Section A: Pure Mathematics [40 Marks]

1 **(i)** By expressing $\frac{1}{k!}-\frac{1}{(k+1)!}$ as a single fraction, find the sum of

$$\frac{2}{3!}+\frac{3}{4!}+\frac{4}{5!}+\ldots+\frac{2n+1}{(2n+2)!}.$$ **[4]**

(ii) Hence show that $\sum_{r=2}^{2n}\frac{2}{(r+2)!}<\frac{1}{6}$. **[2]**

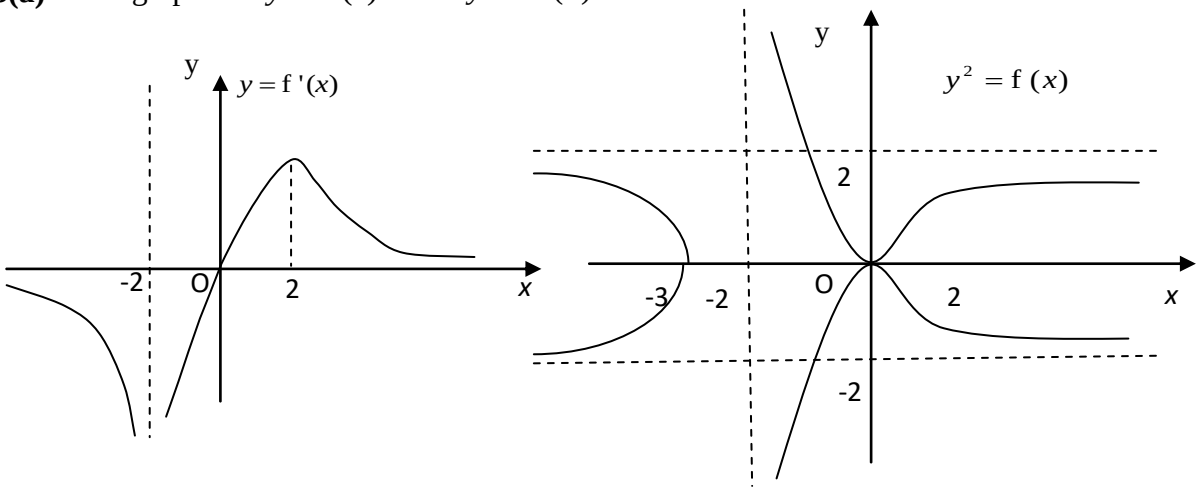
2 Each time a ball falls vertically on to a horizontal floor, it rebounds to three-quarters of the height from which it fell. It is initially dropped from a point 10m above the floor.

(i) Find, and simplify, an expression for the total distance the ball travels when it touches the floor for the $(n+1)^{th}$ time. **[2]**

(ii) Find the number of times the ball has bounced when it has travelled 55m. **[2]**

(iii) When the ball is at the maximum height after bouncing for the 4^{th} time, the floor rises to 2m above the ground. Calculate the total distance it travels before coming to rest. **[3]**

3(a) The graphs of $y=\mathrm{f}'(x)$ and $y^2=\mathrm{f}(x)$ are shown below:



Sketch the graphs of $y = \mathrm{f}(x)$, labelling the asymptotes, point(s) of intersection with the axes and turning points if any. **[2]**

(b) Find the values of the constants A and B such that $\frac{x^2-4x}{(x-2)^2}=\mathrm{A}+\frac{\mathrm{B}}{(x-2)^2}$ for all values of x except $x=2$. Hence state a sequence of transformations by which the graph of $y=\frac{x^2-4x}{(x-2)^2}$ may be obtained from the graph of $y=\frac{1}{x^2}$. **[4]**

4 A virus is found to be present in Country A, containing a population of 10,000 people. The rate at which the number of infected people, *x*, in thousands, is increasing at any time *t* is proportional to the product of the infected people and the number that have yet to be infected. Initially, there are 2,000 people infected. Show that the number of infected people at time *t* is

$$x = \frac{10e^{10kt}}{A + e^{10kt}}$$

where $A > 0$ is a constant to be determined. **[4]**

Sketch, on the same diagram, three solution curves for $k < 0, k = 0, k > 0$. **[2]**

State the range of values of *k* such that the virus will be eliminated. Give a possible reason for that to happen. **[2]**

5(a) The complex numbers *z* and *w* each have modulus 1, and have argument α and β respectively.

Prove that $z - w = 2i\sin(\frac{\alpha - \beta}{2})e^{i(\frac{\alpha+\beta}{2})}$.

Hence find k such that $(z - w)(z^* - w^*) = k(1 - \cos(\alpha - \beta))$, where $k \in \mathbf{Z}$. **[5]**

(b) Solve the equation $\left(\frac{z}{1+\mathrm{i}\sqrt{3}}\right)^3 + 2\left(\frac{1+\mathrm{i}\sqrt{3}}{z^2}\right) = 0$, giving the roots in the form $re^{i\theta}$,

where $r > 0, -\pi < \theta \leq \pi$. **[6]**

Hence show the roots on an Argand diagram. **[2]**

Section B: Statistics [60 Marks]

6 A box contains 9 balls. Out of these 9 balls, there are 3 identical red balls, 2 identical yellow balls and 4 numbered green balls (each labelled with a different number from 1 to 4).

3 balls are to be picked out of the box, and the order in which they are picked out does not matter.

Find the number of possible selections of 3 balls. **[3]**

7 A research company wants to predict the outcome of the Presidential election in America by conducting an opinion poll of the two candidates Mr Obi and Mr Mitt. The company decides that they would conduct the poll by randomly selecting eligible voters from the New York City telephone directory and interviewing those who answered the phone until 1000 eligible voters have been interviewed. The total number of eligible voters in America is estimated to be around 90 million. Assume

that all voters have access to a home telephone.

(i) Explain why the research company needs to obtain a sample of eligible American voters to predict the outcome of the Presidential election. **[1]**

(ii) Name the sampling method used and comment on the appropriateness of the method used. **[2]**

It is suggested that the highest educational qualification of an eligible voter might affect his or her choice of candidate. The percentage of eligible voters in America whose highest educational qualifications of a Bachelor's Degree is 30.44% and the percentage of eligible voters in America whose highest educational qualifications of at least a Master Degree is 10.95%.

(iii) Suggest a method of obtaining a more representative sample and describe how it may be carried out. **[3]**

8 The random variable X is the life span, in hours, of an X-ray light-bulb and X follows a normal distribution with mean μ and standard deviation σ.

It is given that $P(X > 300) = P(X < 250) = 0.2$

(i) Find μ and show that σ is approximately 29.7. **[3]**

(ii) Find the value of c such that $P(X > c) = 0.01$, giving your answer correct to 3 significant figures. **[2]**

9 A box contains 15 balls. Out of these 15 balls, there are 4 identical red balls, 5 identical yellow balls and 6 identical green balls.

A ball is randomly picked from the box. After picking out this ball, all balls of the same colour as the one just drawn are removed from the box.

This process is repeated until the box is empty.

Construct a probability tree showing this information. **[3]**

Find

(i) the probability that the last ball drawn is green. **[2]**

(ii) the probability that the first ball drawn is yellow, given that the last ball drawn is green. **[2]**

10 The number of small defects, X, in a randomly chosen carpet of area 20 m^2 manufactured by a certain company, follows a Poisson distribution with mean λ.

Given that $P(X = 2) = \frac{1}{8} P(X = 0)$, find the value of λ, and hence calculate $P(1 \leq X < 4)$. **[4]**

A house with a floor area 500 m^2 is being fitted with the carpet. Using a suitable approximation, find the probability that there will be more than 10 defects in the carpet of the house. **[3]**

11 The manufacturer of cartons of milk claims that each carton contains a mean volume of at least 2 litres of milk. Upon receiving the milk cartons, the manager of a supermart, Mr Moo measures the volume of milk of 10 randomly chosen cartons are measured and recorded his measurements as follows:

Carton	1	2	3	4	5	6	7	8	9	10
Volume (litres)	1.975	1.950	1.976	1.987	1.980	2.008	1.992	1.989	2.010	1.964

The volume of milk of randomly chosen milk cartons is assumed to be normally distributed.

(i) Test whether the manufacturer has over-claimed the mean volume of milk, using the 10% level of significance. **[4]**

(ii) The manufacturer now claims that the mean volume of milk is 2 litres. Using the *p*-value in **(i)**, explain whether there would be any changes to the outcome of the hypothesis test in **(i)**, if Mr. Moo wants to test whether the manufacturer's new claim is valid at the 10% level of significance. **[2]**

Suppose that the volume of milk is known to have a standard deviation of 0.012 litres. Mr. Moo wants to test at 10% significance level whether the manufacturer has over-claimed the mean volume of milk by using a sample of *n* randomly chosen cartons with a mean volume of 1.998 litres. Find the least number of cartons, *n*, that must be used for this test to conclude that the manufacturer has over claimed the mean volume of milk. **[4]**

12 In a study of the effectiveness of new drug, a researcher recorded the concentration of the drug *y* microgram per litre (mg/*l*) remained in a patient's bloodstream *x* minutes (min) after the drug is given.

The recorded pairs of readings are as follows:

x (min)	5	10	15	20	25	30
y (mg/*l*)	82	56	42	30	24	21

(i) Sketch a scatter diagram for the diagram. **[2]**

(ii) The researcher tried to recall the equation of the regression line of y on x. It was either $y = 79 - 2x$ or $y = -79 + 2x$. Assuming one of these two equations must be the correct equation of the regression line of y on x. Determine without any calculation, which equation is the correct one. **[1]**

One pair of reading (s, t) was lost during filing. It was recorded earlier that the regression line of x on y for seven pairs of readings, was $x = 36.967 - 0.435y$.

Using the regression line of x on y and your choice in **(ii)**,

(iii) Find s and t, correct to the nearest whole number. **[3]**

(iv) Calculate the product moment correlation coefficient for the complete set of seven readings. **[1]**

The researcher proposed another model $e^y = ax^b$ for the relationship between x and y. By converting this new model into a linear relationship between y and ln x, find a and b using the least squares method. **[3]**

13 On the average, 30% of the students in Calculus Madness Institute (CMI) could do the Differential Equation question in Block Test Two. The principal randomly select a class of 30 students to analyse the results.

(i) State, in the context of this question, two assumptions needed to model the results by a binomial distribution. **[2]**

(ii) Find the probability that at least 6 students in that class could do that question. **[2]**

(iii) Find the probability that only 2 students among the first 8 selected students in that class could do the question given that at least 6 students could do that question. **[3]**

(iv) The probability of no more than 5 students could do the question in a randomly selected class exceeds 0.9. Find the largest possible number of students in that class. **[3]**

50 classes with 30 students each are selected randomly. Find the probability that an average of at least 10 students per class could do the question. **[2]**

End of Paper