Year 4 Mathematics 2
Supplementary Worksheet
Differentiation & Applications of Differentiation

Name : ___________________________________ (   ) **Class :** _______ **Date :** __________

1 **(a)** The force F newtons between two magnetic poles is given by the formula $F = \dfrac{1}{500r^2}$, where r m is their distance apart. Find the rate of change of the force when the poles are 0.2 m apart and where the distance between them is increasing at a rate of 0.03 m s^{-1}. [3]

(b) Given $y = \sqrt{x} + \ln\sqrt{x}$, prove that $\dfrac{dy}{dx} = \dfrac{x+\sqrt{x}}{2x\sqrt{x}}$. [2]

(c) Find the equation of the normal to the curve $y = e^{3x+2}$ at the point where $x = -1$, leaving your answer in terms of e. [4]

[*2008 ACS (Barker Road) AMaths P1*]

2 If $x + y = 82$, using differentiation, find the stationary value of xy. Determine the nature of this stationary value. [4]

[*2008 ACS (Barker Road) AMaths P2*]

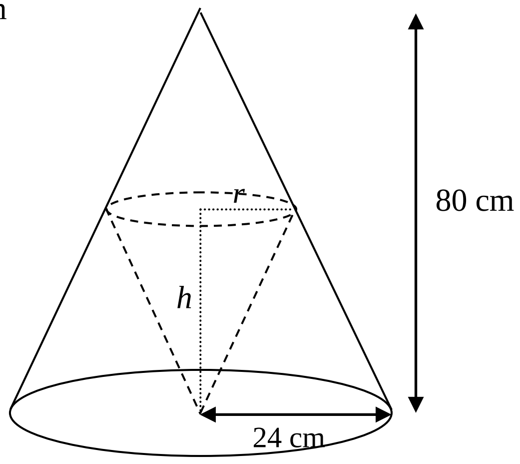
3 Differentiate the following with respect to x.

(i) $y = \tan^3(5x^2 + 1)$, [2]

(ii) $y = \dfrac{1-x}{\sqrt{x^2+1}}$, leaving your answer as a single algebraic fraction. [3]

[*2008 ACS (Barker Road) AMaths P2*]

4 The diagram shows a solid cone with base radius 24 cm and height 80 cm.
A cone of radius r cm and height h cm is to be removed.



(i) Express h in terms of r. [2]

(ii) Hence show that the volume, V cm^3, of the cone to be removed is given by

$$V = \frac{80\pi}{3}r^2 - \frac{10\pi}{9}r^3.$$ [2]

(iii) Calculate the value of r for which V has a stationary value. Hence, find the stationary value of V and determine whether it is a maximum or minimum value. [4]

[*2008 ACS (I) AMaths P2*]

5 A curve has the equation $y = \ln\sqrt{\dfrac{5+3x}{2x-5}}$.

(i) Find the gradient of the curve at the point where the curve meets the x-axis. [4]

(ii) Show that $y = \ln\sqrt{\dfrac{5+3x}{2x-5}}$ has no stationary point for all real values of x. [2]

[*2008 ACS (I) AMaths P1*]

6 P is the point (4, 7) on the curve $y = x^2 - 6x + 15$. Find the equation of the normal at P. The tangent at another point Q is parallel to the normal at P. Calculate the x-coordinate of Q. [5]

[*2008 Anderson Sec AMaths P2*]

7 The vertex of a pyramid is vertically above the centre of its horizontal base. It is given that the volume of the pyramid is $36\sqrt{2}$ cm^3 and that the side of the square base is $2x$ cm.

(a) Show that if A is the area of the sloping triangular face, then $A = \dfrac{\sqrt{1458 + x^6}}{x}$.

(b) Hence find the value of x for which A^2 has a stationary value and determine whether this is a minimum or a maximum. [7]

[*2008 Anderson Sec AMaths P1*]

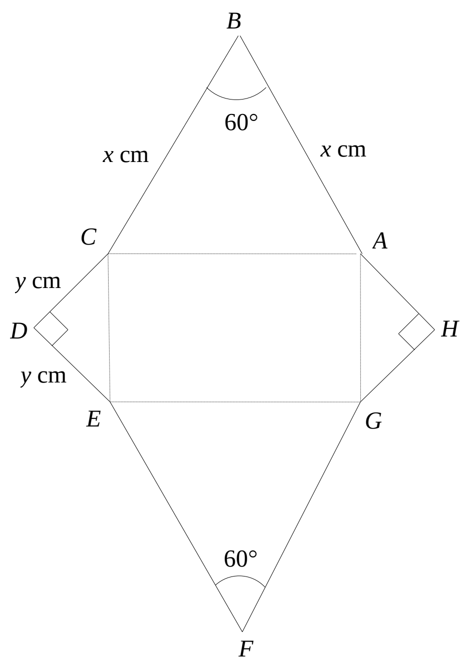
8 Given that $y = \dfrac{e^{2x-3}}{2\left(e^{x-1}\right)^4}$, find $\dfrac{dy}{dx}$. Calculate the rate of change of x at the instant when $x = 3$ and y is changing at the rate of e^{-5} unit s^{-1}. [4]

[*2008 Anglican High AMaths P1*]

9 A piece of wire, 120 cm in length, is bent to form the figure as shown.

Given that $\angle ABC = \angle EFG = 60^\circ$, $\angle CDE = \angle GHA = 90^\circ$,

$AB = BC = EF = FG = x$ cm and $CD = DE = GH = HA = y$ cm.



(a) Show that the area of the figure, P cm^2, is given by

$$P = \left(\frac{\sqrt{3}}{2} + 1 - \sqrt{2}\right)x^2 + (30\sqrt{2} - 60)x + 900. \quad [4]$$

(b) Find the value of x for which P has a stationary value. [2]

Determine whether the stationary value of P is a maximum or minimum. [2]

[*2008 Anglican High AMaths P2*]

10 Show that the gradient function for the curve $y = \dfrac{\tan x}{\cos x}$ is $\dfrac{1+\sin^2 x}{\cos^3 x}$. Hence deduce that the curve has no stationary points for $0 < x < \dfrac{\pi}{2}$. [6]

[*2008 Anglican High AMaths P1*]

11 A piece of chicken drumstick is removed from the refrigerator and is then allowed to thaw. Its temperature, x degrees Celsius after t minutes, is given by the formula $x = 28 - 30e^{-0.3t}$.

(i) Find the initial temperature of the drumstick. [1]

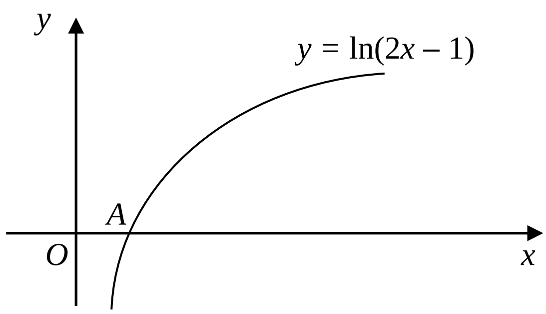
(ii) Find the time taken for the temperature to reach zero degree. [2]

(iii) Find the rate at which temperature is increasing when $t = 5$. [2]

(iv) Sketch the graph of x against t. [3]

[*2008 Balestier Hill Sec AMaths P1*]

12 The diagram shows part of the curve $y = \ln(2x - 1)$ which meets the x-axis at A.

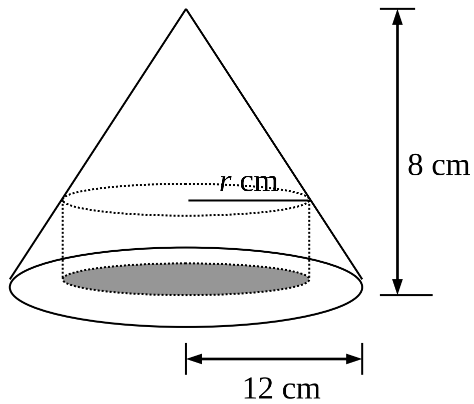


(i) Find the equation of the normal at A. [4]

(ii) Find the coordinates of the point B on the curve such that the tangent drawn at B is parallel to the line $4y = x - 3$. [3]

[*2008 Balestier Hill Sec AMaths P1*]

13 A cylinder is placed inside a right circular cone of radius 12 cm and height 8 cm. The radius of the cylinder is r cm.



(i) Express the height of the cylinder, h cm, in terms of r cm, hence show that its volume, V cm^3 is given by $V = 8\pi r^2 - \dfrac{2}{3}\pi r^3$. [3]

(ii) Given that r varies, find the value of r for which V has a stationary value. [3]

(iii) Find the stationary value of V and determine whether it is a maximum or minimum value. [3]

[*2008 Balestier Hill Sec AMaths P1*]

14 The equation of a curve is $y = xe^{3x}$.

(i) Find an expression for $\dfrac{dy}{dx}$. [2]

The curve has a stationary point at M where $x \neq 0$.

(ii) Find the coordinates of M. [2]

(iii) Determine whether this stationary point is a maximum or a minimum point. [3]

[*2008 Balestier Hill Sec AMaths P1*]

15 Given that $y = e^{3x} - e^{-x} + 1$, find the rate of change of x at the instant when $y = 1$, given that y is changing at the rate of 0.4 units per second at this instant. [4]

[*2008 Catholic High AMaths P1*]

16 Find the x-coordinate of the stationary point of the curve $y = \frac{(x-1)^3}{x+1}$, $x > 0$.

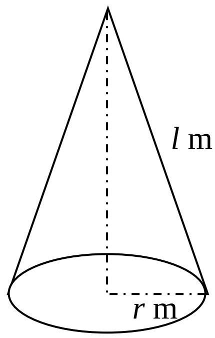
By considering the sign of $\frac{\mathrm{d}y}{\mathrm{d}x}$, or otherwise, determine the nature of the stationary point.

[*2008 Catholic High AMaths P1*]

17 If $y = x^3 - 3x^2 + 4x + 1$, show that y is an increasing function for all real values of x.
Hence, state the minimum value of the gradient of this function. [5]

[*2008 Catholic High AMaths P1*]

18 The diagram shows a roof in the shape of a right circular cone whose radius is r m and its slant height l m. The sloping surface of the roof is covered with a sheet of thin metal whose area is $4\sqrt{3}\pi$ m^2.



[Curved Surface Area of Cone = πrl where l is the slant height of the cone.]

(a) Express l in terms of r and show that the volume of the cone, V cm^3, is given by $V = \frac{\pi}{3} r\sqrt{48 - r^4}$. [3]

(b) Given that r can vary, find

(i) an expression for $\frac{\mathrm{d}V}{\mathrm{d}r}$, [2]

(ii) the value of r for which V has a stationary value. [3]

[*2008 Catholic High AMaths P1*]

19 **(a)** Differentiate with respect to x

(i) $\tan^3 2x$, [2]

(ii) $\ln\left(\frac{1-2x}{x+2}\right)$. [3]

(b) Given that $y = \sqrt{3x+2}$, show that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 \frac{1}{y} = 0$. [4]

[*2008 Catholic High AMaths P2*]

20 A curve has the equation $y = \frac{2(x+3)}{\sqrt{2x-1}}$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{A(x-B)}{\sqrt{(2x-1)^3}}$ where A and B are integers. [4]

(ii) Find the equation of the normal to the curve at the point where $x = 5$. [3]

[*2008 Cedar Girls' Sec AMaths P1*]

21 Under a heating process, the length, x cm, of each side of a metal cube increases at a constant rate. Express the volume, V cm^3, and the surface area, A cm^2, of the cube in terms of x.

Write down expressions for $\frac{dV}{dx}$ and $\frac{dA}{dx}$.

Given that V is increasing at the rate of $1.5 \text{ cm}^3/\text{s}$, find the rate of increase of

(i) x,

(ii) A,

at the instant when $x = 10$. [8]

[*2008 Cedar Girls' Sec AMaths P1*]

22 An object is heated until it reaches a temperature of T degrees Celsius. It is then allowed to cool. Its temperature T degrees Celsius when it is cooling for time t minutes is given by the equation $T = 24 + 16e^{-\frac{3}{4}t}$. Find

(i) the value of t when $T = 32$, [2]

(ii) the value of T when $t = 4$, [1]

(iii) the rate at which T is decreasing when $t = 12$. [2]

State the highest temperature T_h and the approximate temperature T_A when t becomes very large. [2]

[*2008 Cedar Girls' Sec AMaths P2*]

23 **(a)** A curve has equation $y = e^{\tan(1-3x)}$. Find an expression for $\frac{dy}{dx}$ in terms of x.

State whether y is increasing or decreasing and explain your answer clearly. [4]

(b) Given that $y = \ln\left(\frac{x^2+8}{x-1}\right)$, find $\frac{dy}{dx}$ and hence determine the stationary value of y. [4]

[*2008 Cedar Girls' Sec AMaths P2*]

24 A curve has the equation $y = x + \frac{4}{x^2}$.

(a) Find

(i) an expression for $\frac{dy}{dx}$, [1]

(ii) the value of k for which $y = 2x + k$ is a tangent to the curve. [3]

(b) A point $P(x, y)$ moves along the curve $y = x + \frac{4}{x^2}$. When P is at the point where $x = 4$, the x-coordinate is increasing at a rate of 0.02 units per second. Find the corresponding rate of change of y-coordinate of P at this instant. [2]

[*2008 CHIJ Toa Payoh Sec AMaths P1*]

25 The equation of a curve is $y=\dfrac{x}{2-3x}$. Find an expression for $\dfrac{dy}{dx}$. Hence find the equation of the normal to the curve at the point $x=1$. [4]

[*2008 CHIJ Toa Payoh Sec AMaths P2*]

26 The equation of a curve is $y=2x\left(9-\dfrac{x^2}{3}\right)$.

(a) Find expressions for $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [2]

(b) Find the coordinates of the stationary points and determine the nature of the stationary points. [4]

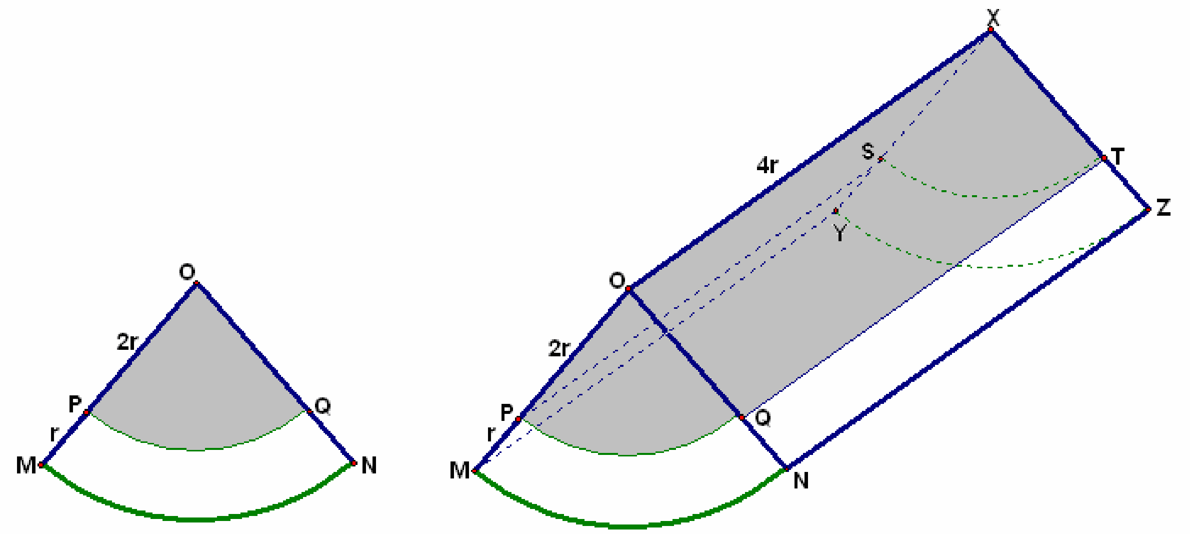
[*2008 CHIJ Toa Payoh Sec AMaths P2*]

27 **(a)** Find the coordinates of the stationary points of the curve $y=x(2-x)^3$. [4]

(b) A curve has the equation $y=\dfrac{x}{x-2}$, $x \neq 2$. Obtain an expression for $\dfrac{dy}{dx}$ and hence explain why $y=\dfrac{x}{x-2}$ is a decreasing function. [2]

[*2008 CHIJ St Nicholas Girls' AMaths P1*]

28 Figure 1 shows two sectors in which PQ and MN are arcs of concentric circles, centre O, $OP = 2r$ cm and $MP = r$ cm.



(i) Given that the perimeter of sector OMN = 90 cm, show that the area, A cm ^{2}, of the unshaded region is given by $A=75r-5r^2$. [4]

(ii) It is given that A and r vary with time. If the area of the unshaded region is increasing at a rate of 10 cm^2/s, find the rate of change of the radius at the instant when r = 5 cm. [3]

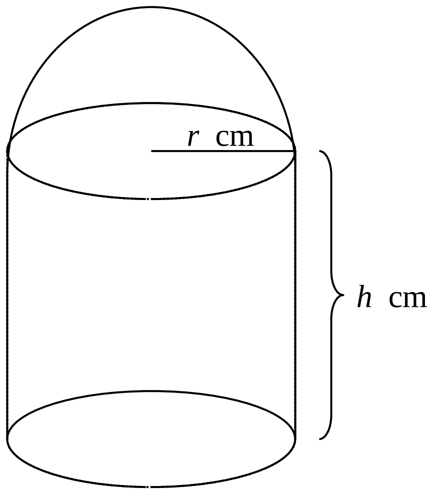
(iii) The sector OMN forms the cross-sectional area of a wooden prism, shown in figure 2. A section of the prism, $POQTXS$, shown shaded, is removed. Given that the length OX of the prism is $4r$ cm and r may vary, find the value of r for which the volume of the remaining solid is a maximum. [5]

[*2008 CHIJ St Nicholas Girls' AMaths P1*]

29 The volume of a container in the shape of an open right circular cylinder of radius r cm and height h cm is 500π cm^3.

(i) Express h in terms of r. [1]

(ii) A hemispherical lid is attached to the container as shown in the diagram below. External surfaces of the container and the lid are painted. It costs 3 cents per cm^2 to paint the cylindrical surface and 4 cents per cm^2 to paint the base and the lid.
Let $C be the total cost of painting the container.
Show that $C = 3\pi\left(\dfrac{r^2}{25}+\dfrac{10}{r}\right)$. [3]



(iii) Find the value of r which gives the minimum value of C and find the minimum cost of painting the container and the lid, giving your answer to the nearest cent. [4]

[*2008 CHIJ Toa Payoh Sec AMaths P1*]

30 A curve has the equation $y = \dfrac{x^2-6}{e^{2x}}$.

(i) Find the coordinates of the points of intersection of curve with the x-axis. [2]

(ii) Find the coordinates of the stationary points of the curve. [5]

(iii) Determine the nature of these stationary points. [2]

[*2008 Clementi Woods Sec AMaths P1*]

31 A curve has equation $y = x\tan x$. Find

(i) $\dfrac{dy}{dx}$, [2]

(ii) the equation of the tangent to the curve at the point where $x = \dfrac{\pi}{4}$, [4]

(iii) the rate of change of y when $x = \dfrac{\pi}{4}$, given that x is decreasing at a rate of 2 units per second. [3]

[*2008 Clementi Woods Sec AMaths P2*]

32 Mr Lim buys a brand new car at a cost of $77 900. The value of this car decreases with time so that its value, $$V$, after t years of use is given by

$$V = 77900\,e^{-kt}$$

where k is a positive constant.
The value of the car is expected to be $43 675 after 5 years of use. Find

(i) the value of k, [2]

(ii) the year in which Mr Lim must sell his car in order not to make a loss of more than $25 000. [3]

(iii) the rate at which the value of the car is decreasing when $t = 10$. [2]

[*2008 Holy Innocents AMaths P2*]

33 A particle moves in a straight line such that its displacement, s metres, from a fixed point at time t seconds, is given by

$$s = \begin{cases} \ln(t+2) & \text{for } 0 \le t \le 4 \\ \frac{1}{3}\ln(t-3) - \ln t + \ln 24 & \text{for } t > 4 \end{cases}$$

Find

(i) the initial velocity of the particle, [2]

(ii) the velocity of the particle when $t = 5$, [2]

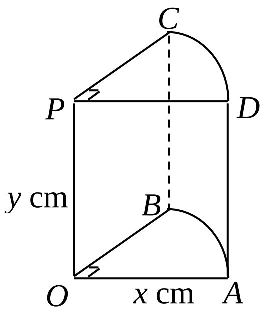
(iii) acceleration of the particle when $t = 5$, [2]

(iv) value of t when the particle is instantaneously at rest, [1]

(v) the distance travelled in the 6th second, correct to 3 significant figures. [2]

[*2008 Clementi Woods Sec AMaths P1*]

34 **(a)** The diagram shows a prism which is such that each cross-section is a sector of a circle of radius x cm, with angle at the centre equal to 90°.



The cross-sections are OAB and PDC where A, B, C, D lie on the curved surface of the prism and the vertical line OP is the intersection of the vertical plane faces $OADP$ and $OBCP$.
The cross-sections are horizontal and y cm apart. Given that the volume of the prism is 25π cm^3, express y in terms of x.
Hence show that the total surface area, A cm^2, of the prism is given by $\frac{\pi}{2}x^2 + \frac{50(4+\pi)}{x}$.

Find

(i) the value of x for which A has a stationary value,

(ii) the stationary value of A .

Determine whether the stationary value of A is a maximum or a minimum. [9]

(b) Given also that the total surface area is increasing at a constant rate of 7 cm^2 s^{-1}, find the rate at which x is changing when $x = 5$. [2]

[*2008 Commonwealth Sec AMaths P1*]

35 A curve has the equation $y = \frac{16-3x}{x+3}$.

(i) Show that, for $x \neq -3$, y is a decreasing function. [3]

(ii) Find the equation of the normal to the curve at a point where $x < 0$ and such that the normal is parallel to the line $y - 4x = 1$. [3]

(iii) Find the rate of change of x at the instant when $x = 2$, given that y is increasing at the rate of 0.5 units per second at this instant. [2]

[*2008 Crescent Girls' AMaths P1*]

36 **(a)** Differentiate with respect to x

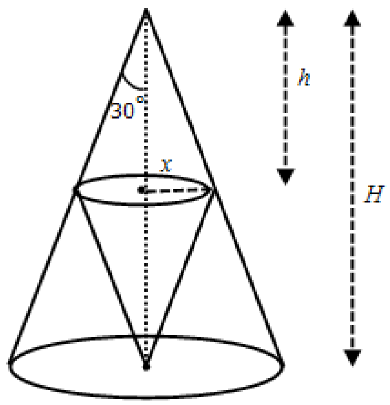
(i) $\cos^3(5x+3)$

(ii) $x^3\ln(1-x)^3$ [4]

(b) Given that $y=\dfrac{(2x-3)^2}{x^2+1}$, find the values of x for which y has a stationary value. [4]

[*2008 Commonwealth Sec AMaths P2 (modified)*]

37 A circular hollow cone of constant height H cm and semi-vertical angle 30° stands on horizontal ground. A smaller upright, inverted cone with base radius x cm just fits into the cone as shown. The distance between the vertex of the circular hollow cone and the base of the smaller cone is h cm.



(i) Express h in terms of x. Hence, show that the volume, V cm^3 of the smaller cone is given by $V=\frac{1}{3}\pi x^2(H-\sqrt{3}x)$. [2]

(ii) Prove that, as x varies, the maximum possible volume of the smaller cone is $\dfrac{4\pi}{243}H^3$ cm^3. [6]

[*2008 Crescent Girls' AMaths P1*]

38 A curve has the equation $y=xe^{mx}$ where m is a constant.

(i) Given that the curve has a stationary point when $x=\dfrac{1}{3}$, find the value of m. [3]

(ii) Hence find $\dfrac{d^2y}{dx^2}$ and determine the nature of the stationary point when $x=\dfrac{1}{3}$. [3]

[*2008 Crescent Girls' AMaths P2*]

39 A curve has the equation $y=\dfrac{x+1}{1-3x}$.

(a) Find the gradient of the curve at the point where it meets the y-axis. [3]

(b) Find the equation of the normal to the curve at the point it meets the y-axis. [2]

(c) Show that the curve has no stationary points. [2]

[*2008 Holy Innocents AMaths P1*]

40 An open rectangular water tank has a length which is three times its width x cm.

(a) If this tank has a capacity of 1152 cm^3, show that its total surface area is

$$A=\frac{3072}{x}+3x^2.$$ [3]

(b) A contractor wants to use as little material as possible to build this water tank. Find the width and surface area of such a water tank. [5]

[*2008 Holy Innocents AMaths P1*]

41 A curve has the equation $y = 2x^3 - 5$.

(i) Find the x-coordinate of the turning point of the curve. [3]

(ii) Determine the nature of the turning point. [3]

[*2008 Holy Innocents AMaths P2*]

42 A particle moves in a straight line so that its displacement, s metre, from a fixed point O is given by $s = t^3 - 9t^2 + 24t$, where t is the time in seconds after the start of motion.

Find

(i) the initial acceleration of the particle, [2]

(ii) the values of t when the particle is instantaneously at rest, [3]

(iii) the average speed of the particle in the first 4 seconds, [3]

[*2008 Holy Innocents AMaths P2 (modified)*]

43 Nicholas wants to construct a closed right cylindrical can of base radius r cm and height h cm with a thin sheet of material of area 600π cm^2. Determine the value of r and of h, in order that Nicholas may construct a cylindrical can with the greatest possible volume. [7]

[*2008 Maris Stella High AMaths P1*]

44 A curve has the equation $y = \dfrac{x-4}{3x+2}$.

(i) Find an expression for $\dfrac{dy}{dx}$ and explain why the curve has no turning points. [3]

(ii) Find the gradient of the curve when $y = 0$. [2]

(iii) Given that y is increasing at the rate of 0.28 units per second at the instant when $x = 2$, find the rate of change of x at this instant. [3]

[*2008 Ngee Ann Sec AMaths P1*]

45 The equation of a curve is $y = e^{2x} \cos x$.

Find the value of x between 0 and π for which y is stationary. Hence, determine the nature of this stationary value of y. [8]

[*2008 Ngee Ann Sec AMaths P1*]

46 Differentiate the following with respect to x

(i) $\ln\sqrt{\dfrac{2x+1}{x-4}}$ [3]

(ii) $e^{\sin^2 x}$ [2]

[*2008 Ngee Ann Sec AMaths P2*]

47 Find, in terms of c, the x-coordinate of the stationary point on the curve $y = \ln\dfrac{3c+x}{(c-x)^2}$, where c is a constant. [4]

[*2008 Tanjong Katong Sec AMaths P2*]

48 A curve has equation $y = \dfrac{ax+2}{3x+1}$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and the range of values of a for which y is an increasing function. [3]

(ii) Given that the line $20y = 64x - 49$ is a normal to the curve at the point where $x = 1$, find the value of a. [3]

[*2008 Tanjong Katong Sec AMaths P1*]

49 A particle P moves in a straight line so that its displacement, s metres , from a fixed point O is given by $s = 4 + 3t^2 - t^3$, where t is the time in seconds measured from the start of the motion. Calculate

(i) the velocity of P after 3 seconds, [2]

(ii) the value of s at the instant when P reverses its direction of motion, [3]

(iii) the acceleration of the particle when $t = 4$. [2]

[*2008 Tanjong Katong Sec AMaths P1*]

50 **(i)** An inverted right conical container of radius 12 cm contains paint to a depth of 28 cm. The paint is then poured into a hemispherical bowl at a constant rate and the transfer is completed in 32 seconds.

Calculate the rate of transfer of the paint in terms of π and in cm^3/s. [2]

(ii) As the paint is being transferred, the volume, $V\ \text{cm}^3$, of paint in the hemispherical bowl, is given by $V = \dfrac{1}{3}\pi h^2(18-h)$, where h cm is the depth of paint in the bowl. Find the rate at which h is increasing when $h = 2$. [3]

[*2008 Tanjong Katong Sec AMaths P1 (modified)*]

51 A certain type of micro-organism, under ideal conditions, multiply themselves according to the formula $N = N_0\,\mathrm{e}^{\frac{t}{3}}$ where N_0 is the initial quantity of micro-organism and t is the time interval in minutes.

For a small isolated sample where the initial quantity $N_0 = 1200$, find

(i) the time taken for the quantity N to be twice the initial quantity, [2]

(ii) the rate of increase of the N at 45 seconds. [2]

[*2008 Tanjong Katong Sec AMaths P2 (modified)*]

52 Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}(\sec x) = \sec x \tan x$.

Given that $y = x - \ln(\sec x + \tan x)$, $0 < x < \frac{1}{2}\pi$,

(i) show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ can be written in the form $a + b\sec x$, where a and b are integers, and [5]

(ii) deduce that y is a decreasing function. [2]

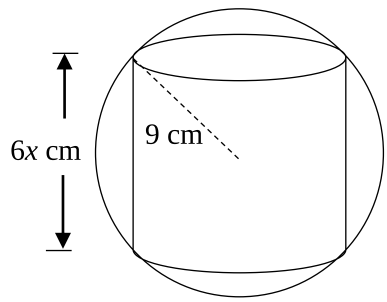
[*2008 Singapore Chinese Girls' AMaths P2 (modified)*]

53 **(a)** A curve has equation $y = 2\cos^2(x + \frac{\pi}{4})$, $0 < x < \frac{1}{2}\pi$. Find

(i) an expression for the gradient of the curve, [2]

(ii) the x-coordinate of the point on the curve at which the gradient of the tangent is 1. [3]

(b) A solid right cylinder is removed from a solid sphere of radius 9 cm as shown. The height of the cylinder is $6x$ cm.



(i) Show that the volume, V cm^3, of the cylinder is given by $V = 486\pi x - 54\pi x^3$. [3]

Given that x can vary,

(ii) find the value of x for which V has a stationary value and determine whether this value of V is a maximum or a minimum. [4]

[*2008 Tanjong Katong Sec AMaths P2 (modified)*]

54 The equation of a curve is $y = ax^2 - 2bx + c$, where $a > 0$ and a, b and c are constants.

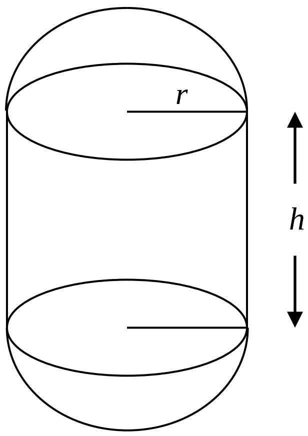
(i) Find in terms of a, b and c the coordinates of the turning point on the curve and state its nature. [4]

(ii) Given that the turning point on the curve lies on the line $y = x$, find an expression for c in terms of a and b. [2]

(iii) Find the value of a, of b and of c when the curve passes through the point (0 , 6) and has a turning point (2 , 2). [4]

[*2008 Singapore Chinese Girls' AMaths P1*]

55 A solid of volume V m^3 is to make from a thin metal sheet. The solid consists of a right circular cylinder and two hemispheres as shown in the figure below. The cylinder is of height h metres and radius r metres .



(i) Express h in terms of r and V. [2]

(ii) The cost of making the cylindrical surface is \$$M$ per square metre while that of making the hemispherical surfaces is \$2M per square metre.

(a) If the total cost for making the solid is \$$C$, show that $C = \frac{16}{3}\pi r^2 M + \frac{2MV}{r}$. [3]

(b) Find the value of r, in terms of V and π, for which C is minimum. [4]

(c) Find the ratio $r : h$ when C is a minimum. [2]

[*2008 Unity Sec AMaths P1 (modified)*]

56 **(i)** A curve has equation $y=\dfrac{4x}{(x+1)(2x+1)}$. Find the x-coordinates of the stationary points on the curve. [3]

(ii) Differentiate with respect to x

(a) $\cos\left(\ln\sqrt{2x}\right)$, [2]

(b) $\ln\left(\sqrt{\cos 2x}\right)$. [2]

[*2008 Unity Sec AMaths P1 (modified)*]

57 Differentiate each of the following with respect to x and simplify your answers.

(a) $\cos^2(3x-5)-\tan 2x$ [2]

(b) $\dfrac{\ln x}{4+\ln x}$ [2]

[*2008 Unity Sec AMaths P2 (modified)*]

58 The height of a cone is 28 cm and the base radius is 8 cm. The cone is inverted, completely filled with water and held such that the circular base is horizontal. Given that water is leaking out of the inverted cone at a constant rate of 20 cm^3/s, find the rate of decrease of height h cm of the water at the instant when h = 7, leaving your answer in terms of π. [6]

[*2008 Unity Sec AMaths P2 (modified)*]

59 A particle M travels in a straight line so that its displacement, s metres, from a fixed point O is given by $s=t+\dfrac{9}{t+1}$, where t is the time in seconds measured from the start of the motion. Calculate

(i) the initial acceleration of M, [3]

(ii) the velocity of M when it is at its starting point again, [4]

(iii) the minimum displacement of M from O. [3]

(iv) the total distance travelled during the first three seconds. [3]

[*2008 Unity Sec AMaths P2 (modified)*]

60 An open cylindrical well has a circular base of radius x m and height y m. Its total surface area is $48\pi\ \text{m}^2$.

(i) Express y in terms of x. [2]

(ii) Show that the volume of the well, $V\ \text{m}^3$, can be expressed as $\dfrac{\pi}{2}\left(48x-x^3\right)$. [2]

Given that x can vary, find

(iii) the value of x for which V is a maximum. [4]

(iv) the maximum value of V. [2]

[*2008 Temasek Sec AMaths P1 (modified)*]

61 A curve has the equation $y = \frac{2x^3}{x-1}$. P is the point on the curve where $x = -1$.

Find the angle that the normal to the curve at P makes with the x–axis. [7]

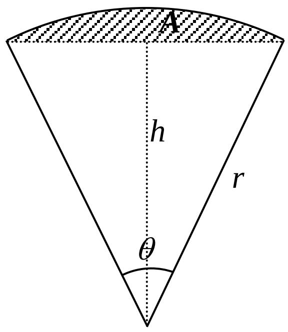
[*2008 Temasek Sec AMaths P2*]

62 **(a)** Water is poured into a container at a rate of 5 $cm^3\ s^{-1}$. The volume of water in the container, V cm^3, is given by $V = h^2 + \frac{1}{3}h$, where h cm is the height of the water level in the container. Find, when $V = 10$,

(i) the value of h,

(ii) the rate at which h is increasing. [5]

(b) The figure shows a sector with angle θ radian and radius r cm. The height of the triangle is h cm and the area, A cm^3, of the shaded segment is $A = \frac{1}{2}r^2(\theta - \sin\theta)$.



(i) Given that the radius of the sector is 12 cm and that θ increases at a constant rate of $\frac{\pi}{60}$ radian s^{-1}. Find the rate of increase of A at the instant when $\theta = \frac{\pi}{3}$.

(ii) θ is now kept constant while r increases at a constant rate of 0.5 cm s^{-1}. Find the rate of increase of h when $\theta = \frac{\pi}{3}$ radians. [6]

[*2008 Temasek Sec AMaths P2 (modified)*]

63 Differentiate and simplify each of the following with respect to x.

(i) $\frac{\ln 3x}{e^x}$, [3]

(ii) $\left(\sqrt{\tfrac{1}{2}x^2 + 7 + 2x^{-1}}\right)^3$ [3]

[*2008 Victoria School AMaths P1*]

64 The concentration of caffeine, C %, in Peter's blood when he drinks a cup of coffee in the morning is represented by $C = 1.2t\,e^{-0.8t}$, where t is the number of hours after drinking a cup of coffee.

(i) Calculate the value of C when $t = 1.5$. [1]

(ii) Find the value of t for which C has a stationary value. [3]

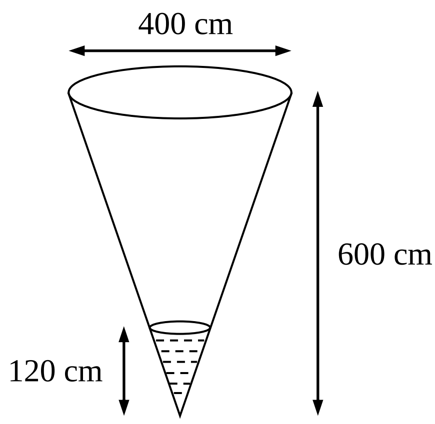
(iii) Find this value of C and determine whether it is a maximum or a minimum. [3]

[*2008 Victoria School AMaths P1*]

65 The equation of a curve is $y = \dfrac{3x}{\sqrt{1+x}}$. Given that the equation of the tangent to the curve at the point $x = 3$ is $15x - 16y = k$, find the value of k. [6]

[*2008 Zhonghua Sec AMaths P1*]

66 The diagram shows an inverted conical tank of height 600 cm. The diameter at the top of the tank is 400 cm. Water is leaking out the tank at a rate of 10 000 cm ^{3} per minute. At the same time, water is pumped into the tank at a constant rate. The water level is rising at a rate of 3 cm per minute when the height of the water is 120 cm.



(i) Show that, when the depth of water in the tank is h cm, the volume of the water in the tank is $\dfrac{\pi h^3}{27}$ cm ^{3}. [3]

(ii) Find the rate at which water is being pumped into the tank. [4]

[*2008 Victoria School AMaths P2*]

67 It is given that $y = (x-1)\sqrt{4x+3}$.

(i) Calculate $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in the form $\dfrac{px+q}{\sqrt{4x+3}}$ where p and q are integers. [2]

(ii) Given that y is changing at the rate of 2.5 units per second when $x = 3$, find the rate of change of x at this instant. [3]

[*2008 Zhonghua Sec AMaths P1*]

68 **(a)** Given that $y = \dfrac{\sin x}{2 - \cos x}$, find the value of x between 0 and 2π for which y is stationary. [4]

(b) A rectangular block with a square base of side $2x$ metres and height h metres, has a total surface area of 6 m^2.

(i) Obtain an expression for h in terms of x. [2]

(ii) Show that the volume, $V\text{ m}^3$, of the block is given by $V = 3x - 4x^3$. [2]

(iii) Calculate the stationary value of V and determine whether it is a maximum or a minimum. [2]

[*2008 Zhonghua Sec AMaths P2*]

69 A curve has the equation $y = \ln(\sin 2x)$ for $0 < x < \dfrac{\pi}{2}$.

(i) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [1]

(ii) Find the x-coordinate of the stationary point of the curve for which $0 < x < \dfrac{\pi}{2}$. [2]

(iii) Determine the nature of this stationary point. [3]

[2009 DHS M2 EOY Q9]

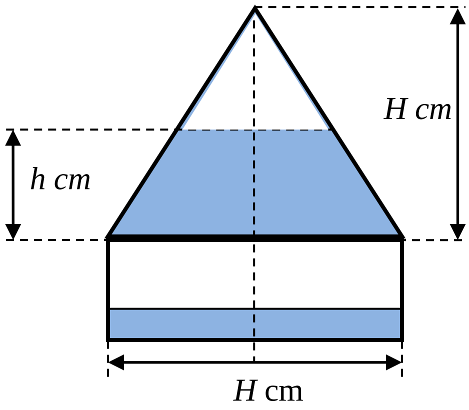
70 A closed circular cylinder of radius r cm and height h cm, has an internal surface area of 320π cm^2 and a volume of V cm^3.

(a) Show that $V = \pi r^2 \left(\frac{160}{r} - r \right)$. [3]

(b) Given that r can vary, find the value of r for which V has a stationary value and determine whether this stationary value is a maximum or a minimum. [4]

[2009 DHS M2 EOY Q10]

71 The diagram shows a vertical cross-section of a vessel in the form of a right circular cone sitting on a circular cylindrical container. The height, H cm, of the conical vessel is equal to its base diameter. Initially, the conical vessel is completely filled with water and the cylindrical container is empty. A small opening at the base of the vessel allows water to leak into the container at a rate of 5 cm^3 s^{-1}.



(i) Given that r cm is the radius as shown in the diagram, show that $r = \frac{1}{2}(H - h)$. [1]

(ii) If V cm^3 is the volume of water remaining in the conical vessel and V_o cm^3 is the initial volume of the water in the vessel, show that $V = V_o - \frac{\pi}{12}(H - h)^3$, where h cm is the depth of the water remaining in the conical vessel. [2]

[Volume of cone = $\frac{1}{3}\pi x^2 y$, where x and y is the radius and height of the cone respectively]

(iii) **(a)** At the instant when the volume of water collected in the cylindrical container is 200 cm^3, show that $H - h = \left(\frac{2400}{\pi} \right)^{\frac{1}{3}}$. [2]

(b) **Hence**, calculate the rate of change of h at this instant. [4]

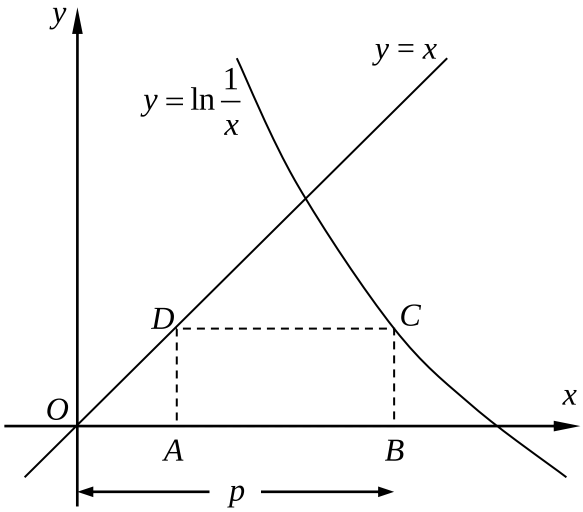
[2009 DHS M2 EOY Q14]

72 **(a)** The equation of a curve is given by $y = \frac{ax^3 - 7x^2 + 5}{x}$, where a is a positive constant. When $x = a$, the gradient of the curve is 1. Find the value of a. [5]

(b) A metal plate expands as it is heated. The surface area, A cm^2, of the metal plate when its temperature is θ °C, is given by $A = 3(2\theta + 6)^{\frac{1}{4}}$. Given that the temperature of the plate is increasing at a constant rate of 3°C / s, find the rate of change of its surface area when the temperature is 5°C. [4]

[2010 DHS M2 EOY Q6 & 7]

73 In the diagram, rectangle *ABCD* is bounded by the lines $y = x$, $y = \ln\frac{1}{x}$, where $x > 0$, and the *x*-axis. Vertices *A* and *B* lie on the *x*-axis, vertex *C* lies on the curve $y = \ln\frac{1}{x}$, and vertex *D* lies on the line $y = x$.



Given that the distance of *OB* is *p* units , show that

(i) the length of *AB* is given by $\left[p - \ln\left(\frac{1}{p}\right)\right]$ units, [2]

(ii) the area, *S* units ^{2}, of rectangle *ABCD* is given by $S = -\left(p + \ln p\right)\ln p$. [2]

Given that *p* can vary,

(iii) show that the root of the equation $p + \ln p^{p+2} = 0$ gives the stationary value of *S*, [4]

(iv) **(*BQ*)** determine whether the stationary value of *S* is a maximum or a minimum. [4]

[2010 DHS M2 EOY Q15]

74 A curve has the equation $y = 2x^2 - \frac{4}{x}$, where $x \neq 0$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [1]

(ii) Find the equation of the normal to the curve at the point *P* where $x = -2$. [4]

A point $(x\ ,\ y)$ moves along the curve in such a way that the *y*-coordinate decreases at a rate of 6.5 units per second.

(iii) Find the rate of change of the *x*-coordinate as the point passes through *P*. [3]

[2011 DHS M2 EOY Q2]

75 The equation of a curve is $y = \frac{2\cos 2x}{\mathrm{e}^x}$ where $0 < x \leq \frac{\pi}{2}$.

(i) Obtain an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(ii) Show that the *x*-coordinate of the stationary point of the curve is approximately 1.33897. [3]

(iii) By considering the sign of the first derivative, determine the nature of this stationary point. [2]

[2011 DHS M2 EOY Q9]

Answers :

1 **(a)** $-0.015\ \text{N s}^{-1}$ **(c)** $3y + \mathrm{e}x = \dfrac{3}{\mathrm{e}} - \mathrm{e}$

2 1681 ; maximum

3 **(i)** $30x\tan^2(5x^2+1)\sec^2(5x^2+1)$ **(ii)** $\dfrac{-1-x}{(x^2+1)^{\frac{3}{2}}}$

4 **(i)** $h = 80 - \dfrac{10}{3}r$ **(iii)** $r = 16$; $V = 7150$; maximum

5 **(i)** $-\dfrac{1}{50}$

6 $y = -\dfrac{1}{2}x + 9$; $x = \dfrac{11}{4}$

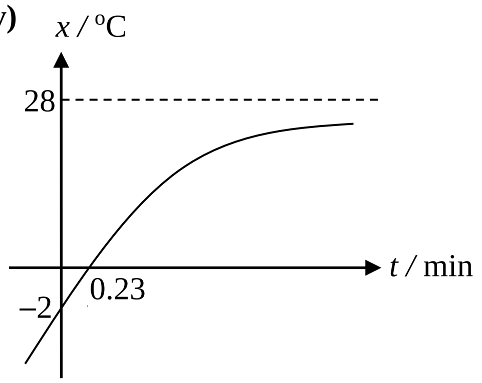
7 **(b)** $x = 3$; minimum

8 $-e^{-2x+1}$; -1 unit s^{-1}

9 **(b)** $x = 19.4$; minimum

11 **(i)** $-2°$ C **(ii)** 0.230 min **(iii)** $2°$ C per min

11 **(iv)**



12 **(i)** $2y + x = 1$ **(ii)** (4.5 , 2.08)

13 **(i)** $h = 8 - \dfrac{2}{3}r$ **(ii)** $r = 8$ **(iii)** 536 cm^3 ; max value

14 **(i)** $\mathrm{e}^{3x}(3x+1)$ **(ii)** $\left(-\dfrac{1}{3}\ ,\ -0.123\right)$ **(iii)** min pt

15 0.1 units per second

16 $x = 1$; stat pt of inflexion

17 1

18 **(a)** $l = \dfrac{4\sqrt{3}}{r}$ **(b)(i)** $\left(\dfrac{16-r^4}{\sqrt{48-r^4}}\right)\pi$ **(ii)** $r = 2$

19 **(a)(i)** $6\tan^2 2x\sec^2 2x$ **(ii)** $\dfrac{-5}{(1-2x)(x+2)}$

20 **(i)** $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2(x-4)}{\sqrt{(2x-1)^3}}$ **(ii)** $81x + 6y - 437 = 0$

21 $V = x^3$; $A = 6x^2$; $\dfrac{\mathrm{d}V}{\mathrm{d}x} = 3x^2$; $\dfrac{\mathrm{d}A}{\mathrm{d}x} = 12x$ **(i)** 0.005 cm/s **(ii)** 0.6 cm^2/s

22 **(i)** $t = 0.924$ **(ii)** $T = 24.8$ **(iii)** $\dfrac{\mathrm{d}T}{\mathrm{d}t} = -0.00148$ °C/min

$T_h = 40°C$; $T_A = 24°C$

23	**(a)**	$\frac{dy}{dx} = -3e^{\tan(1-3x)}\sec^2(1-3x)$; decreasing function	**(b)**	$\frac{dy}{dx} = \frac{x^2-2x-8}{(x^2+1)(x-1)}$; $y = 2.08$				
24	**(a)(i)**	$\frac{dy}{dx} = 1 - \frac{8}{x^3}$	**(ii)**	$k = 3$	**(b)**	0.0175 units/s		
25		$\frac{dy}{dx} = \frac{2}{(2-3x)^2}$; $2y = -x - 1$						
26	**(a)**	$\frac{dy}{dx} = 18 - 2x^2$; $\frac{d^2y}{dx^2} = -4x$			**(b)**	(3, 36) max ; (–3, –36) min		
27	**(a)**	(2 , 0) ; $\left(\frac{1}{2}, 1\frac{11}{16}\right)$	**(b)**	$\frac{dy}{dx} = -\frac{2}{(x-2)^2}$				
28	**(ii)**	0.4 cm/s	**(iii)**	$r = 10$				
29	**(i)**	$h = \frac{500}{r^2}$	**(iii)**	$r = 5$; 2827 cents				
30	**(i)**	$(\sqrt{6}, 0), (-\sqrt{6}, 0)$	**(ii)**	$(-2, -2e^4), (3, \frac{3}{e^6})$	**(iii)**	min pt, max pt		
31	**(i)**	$\tan x + x\sec^2 x$	**(ii)**	$y = \left(1 + \frac{\pi}{2}\right)x - \frac{\pi^2}{8}$	**(iii)**	$-(\pi + 2)$ units per second		
32	**(i)**	0.116	**(ii)**	3rd year	**(iii)**	\$ 2832.79 per year		
33	**(i)**	$\frac{1}{2}$ m/s	**(ii)**	$-\frac{1}{30}$ m/s	**(iii)**	$-\frac{13}{300}$ m/s^2	**(iv)**	0.0472 m
34	**(a)**	$y = \frac{100}{x^2}$	**(i)**	4.84 cm	**(ii)**	111 cm^2, minimum	**(b)**	4.91 cm/s
35	**(ii)**	$2y = 8x + 93$	**(iii)**	– 0.5 units per second				
36	**(a)(i)**	$-15\sin(5x+3)\cos^2(5x+3)$	**(ii)**	$3x^2\left(3\ln(1-x) - \frac{x}{1-x}\right)$	**(b)**	$x = \frac{3}{2}$ or $-\frac{2}{3}$		
37		$h = \sqrt{3}\,x$						
38	**(i)**	$m = -3$	**(ii)**	maximum point				
39	**(a)**	4	**(b)**	$y = -\frac{1}{4}x + 1$				
40	**(b)**	8 cm ; 576 cm^2						
41	**(a)**	$x = 0$	**(b)**	stationary point of inflexion				
42	**(i)**	–18 m/s^2	**(ii)**	$t = 2$ or 4	**(iii)**	6 m/s		
43		$r = 10$; $h = 20$						
44	**(i)**	$\frac{14}{(3x+2)^2}$	**(ii)**	$\frac{1}{14}$	**(iii)**	1.28 units per second		
45		$x = 1.11$; Maximum						
46	**(i)**	$\frac{-9}{2(2x+1)(x-4)}$	**(ii)**	$e^{\sin^2 x}\sin 2x$				
47		$x = -7c$						
48	**(i)**	$\frac{dy}{dx} = \frac{a-6}{(3x+1)^2}$; $a > 0$			**(ii)**	$a = 1$		
49	**(i)**	–9 m/s	**(ii)**	$s = 8$	**(iii)**	–18 m/s^2		
50	**(i)**	42π cm^3/s	**(ii)**	2.1 cm/s				

51	**(i)**	2.08 mins	**(ii)**	514 per min						
52	**(i)**	$\dfrac{dy}{dx} = 1 - \sec x$								
53	**(i)**	$-2\sin\left(2x+\dfrac{\pi}{2}\right)$	**(ii)**	$x = \frac{1}{3}\pi$	**(b)(ii)**	$x = \sqrt{3}$; maximum				
54	**(i)**	$\left(\dfrac{b}{a}\ ,\ c-\dfrac{b^2}{a}\right)$; min	**(ii)**	$c = \dfrac{b(1+b)}{a}$	**(iii)**	$a = 1$; $b = 2$				
55	**(i)**	$h = \dfrac{V}{\pi r^2} - \dfrac{4r}{3}$	**(ii)(b)**	$r = \sqrt[3]{\dfrac{3v}{16\pi}}$	**(c)**	1 : 4				
56	**(i)**	$x = \pm\dfrac{1}{\sqrt{2}}$	**(ii)(a)**	$-\dfrac{1}{2x}\sin\left(\ln\sqrt{2x}\right)$	**(b)**	$-\tan 2x$				
57	**(a)**	$-6\sin(3x-5)\cos(3x-5)-2\sec^2 2x$			**(b)**	$\dfrac{4}{x(4+\ln x)^2}$				
58		$\dfrac{5}{\pi}$ cm s $^{-1}$								
59	**(i)**	18 m/s ^{2}	**(ii)**	$\dfrac{8}{9}$ m/s	**(iii)**	5 m	**(iv)**	4.25 m		
60	**(i)**	$y = \dfrac{48-x^2}{2x}$	**(iii)**	$x = 4$	**(iv)**	max $V = 64\pi$				
61		21.8° (or 0.381 rad)								
62	**(a)(i)**	$h = 3$	**(ii)**	$\dfrac{15}{19}$ cm s $^{-1}$	**(b)(i)**	$\dfrac{3\pi}{5}$ cm ^{2} s $^{-1}$	**(ii)**	$\dfrac{\sqrt{3}}{4}$ cm s $^{-1}$		
63	**(i)**	$\dfrac{1-x\ln 3x}{xe^x}$	**(ii)**	$\dfrac{3}{2}\left(x-\dfrac{2}{x}\right)\sqrt{\dfrac{1}{2}x^2+7+\dfrac{2}{x}}$						
64	**(i)**	0.542	**(ii)**	1.25	**(iii)**	0.552 ; Maximum				
65		–27								
66	**(ii)**	25100 cm ^{3}/min								
67	**(i)**	$\dfrac{6x+1}{\sqrt{4x+3}}$	**(ii)**	0.510 units per second						
68	**(a)**	$\dfrac{\pi}{3}$ or $\dfrac{5\pi}{3}$	**(b)(i)**	$h = \dfrac{3-4x^2}{4x}$	**(ii)**	1 m ^{3} ; maximum				
69	**(i)**	$\dfrac{dy}{dx} = 2\cot 2x$	**(ii)**	$x = \dfrac{\pi}{4}$	**(iii)**	Max				
70	**(b)**	$r = \sqrt{\dfrac{160}{3}}$ **;** Max V								
71	**(iii)(b)**	– 0.0762 cm s $^{-1}$								
72	**(a)**	2.13	**(b)**	$\dfrac{9}{16}$ cm ^{2} / s						
73	**(iv)**	**(BQ)** Maximum								
74	**(i)**	$\dfrac{dy}{dx} = 4x + \dfrac{4}{x^2}$	**(ii)**	$7y = x + 72$	**(iii)**	0.929 unit per second				
75	**(i)**	$\dfrac{dy}{dx} = -\dfrac{2(2\sin 2x + \cos 2x)}{e^x}$			**(iii)**	Minimum				