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MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Advanced Level
Higher 3

CANDIDATE NAME

CENTRE NUMBER S

INDEX NUMBER

PHYSICS

9814/01

Paper 1

October/November 2022

3 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams, graphs or rough working.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

The use of an approved scientific calculator is expected, where appropriate.

Section A
Answer **all** questions.
You are advised to spend about 1 hour and 50 minutes on Section A.

Section B
Answer **two** questions only.
You are advised to spend about 35 minutes on each question in Section B.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **35** printed pages and **5** blank pages.

Singapore Examinations and Assessment Board

Cambridge Assessment International Education

DC (LK/CGW) 303346/5

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Data

speed of light in free space	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
permeability of free space	$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
permittivity of free space	$\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
	$(1/(36\pi)) \times 10^{-9}\,\mathrm{F\,m^{-1}}$
elementary charge	$e = 1.60 \times 10^{-19}\,\mathrm{C}$
the Planck constant	$h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$
unified atomic mass constant	$u = 1.66 \times 10^{-27}\,\mathrm{kg}$
rest mass of electron	$m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$
rest mass of proton	$m_p = 1.67 \times 10^{-27}\,\mathrm{kg}$
molar gas constant	$R = 8.31\,\mathrm{J\,K^{-1}\,mol^{-1}}$
the Avogadro constant	$N_A = 6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
the Boltzmann constant	$k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
gravitational constant	$G = 6.67 \times 10^{-11}\,\mathrm{N\,m^{2}\,kg^{-2}}$
acceleration of free fall	$g = 9.81\,\mathrm{m\,s^{-2}}$

Formulae

uniformly accelerated motion	$s = ut + \frac{1}{2}at^2$
	$v^2 = u^2 + 2as$
moment of inertia of rod through one end	$I = \frac{1}{3}ML^2$
moment of inertia of hollow cylinder through axis	$I = \frac{1}{2}M\left(r_1^{\,2} + r_2^{\,2}\right)$
moment of inertia of solid sphere through centre	$I = \frac{2}{5}MR^2$
moment of inertia of hollow sphere through centre	$I = \frac{2}{3}MR^2$
work done on/by a gas	$W = p\Delta V$
hydrostatic pressure	$p = \rho g h$
gravitational potential	$\phi = -Gm/r$

DO NOT WRITE IN THIS MARGIN

Kepler's third law of planetary motion	T^2	=	$\dfrac{4\pi^2 a^3}{GM}$
temperature	T/K	=	T/°C + 273.15
pressure of an ideal gas	p	=	$\dfrac{1}{3}\dfrac{Nm}{V}\langle c^2 \rangle$
mean translational kinetic energy of an ideal gas molecule	E	=	$\frac{3}{2}kT$
displacement of particle in s.h.m.	x	=	$x_0 \sin \omega t$
velocity of particle in s.h.m.	v	=	$v_0 \cos \omega t$
		=	$\pm\omega\sqrt{(x_0^{\,2} - x^2)}$
electric current	I	=	$Anvq$
resistors in series	R	=	$R_1 + R_2 + \ldots$
resistors in parallel	$1/R$	=	$1/R_1 + 1/R_2 + \ldots$
capacitors in series	$1/C$	=	$1/C_1 + 1/C_2 + \ldots$
capacitors in parallel	C	=	$C_1 + C_2 + \ldots$
energy in a capacitor	U	=	$\frac{1}{2}CV^2$
electric potential	V	=	$\dfrac{Q}{4\pi\varepsilon_0 r}$
electric field strength due to a long straight wire	E	=	$\dfrac{\lambda}{2\pi\varepsilon_0 r}$
electric field strength due to a large sheet	E	=	$\dfrac{\sigma}{2\varepsilon_0}$
alternating current/voltage	x	=	$x_0 \sin \omega t$
magnetic flux density due to a long straight wire	B	=	$\dfrac{\mu_0 I}{2\pi d}$
magnetic flux density due to a flat circular coil	B	=	$\dfrac{\mu_0 N I}{2r}$
magnetic flux density due to a long solenoid	B	=	$\mu_0 n I$
energy in an inductor	U	=	$\frac{1}{2}LI^2$
RL series circuits	τ	=	$\dfrac{L}{R}$
RLC series circuits (underdamped)	ω	=	$\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}$
radioactive decay	x	=	$x_0 \exp(-\lambda t)$
decay constant	λ	=	$\dfrac{\ln 2}{t_{\frac{1}{2}}}$

Section A

Answer **all** questions in this section.

You are advised to spend about 1 hour 50 minutes on this section.

1 A uniform rod **AB** has mass M and length L.

The rod can rotate freely about a hinge at **A**.

A student holds the rod at **B** so that the rod makes an angle θ to the vertical, as shown in Fig. 1.1.

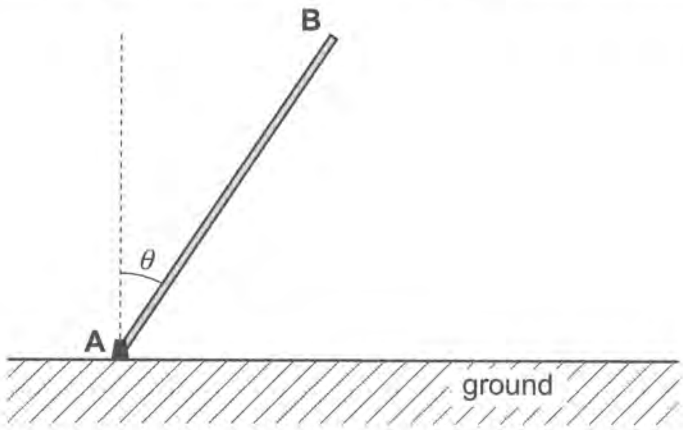

Fig. 1.1

The student releases the rod.

(a) (i) Show that the initial angular acceleration of the rod α is given by:

$$\alpha = k\frac{g\sin\theta}{L}$$

where k is a constant and g is the acceleration of free fall.

[2]

(ii) State the value of k.

k = ... [1]

(b) Determine an expression for the angular velocity of the rod just before it hits the ground in terms of g, L and θ.

[3]

[Total: 6]

2 A coaxial cable is used for the transmission of high frequency electrical signals such as television signals.

Fig. 2.1 shows the typical construction of a coaxial cable and the radii of the different layers.

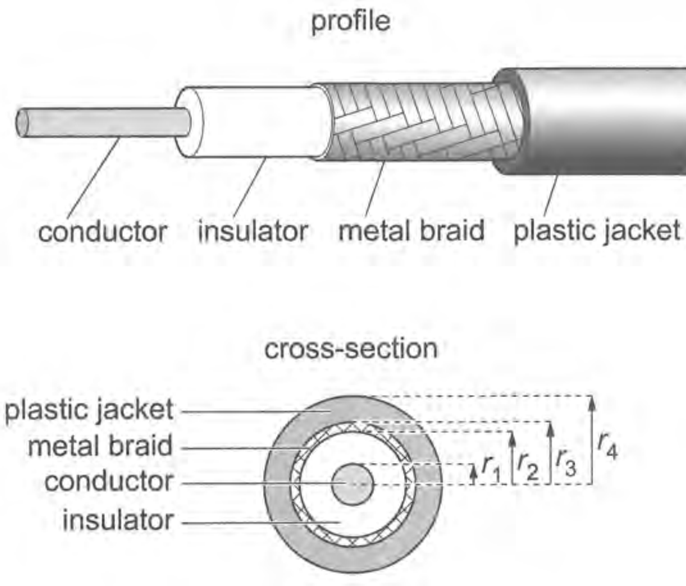

Fig. 2.1

When there is a current I in the conductor in one direction, there is a current I in the metal braid in the opposite direction.

(a) State Ampère's Law in integral form, defining all terms.

....

....

....

.... [2]

(b) The current per unit cross-sectional area is a quantity known as the current density J.

The current density in the conductor is J_1 and the current density in the metal braid is J_2.

Assume that J_1 and J_2 are each constant.

(i) Determine an expression for J_2 in terms of J_1 and the radii shown in Fig. 2.1.

[2]

(ii) Derive an expression for the magnitude of the magnetic flux density B in terms of the current density J_1 at:

1. the conductor-insulator boundary

[1]

2. the insulator-metal braid boundary.

[1]

(iii) Use Fig. 2.2 to sketch how the magnetic flux density B varies between zero and a maximum value B_{max} with distance x from the centre of the coaxial cable.

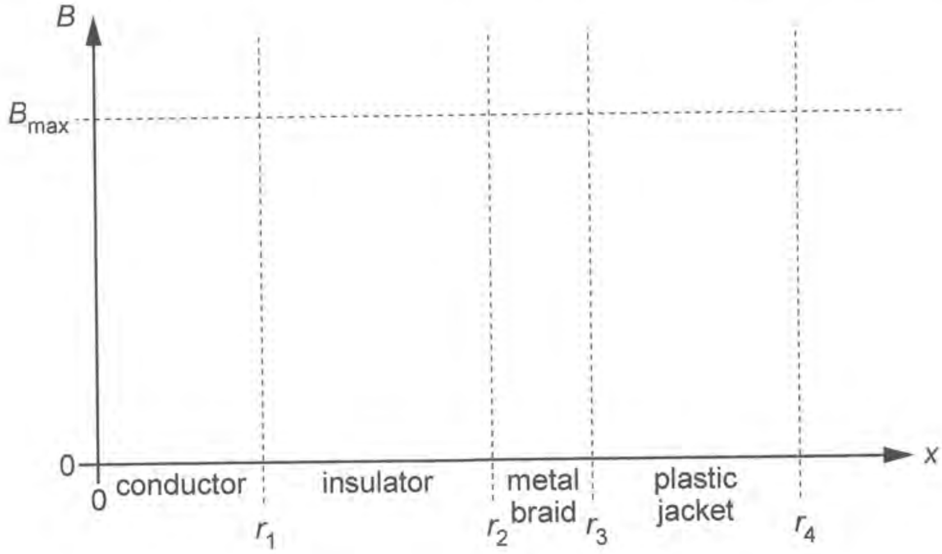

Fig. 2.2

[4]

(c) Standard electrical transmission cables are made of two insulated wires, as shown in Fig. 2.3.

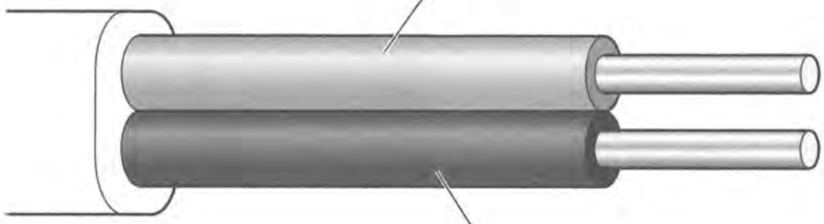

Fig. 2.3

Standard electrical transmission cables are less expensive than coaxial cables.

Suggest why standard electrical transmission cables are **not** used for high frequency electrical signals.

....

....

.... [1]

[Total: 11]

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3 The range of a projectile is the horizontal distance the projectile travels.

(a) Fig. 3.1 shows the range R of a projectile launched from horizontal ground with an initial speed u at an angle θ above the horizontal.

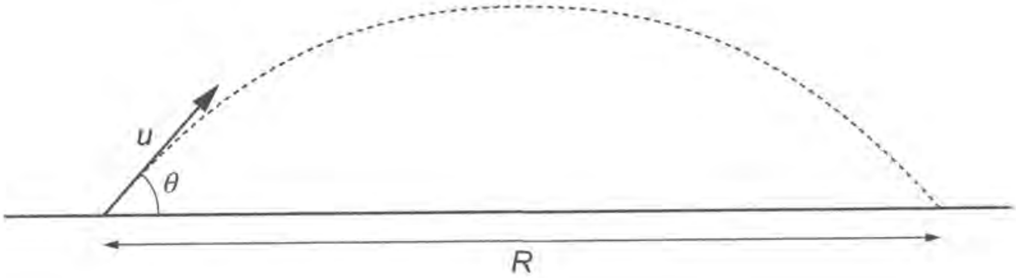

Fig. 3.1

(i) Show that:

$$R = \frac{u^2}{g} \sin 2\theta$$

where g is the acceleration of free fall.

Ignore the effects of air resistance.

Hint:
You may find it helpful to use the trigonometric identity: $\sin 2x = 2 \sin x \cos x$.

[3]

(ii) Show that the angle which gives the maximum range, θ_{max}, is 45°.

[1]

(iii) The initial speed of the projectile $u = 150\,\mathrm{m\,s^{-1}}$.

Calculate the maximum range of the projectile R_{max}.

R_{max} = ... km [1]

(b) Fig. 3.2 shows the range R of the projectile launched from a slope with an initial speed u at an angle θ above the horizontal.

The slope is of constant gradient and makes an angle ϕ with the horizontal.

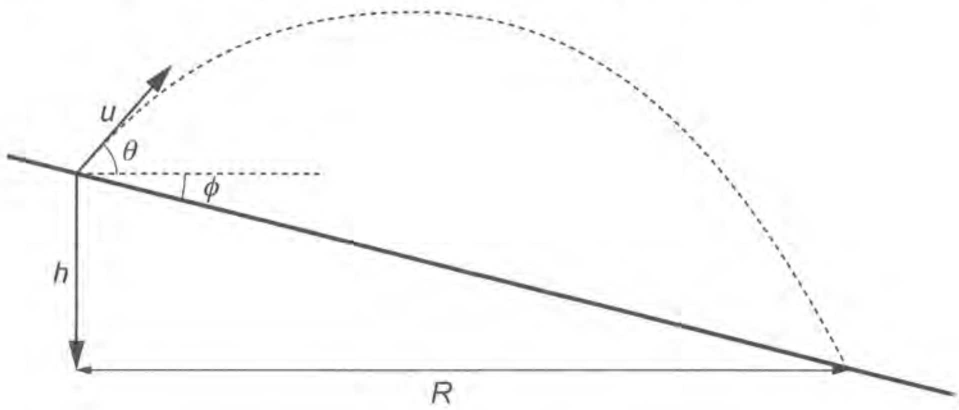

Fig. 3.2

(i) State an expression for the time-of-flight t of the projectile in terms of R, u and θ.

[1]

(ii) State an expression for the vertical displacement h of the projectile in terms of R and ϕ.

[1]

(iii) By using your expressions in **(b)(i)** and **(b)(ii)** or otherwise, show that the general range equation takes the form:

$$R = \frac{u^2}{g}\,[\sin 2\theta + (1 + \cos 2\theta)\tan\phi].$$

Hint:
You may find it helpful to use the trigonometric identity: $2\cos^2 x = 1 + \cos 2x$.

[3]

(iv) Show that the equation in **(a)(i)** is a special case of the general range equation given in **(b)(iii)**.

[1]

(v) For a given value of ϕ, determine the relationship between θ and ϕ that maximises the range.

Hint:
You may find it helpful to use one of the trigonometric identities:

$a\sin x + b\cos x = \sqrt{a^2 + b^2}\sin\left(x + \tan^{-1}\frac{b}{a}\right)$

$\frac{1}{\tan x} = \tan(\frac{\pi}{2} - x)$.

[3]

[Total: 14]

4 A student connects two identical light springs of spring constant k and original length l_0 in series with a mass M, as shown in Fig. 4.1.

When the system is in equilibrium, the length of each extended spring is $\frac{L}{2}$, and the total length of the system is L.

The student then cuts two pieces of light inextensible string, each of length $\frac{L}{2}$, and attaches them to the mass and springs, as shown in Fig. 4.2.

The student holds mass M so that the system has the same total length L as the system shown in Fig. 4.1.

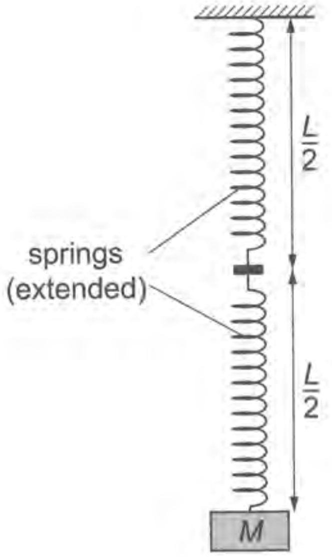

Fig. 4.1

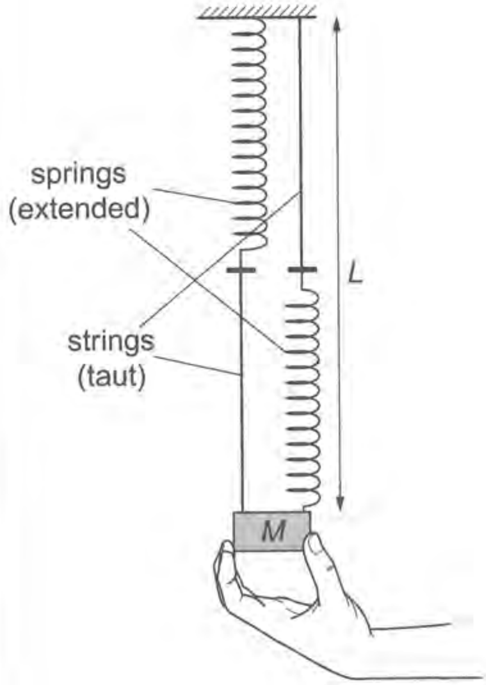

Fig. 4.2

(a) (i) The student releases mass M in Fig. 4.2.

State whether mass M initially moves up or down. Explain your answer.

mass M moves ....

explanation ....

....

....

.... [2]

(ii) The system in Fig. 4.2 is allowed to reach its equilibrium position. The total length of the system is L_1.

Show that:

$$L_1 = 2l_0 + \frac{3Mg}{2k}$$

where g is the acceleration of free fall.

[2]

(b) The student changes the mass of the system in Fig. 4.2 to $\frac{5}{2}M$.

At its new equilibrium position, the total length of the system becomes $L_2 = \frac{17}{14}L_1$.

(i) Show that the spring constant k is given by:

$$k = \frac{Mg}{l_0}.$$

[3]

(ii) Hence determine L_1 in terms of L.

[2]

[Total: 9]

5 In 1798, Henry Cavendish performed an experiment to determine a value for the average density of the Earth.

The experiment can also be used to determine G, the gravitational constant.

Cavendish carried out the measurements using the method and torsion balance apparatus devised by John Michell in 1783.

Fig. 5.1 shows the torsion balance in its equilibrium position.

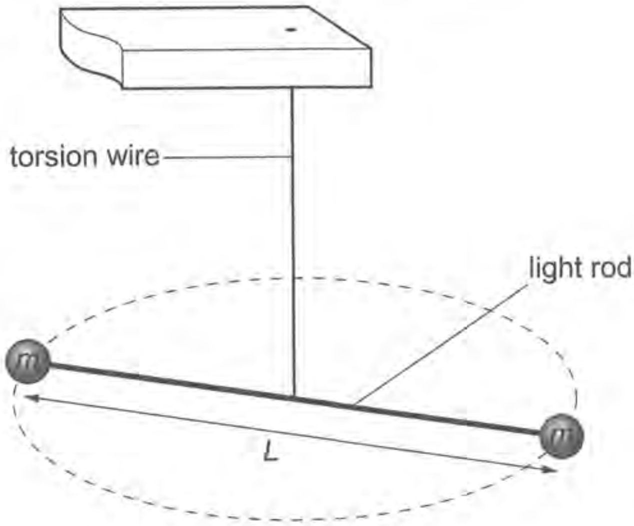

Fig. 5.1 (not to scale)

A stiff torsion wire was used to suspend a light horizontal rod from its midpoint.

Small lead spheres of mass m = 0.730 kg and diameter d = 50 mm were attached to the ends of the light rod.

The centre-to-centre separation of the spheres was L = 1.80 m.

(a) **(i)** The rod was turned by a small angle θ from its equilibrium position in a horizontal plane, as shown in Fig. 5.2.

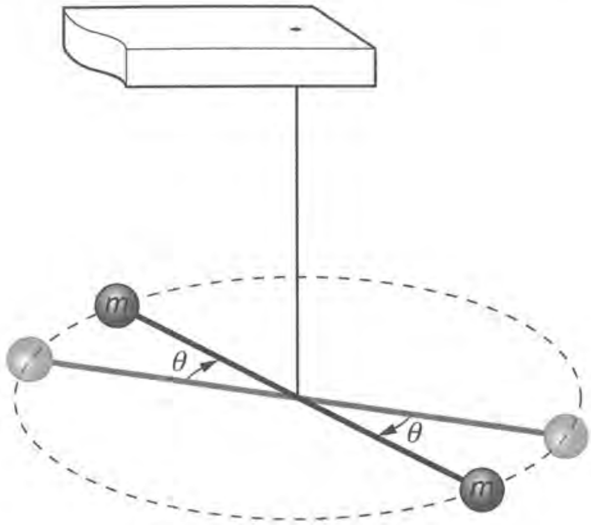

Fig. 5.2 (not to scale)

The torque τ exerted on the rod by the torsion wire was given by the angular form of Hooke's law:

$$\tau = \kappa\theta$$

where κ is the torsion coefficient of the torsion wire.

When released, the rod oscillated with simple harmonic motion about the equilibrium position with a period T.

Show that the torsion coefficient κ is related to the period T by the equation:

$$\kappa = \frac{2\pi^2 mL^2}{T^2}.$$

[4]

(ii) The rod was returned to its equilibrium position.

Next, Cavendish placed large lead spheres of mass M = 158 kg and diameter D = 30.0 cm near the small lead spheres.

A gravitational attraction force F between each pair of spheres caused the rod to rotate through an angle θ_1, as shown in Fig. 5.3.

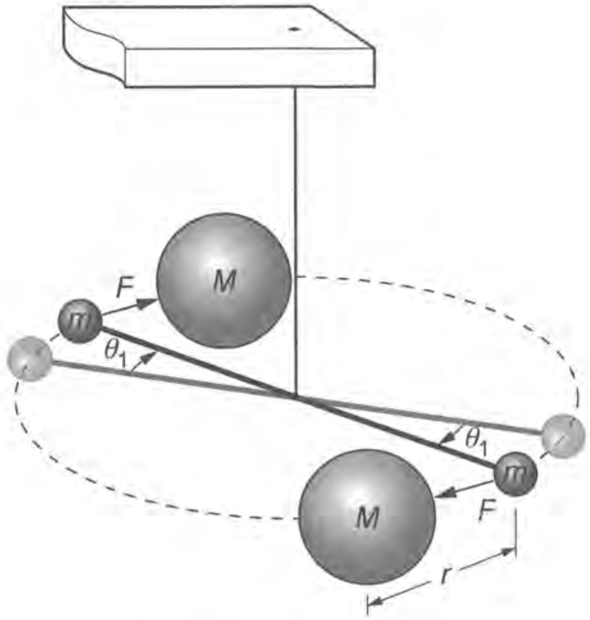

Fig. 5.3 (not to scale)

At this angle, the torque caused by the gravitational attraction was equal to the opposing torque caused by the torsion in the wire.

The centre-to-centre separation of one large lead sphere and the small lead sphere next to it was r.

The air gap between each large lead sphere and the small lead sphere next to it was 1.0 mm.

Determine the value and unit of r.

r = ... unit .... [1]

(iii) The angle of rotation θ_1 was too small to be measured directly.

Using a vernier scale, Cavendish was able to determine the displacement of the smaller lead spheres to be 4.1 mm ± 0.1 mm.

The centre-to-centre separation of the two small lead spheres was measured to be 1.80 m ± 0.01 m.

Determine the angle θ_1 and its corresponding uncertainty.

θ_1 = ... ± .... rad [3]

(iv) The large lead spheres were then removed, and the system oscillated with simple harmonic motion as before.

Show that the gravitational constant G is related to the period T by the relationship:

$$G = \frac{2\pi^2 L r^2 \theta_1}{MT^2}.$$

[2]

(b) At an evening lecture, a university professor performs a modern version of the Cavendish experiment. The professor uses the same torsion balance and method but adds a mirror, laser and screen to measure the rotation.

Fig. 5.4 shows the torsion balance in its equilibrium position.

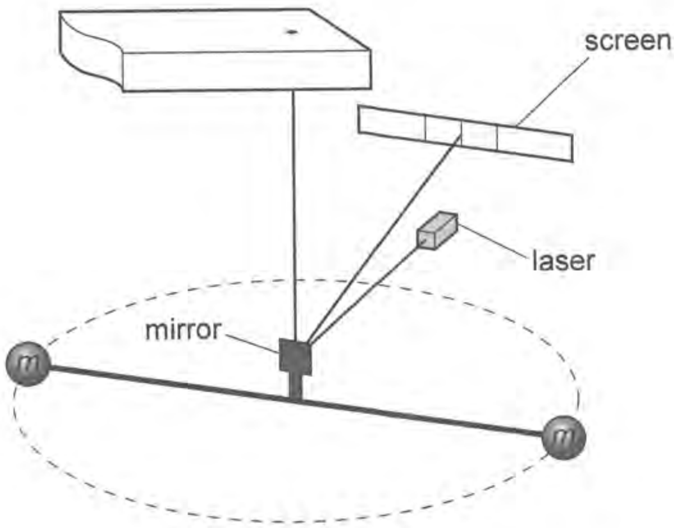

Fig. 5.4 (not to scale)

When the professor brings the large lead spheres near to the small lead spheres, the light rod rotates and the laser beam is deflected, as shown in Fig. 5.5.

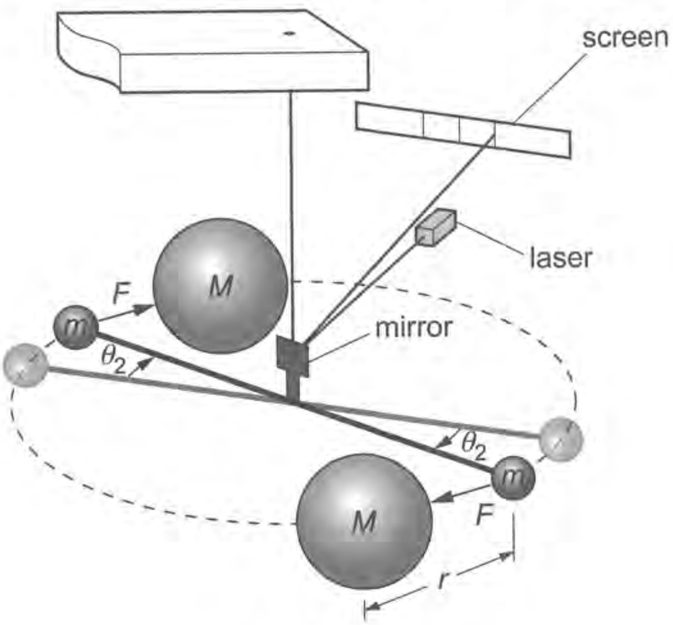

Fig. 5.5 (not to scale)

(i) The mirror is a distance of 12.00 m from the screen. The professor measures the deflection of the laser beam on the screen as 15.6 cm.

Determine the corresponding angle of rotation θ_2 in degrees.

θ_2 = ... ° [2]

(ii) The professor removes the large lead spheres and simultaneously starts a timer.

The professor records a time of 10 minutes 22 seconds for the laser beam to move from the maximum deflection, through the equilibrium position to the opposite maximum deflection, and back to the equilibrium position again.

Use this measurement and the expression in **(a)(iv)** to determine a value for the gravitational constant G.

G = ... $m^3 kg^{-1} s^{-2}$ [4]

(iii) The professor switches the laser off but leaves the system oscillating and the timer running overnight.

The next morning, the professor switches the laser back on and records the time when the laser beam next passes through the equilibrium position, travelling from the right, as t = 13 hours 48 minutes 18 seconds.

Use this value of t to determine a more accurate value for the period T.

T = ... s [3]

(iv) The professor calculates a new value for G using the value for T from **(b)(iii)**.

This new value for G is slightly lower than the accepted value of
$G = 6.674 \times 10^{-11}\,m^3\,kg^{-1}\,s^{-2}$.

Suggest the main reason for this difference.

....

....

....

.... [1]

[Total: 20]

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Section B

Answer **two** questions from this section.

You are advised to spend about 35 minutes on each question.

6 **(a)** State Faraday's law and Lenz's law.

Faraday's law ....

....

Lenz's law ....

.... [2]

(b) A capacitor of capacitance C is fully charged to have a charge Q_0 on one of its plates.

An inductor of inductance L and a switch S are connected to the charged capacitor to form a series circuit, as shown in Fig. 6.1.

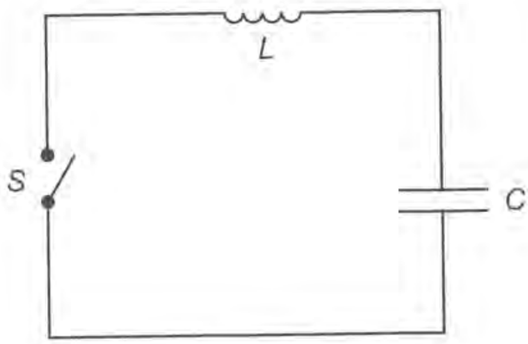

Fig. 6.1

At time $t = 0$, the switch is closed.

(i) By differentiating the total energy in the circuit with respect to time, show that charge Q on one of the capacitor plates obeys the second order differential equation:

$$L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0.$$

[3]

(ii) Hence determine an expression for Q as a function of time.

[2]

(c) Initially, the capacitor in Fig. 6.1 is fully charged with a charge of 1.5 mC. The inductor has an inductance of 4.0 mH, and the capacitor has a capacitance of 50 μF.

At time $t = 0$, the switch is closed.

(i) Calculate the time at which the capacitor is next fully charged.

t = ... s [2]

(ii) On Fig. 6.2, sketch a graph for time $t = 0$ to $t = 6.0$ ms to show the variation of the charge Q on one of the capacitor plates.

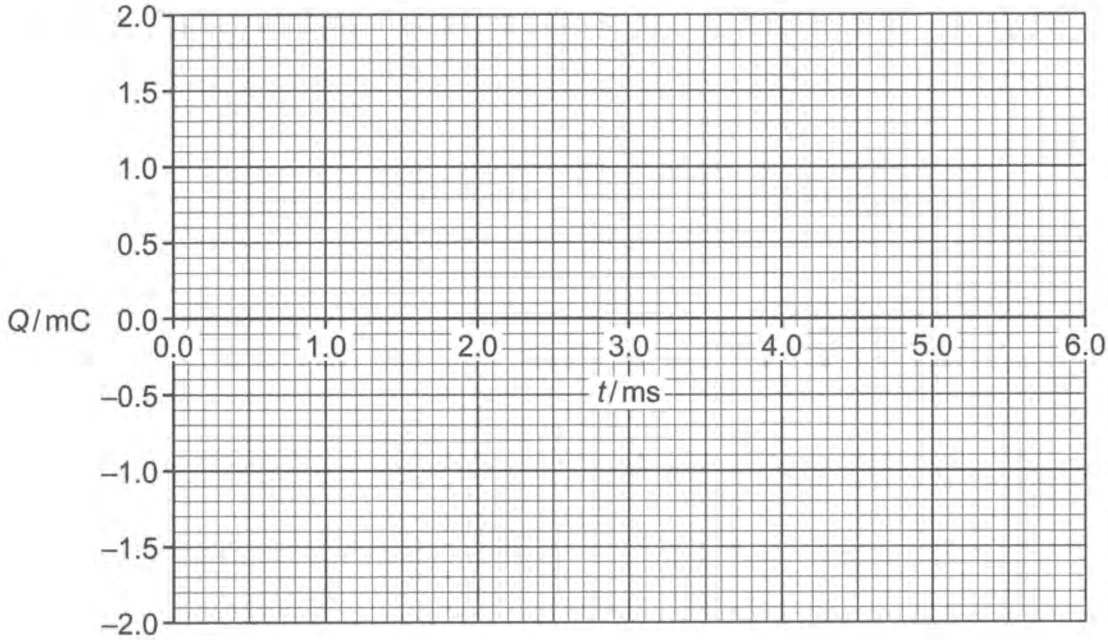

Fig. 6.2

[2]

(iii) Calculate the first time at which the energy stored in the capacitor is equal to the energy stored in the inductor.

t = ... s [2]

(iv) On Fig. 6.3, sketch a graph to show the variation with time of the energy E stored in:

- the capacitor and label this graph E_C
- the inductor and label this graph E_L.

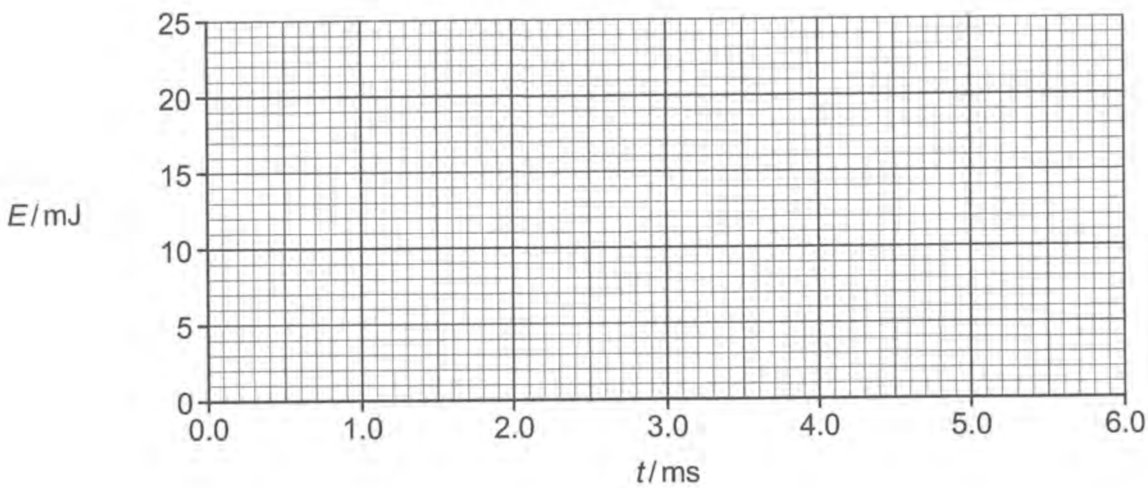

Fig. 6.3

[3]

(v) A resistor is added in series with the inductor and the capacitor.

1. Sketch on Fig. 6.4 how the current I in the resistor varies with time.

   Show at least four cycles of the capacitor discharging and charging. You do **not** need to add values to your graph.

Fig. 6.4

[2]

2. Without calculation, state how the period in Fig. 6.4 compares with the period in Fig. 6.2. Give a reason for your answer.

   ....

   ....

   .... [2]

[Total: 20]

7 Fig. 7.1 shows a side view and top view of a helicopter hovering stationary in mid-air.

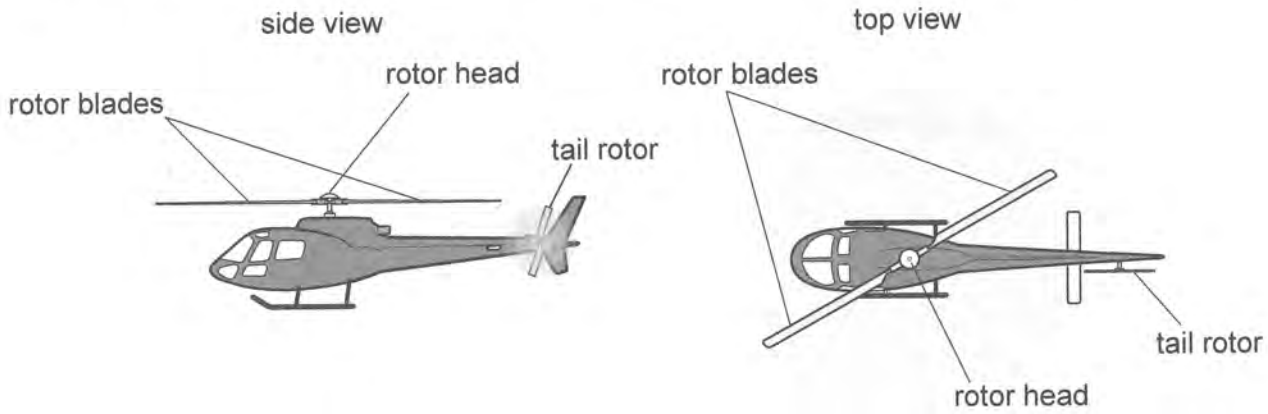

Fig. 7.1

The two rotor blades of the helicopter can be modelled as uniform thin rectangular plates of length R and width c, as shown in Fig. 7.2.

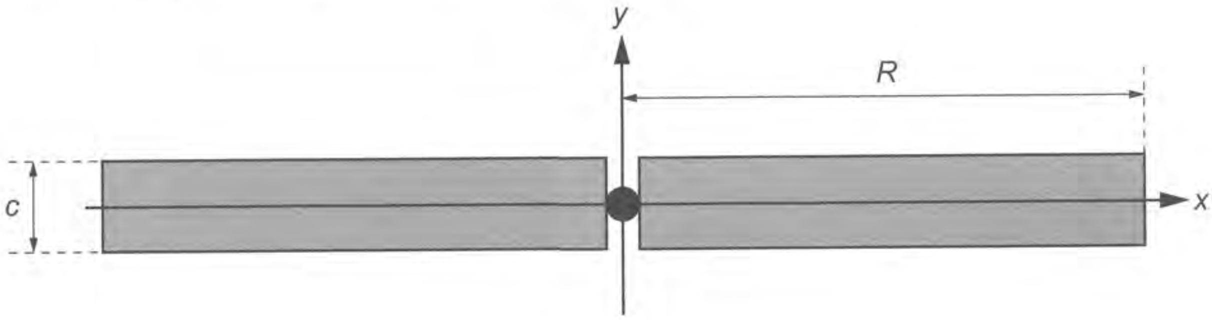

Fig. 7.2 (not to scale)

The gap between the two rotor blades is negligible when compared with the total length of the blades.

(a) **(i)** Show, from first principles, that the moment of inertia I_x of the rotor blades about the x-axis in Fig. 7.2 is given by:

$$I_x = \frac{1}{12}Mc^2$$

where M is the **total** mass of the rotor blades.

[3]

(ii) Similarly, determine the expression for the moment of inertia about the y-axis, I_y.

I_y = ... [1]

(iii) The perpendicular axis theorem states that the moment of inertia about the z-axis for this case is given by:

$$I_z = I_x + I_y.$$

Show that:

$$I_z = \frac{1}{12}M(4R^2 + c^2).$$

[1]

(iv) The rotor blades are made of a composite material of density $\rho_c = 1470\,\text{kg}\,\text{m}^{-3}$. Each blade has length $R = 3.84\,\text{m}$, width $c = 20\,\text{cm}$ and thickness $t = 3.0\,\text{cm}$.

Determine the value of I_z.

I_z = ... $\text{kg}\,\text{m}^2$ [1]

(v) During take-off, it takes 9.0 s for the angular velocity of the rotor blades to increase from 20 rpm (revolutions per minute) to 320 rpm at a constant angular acceleration.

Calculate the torque required. State the unit.

torque = ... unit .... [2]

(b) The total lift force on the helicopter can be calculated by summing the lift on infinitesimal elements of the blade.

Fig. 7.3 shows an element of the rotor blade with width c and length dr at a distance r from the axis of rotation.

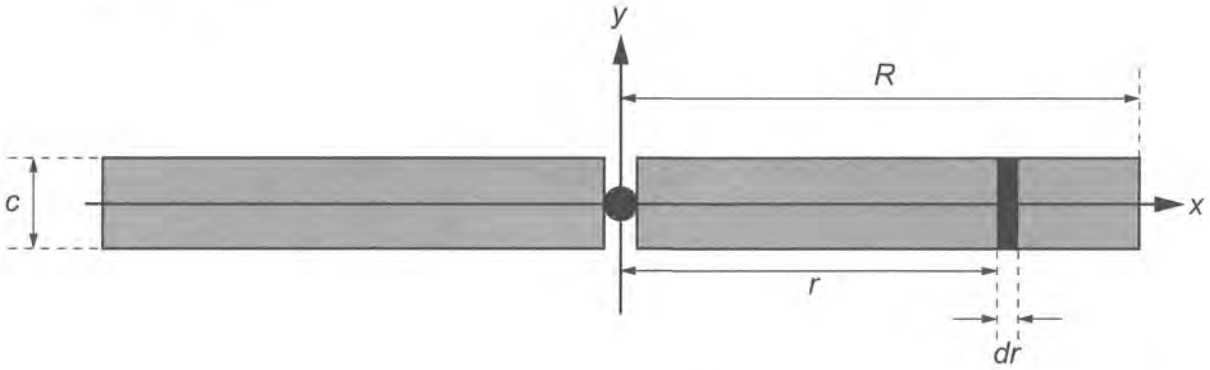

Fig. 7.3 (not to scale)

The lift dL on the blade element is given by the equation:

$$dL = \frac{1}{2} C_L \rho (r\omega)^2 c dr$$

where:

C_L is the coefficient of lift
ρ is the density of air and can be taken to be 1.25 kg m^{-3}
ω is the angular velocity of the rotor blades.

(i) Show that the coefficient of lift C_L is dimensionless.

[1]

(ii) Show that the total lift force L generated by the two rotor blades is given by:

$$L = \frac{1}{3} C_L \rho \omega^2 c R^3.$$

[3]

(c) Coning is an effect where the rotor blades of the helicopter tilt upwards from the horizontal, as shown in Fig. 7.4.

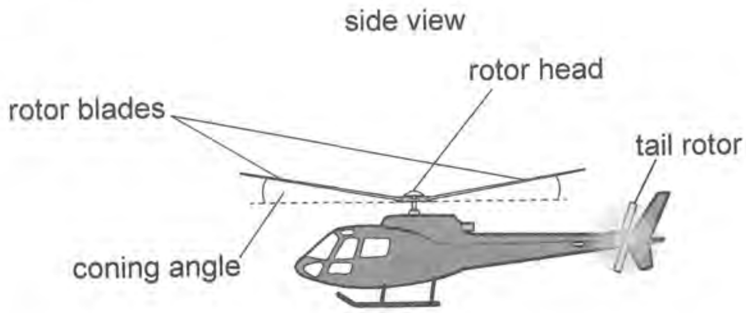

Fig. 7.4

(i) Explain why coning occurs.

....

....

.... [1]

(ii) The total mass of the helicopter, including blades, is 2500 kg.

Calculate the angular velocity ω of the rotor blades, in rpm, required for the helicopter to hover at a constant height above the ground with a coning angle of 10.0°.

Use a typical value for C_L of 0.400.

ω = ... rpm [3]

(d) Fig. 7.1 shows a top view of the helicopter. The rotation of the tail rotor blades pushes air in a direction shown in the top view of Fig. 7.1 as down the page.

Explain:
- why the tail rotor is necessary for stable flight
- whether the main rotor blades rotate clockwise or anticlockwise.

....

....

....

....

....

.... [3]

(e) Fig. 7.5 shows the design of *Ingenuity*, which first flew on the planet Mars in April 2021.

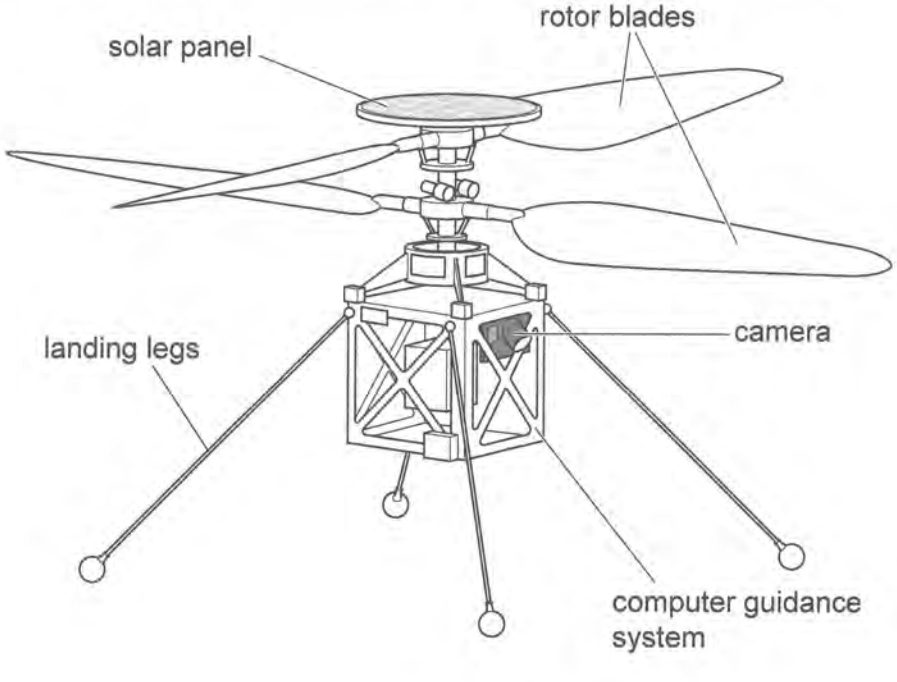

Fig. 7.5

In this design, there is no tail rotor.

Suggest how this design keeps the helicopter in stable flight.

....

....

.... [1]

[Total: 20]

8 A network of four capacitors of capacitance C, $2C$, $3C$ and $4C$ is constructed and placed into an electrical circuit, as shown in Fig. 8.1.

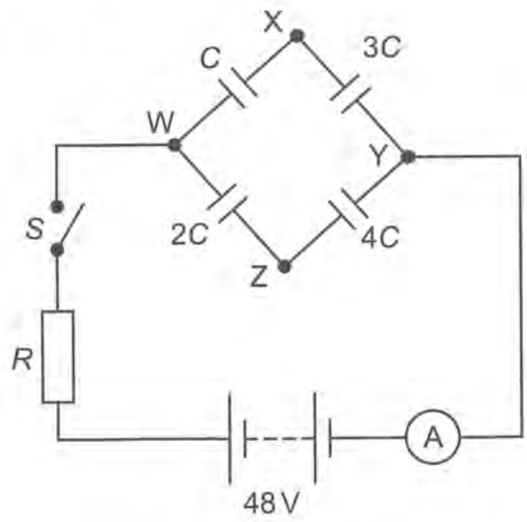

Fig. 8.1

The capacitors are initially uncharged and switch S is open.

At time $t = 0$, switch S is closed and an initial current I_0 is shown on the ammeter.

At time $t = T$, the current shown on the ammeter is $\frac{I_0}{2}$ and the capacitor of capacitance C stores a charge of magnitude q on each plate.

(a) **(i)** Determine the potential difference (p.d.) across each of the capacitors at time $t = T$.

p.d. across capacitor of capacitance C = ... V

p.d. across capacitor of capacitance $2C$ = ... V

p.d. across capacitor of capacitance $3C$ = ... V

p.d. across capacitor of capacitance $4C$ = ... V
[3]

(ii) Show that the equivalent capacitance C_{eq} between points W and Y of the network is just over $2C$.

[2]

(iii) By considering the potential differences in the circuit, show that the total charge Q on the capacitor network at time t is given by the equation:

$$Q = Q_0\left(1 - e^{-\frac{t}{RC_{eq}}}\right)$$

where Q_0 is the maximum charge on the capacitor network.

[4]

(iv) The capacitance $4C$ is $500\,\mu F$. The resistor has a resistance $R = 130\,k\Omega$.

By differentiating the equation in **(a)(iii)** with respect to time, or otherwise, calculate the value of the time T.

$T =$ .... s [4]

(v) Use your answer to **(a)(i)** and information from **(a)(iv)** to determine the value of q.

$q =$ .... C [1]

(vi) Determine the energy stored in the capacitor network at time $t = T$.

energy stored = ... J [3]

(b) Twelve identical capacitors each of capacitance 25 μF are connected so that they form the sides of a cube, as shown in Fig. 8.2.

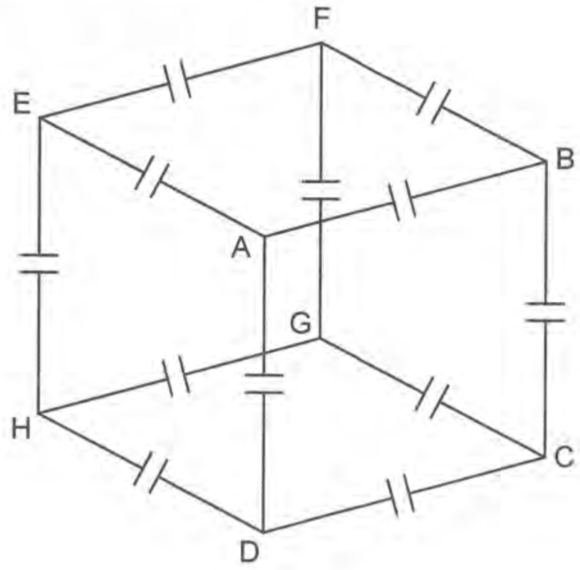

Fig. 8.2

A potential difference is applied between A and G.

(i) State the letters of three points in the circuit that have the same potential.

....

[1]

(ii) Calculate the total capacitance C_T of the cube of capacitors.

C_T = ... μF [2]

[Total: 20]