between the two bodies.	[1]
(ii)	There is a net transfer of heat from A (the one at the higher temp) to B.	[1]
(b)	The temperatures are quoted in degrees Celsius. In order to compare the temperature of the system, we need to use the absolute temperature scale, (i.e. Kelvin scale). So in this case, it is not twice since (30 + 273.15) K is *not* twice that of (15 + 273.15) K. Therefore, one *can* only say that "it is hotter today than yesterday".	[1]
2	T/ K = θ/°C + 273.15 K = 201.84 + 273.15 K = 474.99 K	[1]
3	Absolute zero is the temperature at which all substances have a minimum internal energy (NOT zero energy[2]). On the thermodynamic scale, it is assigned a value of 0 K.	[1]
4 (a)(i)	The Avogadro constant, N_A, is defined as the number of atoms in 0.012 kg of carbon-12. ($N_A = 6.02 \times 10^{23}$ mol^{-1})	[1]
(ii)	An ideal gas is one that obeys the equation $pV = nRT$ for all values of pressure, volume and temperature, where *p*: pressure (Pa), *V*: volume (m^3), *n*: amount of gas (mol), R: molar gas constant (8.31 J K^{-1} mol^{-1}), *T*: *thermodynamic* temperature (K).	[1] [1]
(iii)	The absolute scale of temperature is a theoretical scale that is independent of the properties of any particular substance.	[1]
(b)	Molecular mass is the mass of one molecule. The molar mass (kg mol^{-1}) is the mass per mole of a substance {NOT: mass of one mole}. The relative atomic mass (no units), A_r, of an atom is defined by the following equation: $A_r = \dfrac{\text{Mass of an atom}}{\frac{1}{12} x \text{ the mass of a carbon} - 12 \text{ atom}}$ *NB: Students are not required to state the definition of relative atomic mass but are required to know how to deduce the molar mass of a substance given its relative atomic mass.*	[1] [1] [1]
5	Using $pV = nRT \Rightarrow$ Initial pressure $p = \dfrac{nRT}{V}$ Final pressure, p′ $= \dfrac{n'RT'}{V'} = \dfrac{(1.02n)R(1.01T)}{(1.002)V}$ $= \dfrac{nRT(1.02)(1.01)}{V(1.002)} = 1.028p$ Therefore the percentage increase in *p* is 2.8 %.	 [1] [1]
11(a)	Mean speed $= \dfrac{300+100+100+300+500}{5} = 260 \text{ m s}^{-1}$	[1]
(b)	Mean-square-speed $= \dfrac{(-300)^2+(-100)^2+100^2+300^2+500^2}{5} = 90000 \text{m}^2 \text{ s}^{-2}$	[1]

[2] Originally, it was thought that all atomic motions would cease at absolute zero. The development of *quantum mechanics* (see chapter 18) showed that all motion does not cease; the atoms vibrate with the minimum possible motion.

Level 2 Solutions

6	From $pV = n\mathrm{R}T \Rightarrow T = \dfrac{pV}{n\mathrm{R}}$ Path **L → M**: When V increases with *p* & *n* const, *T* increases proportionately, since $T \propto V$. Hence ans is B. Path **M → N**: $T \propto pV$, i.e. *T* does not vary linearly with *V* now since $p \neq$ constant. Comparing value of pV at M $(2.0\times 10^6 \times 0.003)$ with pV at N$(0.8\times 10^6 \times 0.005)$, $\Rightarrow T_M > T_N$ {Confirms elimination of (C) & (D)} 	 [1] [1]
7	 average kinetic energy of the system $\propto$ T Over the cycle ABCA, the system returns to the original state, So $\Delta T_{cycle} = 0$ so net increase in average kinetic energy of the system = 0 Net Increase in *U* = 0 as it returns to its original state because $\Delta T_{cycle} = 0$ & $U \propto T$.	[2] [1]
8 (a)(i)	$pV = nRT$ $\Rightarrow \Delta n = \dfrac{\Delta pV}{RT}$ $= \dfrac{(3.23 \times 10^5 - 2.62 \times 10^5) \times 0.0120}{8.31 \times (25 + 273.15)}$ = **0.295 mol**	[1] [1] [1]

(ii)	To supply 4 tyres, amount of air to be pumped out from the portable supply, Δn = 4 x Ans (a)(i) = 1.18 mol. The subsequent decrease in pressure, Δp, of this supply $= \dfrac{\Delta nRT}{V} = \dfrac{(1.18)(8.31)(25 + 273.15)}{0.0108} = 2.70 \times 10^5$ Pa Therefore, the pressure remaining in portable supply $= p_i - \Delta p = 8.72 \times 10^5 - 2.70 \times 10^5 = 6.01 \times 10^5$ Pa, without falling below 3.23×10^5 Pa.	[1] [1] [1]
(b)(i)	Average kinetic energy of one molecule of gas $= \dfrac{3}{2}kT$ $=$ **6.17×10^{-21} J**.	[1] [1]
(ii)	Average kinetic energy of one mole of gas = N_A x Ans (b)(i) = **3710 J**	[1]
(iii)	Increase in total kinetic energy = Ans (a)(i) x Ans (b)(ii) = **1100 J**	[1] [1]
9 (a)(i)	From the ideal gas equation, $pV = nRT$ where $n = \dfrac{\text{mass}}{\text{molar mass}}$ $1.0 \times 10^5 \times 0.064 = \dfrac{mass}{0.03}(8.31)(27 + 273.15)$ $\Rightarrow$ mass = 0.0770 kg	[2] [1]
(ii)	From $p_f V_f = n_f R T_f$ $(1.0 \times 10^5)(0.064) = n_f (8.31)(180 + 273.15)$ Hence n_f = 1.70 mol = (1.70 x 0.030) kg = 0.051 kg = m_f $\Delta m = m_i - m_f$ = 0.077 – 0.051 = 0.026 kg	[1] [1]
10	PV = nRT n = PV/RT Total amt of gas n_{tot} $= n_1 + n_2$ $= PV_1/RT + PV_2/RT$ $= 1.01 \times 10^5 \times (400 \times 10^{-6} + 200 \times 10^{-6})/(8.31 \times 293.15)$ = 0.02488 mol $n_{tot} = n_1 + n_2$ $n_{tot} = P_{com} V_1/RT_1 + P_{com} V_2/RT_2$ $0.02488 = P_{com} \times 400 \times 10^{-6}/(8.31 \times 373.15) + P_{com} \times 200 \times 10^{-6}/(8.31 \times 273.15)$ [1] $0.02488 = 2.171 \times 10^{-7} \times P_{com}$ $P_{com} = 1.15 \times 10^5$ Pa	[1] [1]
12	As the temperature in the cylinder increases, the average kinetic energy of the gas molecules increases. Thus the root-mean-square speed of the molecules also increases. With greater speed, when a particle collides against the wall, the momentum change per collision increased, exerting a greater force on the wall. As a result, the volume increases causing the collision frequency to decrease and the pressure to stay constant. {From examiner's report: Any explanation using $pV = nRT$ does not answer the question, since there is no reference to the forces exerted by molecules.}	[1] [1] [1]

13(a)	Thermal equilibrium means that X and Y are at the same temperature. Since translational kinetic energy is directly proportional to thermodynamic temperature, X has the same mean translational kinetic energy as Y, 6.0 x 10^{-21} J	[1]
(b)	$\frac{m_x<c_X^2>}{2} = \frac{m_y<c_y^2>}{2}$, $\frac{\sqrt{<c_y^2>}}{\sqrt{<c_X^2>}} = \sqrt{\frac{m_x}{m_y}} = \sqrt{\frac{1}{2}}$ = 0.707	[1] [1]
14(i)	For an ideal gas: $P = \frac{Nm<c^2>}{3V}$ $\Rightarrow P = \frac{1}{3}\rho < c^2 >$ Since $\frac{1}{2}m < c^2 > = \frac{3}{2}kT$, for the same temperature, $< c^2 >$ is constant. $\Rightarrow P \propto \rho$ Since p is proportional to density of gas for a fixed temperature, it behaves like an ideal gas. Alternatively, thinking from pV = nRT pV = $\frac{Mass}{Molar\ mass}$RT p = $\frac{Mass}{V}\frac{1}{Molar\ mass}$RT p =ρ $\frac{1}{Molar\ mass}$ R T p ∝ ρ at a fixed temperature	[1]
(ii)	Pick a suitable point from the graph. E.g. (P = 1.5 x 10^5 and ρ= 1.75 at 300 K) $P = \frac{1}{3}\rho < c^2 >$ 1.5 x 10^5 = 1/3 x 1.75 $<c^2>$ rms speed = 507 ms^{-1}	[1] [1] [1]
(iii)	For the same Pressure, the density value at T is at a lower value than at 300K, which means that the mean square speed is higher at T, ($P = \frac{1}{3}\rho_1 < c_1^2 > = \frac{1}{3}\rho_2 < c_2^2 >$)) so temperature T is higher. Or for the same density, the Pressure exerted at T is higher which implies that T is higher	[1] [1] [1] [1]
(iv)	At density of 1.0, P_T = 1.5 and P_{300} = 0.85 Since $\frac{P_T}{P_{300}} = \frac{T}{300}$ T = 300 x 1.5/0.85 = 529 K	[1] [1] [1]
15.(a)	average force = Δp/Δt = 6.0x10^{-23}/1x10^{-3} = 6 x 10^{-20}N	[1]
(b)	Average during the 1ms contact = 6 x 10^{-20} N Average force during the remaining 9 ms = 0 Average force during the 10 ms = 6 x10^{-20}/(1+9) = 0.6 x 10^{-20} N Value is obtained = Area under graph for 1 collision / 10 ms	 [1]

(c)	The value is (b) is used.	[1]
16. (a)	Since ½ m $<c^2>$ α T and they are at the same temperature, $\frac{1}{2} m_N <c_N^2> = \frac{1}{2} m_O <c_O^2>$ $\frac{\langle c_N{}^2 \rangle}{\langle c_O{}^2 \rangle} = \frac{m_O}{m_N}$ Ratio $= \frac{\sqrt{32}}{\sqrt{28}}$ =1.07	[1] [1]
(b)	Since ½ m $<c^2>$ α T and they have the same mass, $\frac{(1)}{(2)} : \frac{\langle c_{10}{}^2 \rangle}{\langle c_{100}{}^2 \rangle} = \frac{T_{10}}{T_{100}}$ Ratio = $= \frac{\sqrt{283.15}}{\sqrt{373.15}}$ = 0.871	[1] [1]
17(a)(i)	$\Delta p = mv - mu = mu - (-mu) = 2mu$	[1]
(ii)	time taken = distance travelled between successive collisions / speed of molecule = 2L / u	[1]
(iii)	$\Delta p/\Delta t = 2mu/(2L/u) = mu^2/L$ (shown)	[1]
(b) (i)	From ideal gas and given equation: $pV = {}^1/_3 Nm<c^2> = nRT$ (or NkT) Multiply both sides by $^3/_2$: $\frac{1}{2} Nm<c^2> = {}^3/_2\, nRT$ (or $^3/_2 NkT$) where ½ $m<c^2>$ is the av. KE of a molecule Since *n*, N and *R* (or *k*) are constant, hence ½ $m<c^2> \propto T$	[1] [1] [1]

ASSIGNMENT

1(a)	pV = nRT p = nRT/V = (1.00)(8.31)(273.15 + 30.0) / (1.00) = 2.52 × 10^3 Pa	 [1] [1]
(b)	pV = NkT N = pV/kT = (1.01 × 10^5)(2.00) / (1.38 × 10^{-23}) (273.15 + 100) = 3.92 × 10^{25}	 [1] [1]
2	Let the temperature at A be T_i pV = nRT (1.00 × 10^3)(4.50) = (1.50)(8.31) T_i T_i = 361 K Let the temperature at B be T_f pV = nRT (5.00 × 10^3)(3.00) = (1.50)(8.31) T_f T_f = 1.20 × 10^3 K ΔT = 1.20 × 10^3 - 361 = 839 K	 [1] [1]
3	½ m <c^2> = 3/2 kT <c^2> = 3 kT/ m $c_{rms} = \sqrt{3\,\mathrm{kT/m}}$ $c_{rms} \propto \sqrt{T}$ $\frac{c_{177^oC}}{c_{50.0\,K}} = \sqrt{\frac{177+273.15}{50.0}}$ $\frac{c_{177^oC}}{800} = \sqrt{\frac{177+273.15}{50.0}}$ c_{177^oC} = 2.40 × 10^3 m s^{-1}	 [1] [1]
4	½ m <c^2> = 3/2 kT <c^2> = 3 kT/ m $c_{rms} = \sqrt{3\,\mathrm{kT/m}}$ If the rms speeds are to be the same, $(c_{rms})_{nitrogen} = (c_{rms})_{hydrogen}$ $\sqrt{\frac{3kT_{nitrogen}}{m_{nitrogen}}} = \sqrt{\frac{3kT_{\text{hydrogen}}}{m_{\text{hydrogen}}}}$ $\frac{T_{nitrogen}}{m_{nitrogen}} = \frac{T_{\text{hydrogen}}}{m_{\text{hydrogen}}}$ Mass of 1 nitrogen molecule is $\frac{28.01}{N_A}$ g while mass of 1 hydrogen molecule is $\frac{2.02}{N_A}$ g. $\frac{T_{nitrogen}}{m_{nitrogen}} = \frac{T_{\text{hydrogen}}}{m_{\text{hydrogen}}}$ $\frac{T_{nitrogen}}{28.01/N_A} = \frac{27.0 + 273.15}{2.02/N_A}$ $T_{nitrogen}$ = 4.16 × 10^3 K	 [1]

- End of tutorial solutions -