INDEX NUMBER

Anglo - Chinese School

(Independent)

FINAL EXAMINATION 2022

YEAR THREE EXPRESS

ADDITIONAL MATHEMATICS

PAPER 2 4049/02

Monday **10 October 2022** **1 hour 30 minutes**

Candidates answer on the Question Paper.
No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your index number in the space at the top of this page.
Write in dark blue or black pen.
You may use an HD pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 60.

For Examiner's Use
60

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

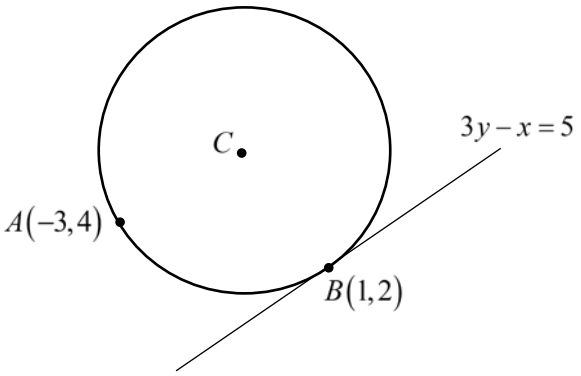
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer **all** the questions.

1 The function $f(x) = 2x^3 + a\,x^2 + b\,x - 12$ is divisible by $(x-3)$ and leaves a reminder of -14 when divided by $(2x+1)$. Find the value of *a* and of *b*. [5]

2 In the diagram, the circle with centre C passes through the point $A(-3,4)$ and touches the line $3y - x = 5$ at the point $B(1,2)$. Find



(a) the equation of the line BC, [3]

(b) the coordinates of C, [5]

(c) the equation of the circle. [2]

3 Given that $4x^3+16x^2+13x+3\equiv(x-m)g(x)$, where $g(x)$ is a polynomial and m is an integer.

(a) Find the value of m. [2]

(b) Find $g(x)$. [2]

(c) Hence, or otherwise solve for $4x^3+16x^2+13x+3=0$. [1]

(d) Explain why the equation $4x^6+16x^4+13x^2+3=0$ has no real solutions. [2]

4 **(a)** Find the values of x and y which satisfy the equations,

$$9^x = 3^y,$$
$$2^x + 1 = 72\left(2^y\right).$$

[5]

(b) Solve the equation $1 + \log_4 a + \log_2 8 = \log_4 (a + 3)$. [4]

5 **(a)** Given that $\frac{(\log_x y)^4}{\log_y x} + 32 = 0$, express y in terms of x. [4]

(b) **(i)** Simplify $\frac{\left(9^n\right)\left(25^n\right)}{15^{2n} + \left(5^{2n+2}\right)\left(3^{2n}\right)}$. [4]

(ii) Hence find the value of x if $\frac{15^{2n} + \left(5^{2n+2}\right)\left(3^{2n}\right)}{\left(9^n\right)\left(25^n\right)} = 3x - 1$. [2]

6 **(a)** Prove that $\dfrac{\cos\theta}{1-\sin\theta}-\dfrac{1}{\cos\theta}=\tan\theta$. [3]

(b) Hence solve the equation $\dfrac{\cos\theta}{1-\sin\theta}-\dfrac{1}{\cos\theta}=3\cot\theta$ for $0°\le\theta\le360°$. [3]

7 **(a)** Obtain the first 3 terms in the expansion of $(2+x)^6$ in ascending powers of x. [2]

(b) Find the term in x in $\left(1+\frac{1}{x}\right)(2+x)^6$. [2]

8 The table below shows experimental values of two variables x and y. The variables x and y are related by the equation $y = px^q$, where p and q are constants.

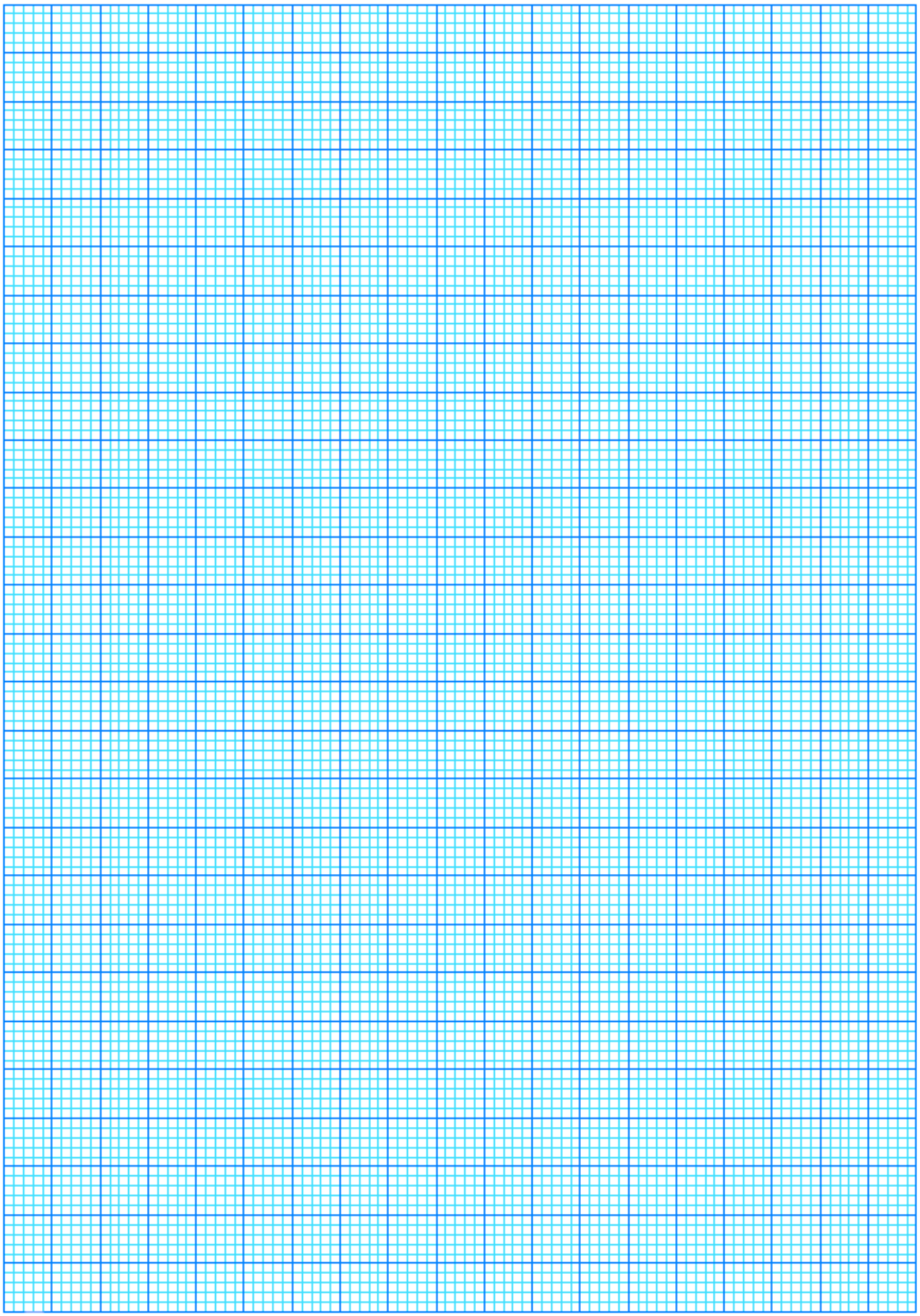
x	10	50	300	800	3100
y	350	144.5	60.1	36.6	18.2

(a) On the grid opposite, draw a straight line graph of $\ln y$ against $\ln x$. [2]

(b) Use your graph to estimate the value of p and of q. [4]

An equation of $y = \frac{e^8}{x^2}$ was given.

(c) Draw the graph of $y = \frac{e^8}{x^2}$ on the same axes and find the value of x of the point of intersection. [3]



Answers

1. $a=-5,\ b=1$

2. (a) $y=-3x+5$ (b) $C(0,5)$ (c) $x^2+(y-5)^2=10$ or $x^2+y^2-10y+15=0$

3. (a) $m=-3$ (b) $g(x)=(2x+1)^2$ (c) $x=-3,\ -\frac{1}{2}$

4. (a) $x=-3,\ y=-6$ (b) $a=\frac{1}{85}$

5. (a) $y=\frac{1}{x^2}$ (bi) $\frac{1}{26}$ (bii) $x=9$

6. (b) $\theta=60°,120°,240°,300°$

7. (a) $64+192x+240x^2+\ldots$ (b) $432x$

8. (b) $p\simeq 1100,\quad q\simeq -0.516$ (c) $x\simeq 1.92$