

Year 4 Mathematics 2 Applications of Integration Area of a Region Supplementary Worksheet

 Name :
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)
 Class :

 Date :

- 1 (a) Evaluate $\int_{0.1}^{0.6} 4\cos^2 2x \, dx$, giving your answer correct to 2 decimal places.
 - (b) The diagram shows part of the graphs of $y = x^2 + 1$ and $y = \cos 2x$.



Find

- (i) the coordinates of A, B and C,
- (ii) the area of the shaded region giving your answer correct to 2 decimal places.

[2008 CHIJ Toa Payoh Sec AMaths P2]

2(a) The diagram shows part of the curve y = g(x).



Copy this diagram and use it to explain why $4 < \int_{1}^{3} g(x) dx < 6$.

(b) Given that $\int_{1}^{3} f(x) dx = 5$, evaluate $\int_{2}^{1} \left(\frac{1}{x} - f(x)\right) dx + \int_{2}^{3} f(x) dx$.

[2008 Commonwealth Sec AMaths P1]

3 The diagram shows part of the graph of $y = e^{2x}$ and of $y = 1 - \sec^2 x$. *OABC* is a rectangle where *A* is on the *y*-axis, *B* is on the curve $y = e^{2x}$ and *C* is (ln 2, 0).



- (i) Find the area of the shaded region, which is bounded by the two curves, the *y*-axis and the line $x = \ln 2$.
- (ii) Region *P* is bounded by the curve $y = e^{2x}$, the line *AB* and the *y*-axis. Given that the area of region *P* may be expressed as $\int_{1}^{k} f(y) dy$, find the value of *k* and function f(y).

[2008 Commonwealth Sec AMaths P2 (modified)]

4(i) The diagram below shows part of the curve $y = \sin 2x$ and the line $y = \frac{1}{2}$. Find the area of the shaded region.



(ii) The diagram below shows part of the graph $y = \ln x^2$, cutting the *x*-axis at (1, 0). The line y = 2 intersects the curve at *P*. A line is drawn from *P*, parallel to the *y*-axis, to meet the *x*-axis at *Q*.



- (a) Find the *x*-coordinate of *Q*.
- (b) Differentiate $2x \ln x$ with respect to x.
- (c) Hence find the area of the shaded region.

[2008 Crescent Girls' AMaths P2 (modified)]



The diagram above shows part of the curve $y = x + \frac{4}{x}$ passing through the points

P, Q and R.

5

- (i) The curve has a minimum point at *P*. Find the coordinates of *P*.
- (ii) Given that the gradient of the line PQ is -1, find the coordinates of Q.
- (iii) Calculate the area of the shaded region.

[2008 Holy Innocents High AMaths P2]

6 The diagram shows part of the curve $y = \left(\frac{x}{2} + 2\right)\sqrt{\frac{x}{2} + 2}$ and the straight line *BC* is the normal to the curve at the point *C*(-2, 1).



- (i) Find the equation of the line *BC*.
- (ii) Show that $OA = \frac{8\sqrt{2}}{5}BC$.
- (iii) Find the area of the shaded region.

[2008 Singapore Chinese Girls' AMaths P2]

7(a) Given that
$$y = x^2 \sqrt{x+1}$$
, show that $\frac{dy}{dx} = \frac{5x^2 + 4x}{2\sqrt{x+1}}$.
Hence, evaluate $\int_{0}^{3} \frac{15x^2 + 12x}{\sqrt{x+1}} dx$.

(b) The diagram shows part of the curve $y = \sin 2x$. The line *OA* intersects the curve at *A* where the $\sqrt{3}$

y-coordinate is $\frac{\sqrt{3}}{2}$.

- (i) Find the *x*-coordinate of *A* in terms of π .
- (ii) Find the area of the shaded region.



[1]

[4]

^{[2008} Tanjong Katong Sec AMaths P2]

8 In the diagram, the curve $y^2 = 2x - 3$ and the straight line x + y = 3 intersect at two points *A* and *B*.



Find

- (a) the coordinates of A and of B,
- (b) the area of the shaded region.

[2008 Temasek Sec AMaths P2]

9 The figure below shows parts of the curve $2y = 4 - x^2$ and $2x = (y+2)^2$.



Calculate

- (i) the coordinates of A, B and C,
- (ii) the shaded area enclosed by the curves and the *y*-axis.

[2008 Unity Sec AMaths P2]

10 The diagram shows part of the curve of $y = 3e^{-\frac{1}{2}x} + e^{\frac{1}{2}x}$.



- (i) Show that the exact value of the *y*-coordinate of the stationary point of the curve is $2\sqrt{3}$.
- (ii) Calculate the area enclosed by the curve, the *x*-axis and the lines x = 0 and x = 1. [2008 Zhonghua Sec AMaths P1 (modified)]
- 11 The diagram shows part of the curve $y = 10 \frac{32}{x^2}$ and two parallel lines *OR* and *PQ*. The line *OR* intersects the curve at the point *R*(2, 2) and the line *PQ* is a tangent to the curve at the point *Q*.



Find

- (a) the gradient of *OR*,
- (b) the coordinates of Q,
- (c) the area of the shaded region *OPQR*.

[2008 Zhonghua Sec AMaths P2 (modified)]

Answers

1(a)	1.14	(b)(i)	$A(0,1), B(\frac{\pi}{4},0)$	$\Big), C\Big($	$\left(\frac{\pi}{4}, \frac{\pi^2}{16} + 1\right)$	(ii)	0.45 unit ²
2(b)	4.31			11			
3(1)	1.64 units ²	(11	k = 4; f(y)	$=\frac{1}{2}\ln$	у		
4(i)	0.342 units^2	(ii)(a)	e	(b)	$2(\ln x+1)$	(c)	2 units ²
5(i)	(2,4)	(ii)	(1,5)	(iii)	42.3 units^2		
6(i)	3y + 4x + 5 = 0	(iii)	3.35 units^2				
7(a)	108	(b)(i)	$\frac{1}{6}\pi$	(ii)	0.0232 units ²		
8 (a)	A(2, 1); B(6, -3)			(b)	$5\frac{1}{3}$ units ²		
9(i)	A(0, 2); B(2, 0);	C(0, -2))	(ii)	4 units ²		
10(ii)	3.66 units ²						
11(a)	1	(b)	(4, 8)	(c)	10 units ²		