Chapter 13

CURRENT OF ELECTRICITY

Content

- Electric current
- Potential difference (p.d.)
- Resistance and resistivity
- Sources of electromotive force (e.m.f.)

Learning Outcomes

Candidates should be able to:

(a)	show an understanding that electric current is the rate of flow of charge.	□	□	□
(b)	derive and use the equation $I = nAvq$ for a current-carrying conductor, where n is the number density of charge carriers and v is the drift velocity.	□	□	□
(c)	recall and solve problems using the equation $Q = It$	□	□	□
(d)	recall and solve problems using $V = \frac{W}{Q}$	□	□	□
(e)	recall and solve problems using $P = IV$, $P = I^2R$ and $P = \frac{V^2}{R}$	□	□	□
(f)	recall and solve problems using $V = IR$	□	□	□
(g)	sketch and explain the I−V characteristics of various electrical components such as an ohmic resistor, a semiconductor diode, a filament lamp and a negative temperature coefficient (NTC) thermistor.	□	□	□
(h)	sketch the resistance-temperature characteristics of an NTC thermistor	□	□	□
(i)	recall and solve problems using $R = \frac{\rho \ell}{A}$	□	□	□
(j)	distinguish between electromotive force (e.m.f.) and potential difference (p.d.) using energy considerations.	□	□	□
(k)	show an understanding of the effects of the internal resistance of a source of e.m.f. on the terminal potential difference and output power.	□	□	□

The term “electricity” describes the set of physical phenomena associated with the presence and flow of electric charge. These phenomena include but are not limited to lightning, static electricity, electric current, and electromagnetic induction. Electricity is involved in powering household appliances, transportation devices, and industrial machineries; it is also used to transfer energy over large distances; and to send information via telecommunications.

This chapter introduces concepts required for the following chapters: D.C. Circuits, Electric Fields, Electromagnetism, Electromagnetic Induction and Alternating Currents.

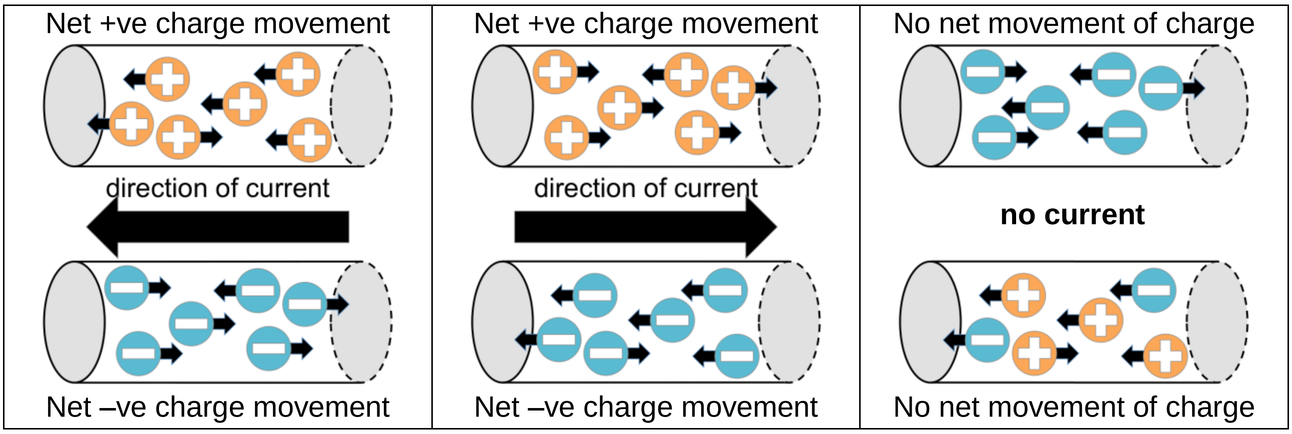
1 ELECTRIC CURRENT AND CHARGE

Charge carriers (or charged particles) such as electrons, protons and ions carry either a positive or negative charge. An electric current is said to be present when there is a **net** flow of charge.

1.1 ELECTRIC CURRENT

Symbol: I SI unit: Ampere (A)
Definition of Electric Current Electric current is the rate of flow of charged particles.
Calculating Current $$\boxed{I = \frac{dQ}{dt}}$$ Where I is the current in amperes, Q is the charge in coulombs and t is the time elapsed in seconds. If the current is constant, $I = \frac{Q}{t}$

Conventionally, the direction of current is the direction in which there is a net flow of **positive charge carriers**; it is opposite to the direction of net flow of negative charge carriers. The diagram below shows net movement of charge and the corresponding direction of current.



Drift velocity

When no potential difference is applied across a conductor, its mobile charge carriers undergo random motion similar to gas molecules in the air as shown in Fig. 3a.

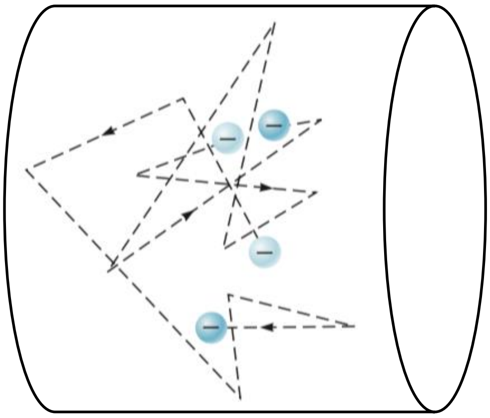


Fig. 3a: Random motion

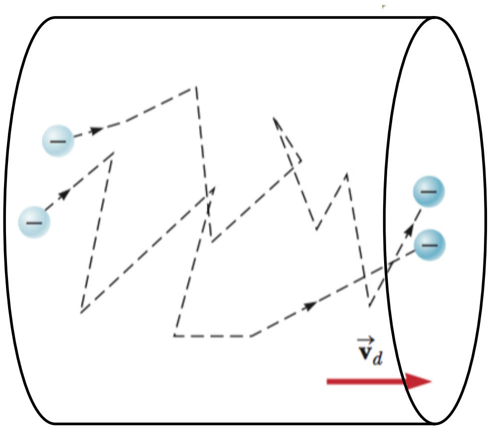


Fig. 3b: Random motion with axial drift (potential difference applied)

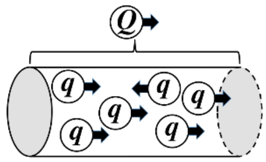
The presence of a potential difference across the conductor alters this random motion such that while moving randomly, the mobile charge carriers also drift axially along the conductor as shown in Fig. 3b. This rate of change of axial displacement with respect to time is known as its **drift velocity, v_d.**

To relate current I to the flow of individual charges q, consider a small flow of charge ΔQ through a small segment of conductor with volume ΔV in a time interval Δt.

The current in this segment is given by $I = \dfrac{\Delta Q}{\Delta t}$.

$\Delta Q = (\Delta N)q$ — The charge ΔQ is given by the product of the number of charge carriers ΔN and the charge q carried by each carrier, thus

$$I = \frac{(\Delta N)q}{\Delta t}$$

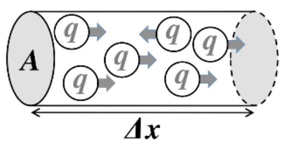


$\Delta N = n\Delta V$ — Let n be the number density of charge carriers (i.e. number of charge carriers per unit volume). The number of charge carriers ΔN is the product of the number density of charge carriers n and the volume of the segment ΔV, thus

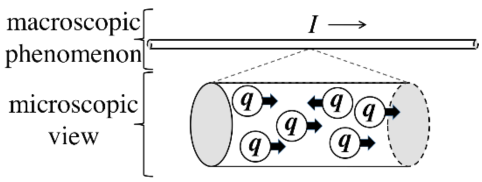
$$I = \frac{(n\Delta V)q}{\Delta t}$$

$\Delta V = A\Delta x$ — The volume of this segment of conductor ΔV is the product of its length Δx and cross-sectional area A, thus

$$I = \frac{(nA\Delta x)q}{\Delta t}$$



Since $\dfrac{\Delta x}{\Delta t}$ is the average distance travelled by the charge carriers per unit time, we denote it as $\boldsymbol{v_d}$, the **drift velocity** of the charge carriers.



Hence, we can relate I and q as follows:

Relationship Between Current And Drift Velocity

The current carried by a conductor, I, is given by

$$\boxed{I = nAv_d q}$$

where n is the number density of charge carriers (number of charge carriers per unit volume),
A is the cross-sectional area of the conductor,
v_d is the drift velocity, and
q is the charge carried by each charge carrier.

1.2 ELECTRIC CHARGE

Electric charge is the physical quantity of matter that causes it to experience an electric force when it is in the vicinity of other electrically charged matter. The nature of this electric force will be further elaborated in the topic on Chapter 14 *Electric Field*.

Symbol: Q
SI unit: Coulomb (C)

Definition of electric charge

The charge which flows past a point in time t is the product of current and time, if the current is steady.

Definition of unit coulomb (not in syllabus)

One coulomb is the amount of charge which flows past a point when a steady current of one ampere flows for one second.

Calculating Total Charge from Current

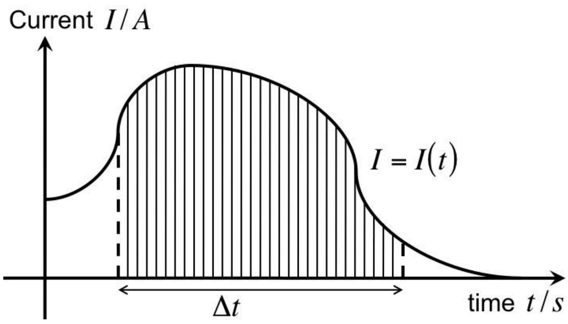
In general,

$$\boxed{Q = \int I \, dt}$$

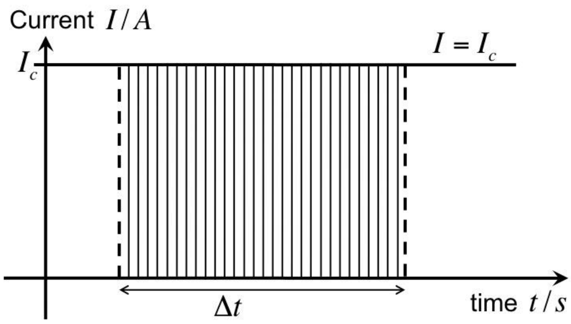
If the current is constant, $Q = It$

Graphical Representation

Consider a current, I, at a point in space over time, t.



Varying current



Constant current

The charge Q which flows past this point in the time interval Δt is determined by the **area** bound by the t-axis and the I–t curve.

Charge is quantised, which means it cannot take on arbitrary values. The smallest quantity of charge that can be carried is 1.60×10^{-19} C, a value known as e, the elementary positive charge. A proton carries a charge of e, an electron carries a charge of $-e$; a helium nucleus carries a charge of $2e$, and so on.

Example 1

A charging iPhone 7 draws a steady current of 1.00 A from the iPhone charger.

(a) Given that the capacity of the iPhone 7's battery is 7.06×10^{3} C, determine how long it would take for the battery to charge fully.

(b) The copper charging wire has a diameter of 0.511 mm, and the number density of charge carriers in the wire is 8.5×10^{28} m^{-3}. Assuming that each charge carrier carries a charge of $e = 1.60 \times 10^{-19}$ C, determine the drift velocity of the charge carriers.

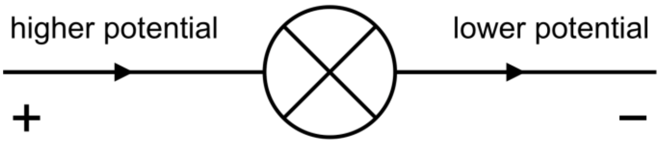
The amount of charge entering and leaving each junction is the same (or conserved) i.e. charges do not accumulate or disappear. This implies that the current remains constant along one single conductor. Similar to energy, charges obey the law of conservation: charges cannot be destroyed or created.

2 ELECTRIC POTENTIAL ENERGY AND POWER

When connected in a circuit, charge carriers possess electric potential energy (or electrical energy). The electric potential energy of a charge carrier may change as other forms of energy (such as heat or chemical energy) are converted from it or to it. This section deals with the energy considerations in various parts of a closed circuit.

2.1 ELECTRIC POTENTIAL

When such a circuit is connected to an electrical device (such as a bulb or television), current flows from a point of higher electric potential to a point of lower electric potential across the device.



In a circuit, the reference zero potential can be chosen at any point, as the **difference in potential** is the quantity that is physically meaningful. Without a **potential difference**, no charge can flow.

2.2 ELECTRIC POTENTIAL DIFFERENCE

When a charge carrier passes through an electrical component (or a resistive load), its electric potential energy is **converted to** other forms of energy (e.g. heat and light). This results in the decrease in the electric potential energy of the charge carrier and the decrease in electric potential at the point after the component.

Symbol: V SI unit: Volt (V)
Definition of Electric Potential Difference (p.d.) The potential difference across a device is the electrical energy **converted to** other forms of energy, per unit charge passing through the device. **Definition of the <u>unit</u> volt (not in syllabus)** One *volt* is the potential difference across a device when the *amount* of electrical energy **converted to** other forms of energy per unit charge passing through the device is *one joule per coulomb*.
Calculating Electric Potential Difference $$V = \frac{W}{Q}$$ $[V] = \text{V (volts)} = \text{J C}^{-1}$ where W is the amount of energy converted **from electric potential energy to other forms of energy** (in joule), Q is the amount of charge passing through the device (in coulomb).

Example 2 (*Self-Attempt*)

An electric clothes iron was connected to a 230 V voltage supply, during which it produced 720 kJ of heat.

(a) Calculate the total amount of charge that flowed through the heater.

(b) State an assumption that you made in your calculations.

DID YOU KNOW?

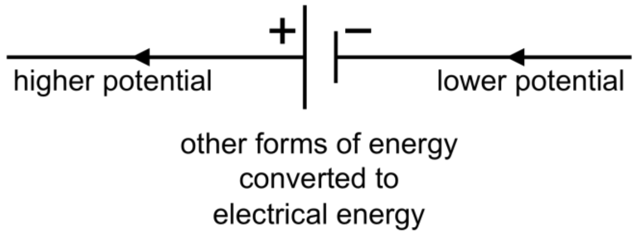
Electricity in Singapore is supplied at a nominal voltage of 230 V (±6%). This is sometimes stated as 220/240 V.

2.3 ELECTROMOTIVE FORCE

Electric potential energy (or electrical energy) in a system comes from a *source*. The quantity of **energy** supplied by the source is termed the *electromotive force* (e.m.f.).

Symbol: ε SI unit: Volt (V)
Definition of Electromotive Force (e.m.f.) The *electromotive force* of a *source* is the electrical energy, **converted from** other forms of energy per unit charge, transferred by the source in driving unit charge round a complete circuit.
Determining Electromotive Force $\boxed{\varepsilon = \dfrac{W}{Q} \quad [\varepsilon] = \text{V (volts)} = \text{J C}^{-1}}$ where ε is the electromotive force (in volt), W is the amount of energy converted **from other forms of energy to electric potential energy** (in joule), Q is the amount of charge passing through the source (in coulomb).

When a closed circuit is connected to a source of e.m.f., the part of the circuit connected to the positive terminal is at a higher potential while the part of the circuit connected to the negative terminal is at a lower potential. This results in a current along the closed circuit (not shown in diagram) in the direction towards the lower potential.



Some examples of e.m.f. sources are:

1) Battery — converts chemical potential energy to electrical energy

2) Solar cell — converts light energy to electrical energy

3) Thermocouple — converts thermal energy to electrical energy

4) Dynamo/generator — converts mechanical (kinetic) energy to electrical energy

2.4 ELECTRIC POWER

Electrical devices *do work* by converting *electrical energy to other forms of energy*. The more electrical energy a device converts **per unit time**, the more power it produces.

Symbol: P SI unit: Watt (W)
Definition of Power Power is the rate at which work is done. (Refer to Chapter 5 *Work, Energy and Power*.)
Calculating Power $$P = \frac{dW}{dt} = IV = I^2R = \frac{V^2}{R}$$ where W is the amount of energy converted **from electric potential energy to other forms of energy** (in joules). See Appendix A for the derivation of this equation.

Example 3

A 230 V storage heater, drawing a current of 6.60 A, requires 18 minutes to fully heat up a tank of water for bathing. Determine

(a) the power rating of the heater, and
(b) the thermal energy gained by the water.

Similarly, e.m.f. sources do work by converting *other forms of energy to electrical energy*. The more energy it converts per unit time, the more electrical power it supplies $P = I\varepsilon$.

2.5 DISTINGUISHING BETWEEN P.D. AND E.M.F.

The term *voltage* is often used interchangeably with *potential difference* and *electromotive force*. However, the two latter terms cannot be used interchangeably as they refer to different physical quantities. P.d. and e.m.f both refer to the amount of work done per unit charge and share the same unit, the volt. However, they are fundamentally different in their energy considerations.

P.d. (across two points)	**E.m.f.** (of a source)
is the electrical energy **converted to** other forms of energy, per unit charge passing through the device.	is the electrical energy, **converted from** other forms of energy per unit charge, transferred by the source in driving unit charge round a complete circuit.
$V = \dfrac{W}{Q}$ W : amount of electric potential energy **converted to** electric potential energy	$\varepsilon = \dfrac{W}{Q}$ W : amount of electric potential energy **converted from** other forms of energy
P.d. is zero when there is no current in the circuit. (IR = 0) P.d. depends on the total resistance of the components when there is current in the circuit.	E.m.f. does not depend on the current in the circuit or the total resistance of the circuit.
$P = IV = I^2R = \dfrac{V^2}{R}$ can be used to find the power delivered by a component when there is a p.d. across it.	$P = I\varepsilon$ can be used to find the power delivered by the source when there is a current in the circuit.

3 RESISTANCE AND RESISTIVITY

Resistance is the physical quantity of matter that opposes the flow of electric charge. The higher the resistance, the lower the mobility of electric charge in the matter.

Symbol: *R*
SI unit: Ohm (Ω)

Definition of Resistance

The resistance of a conductor is defined as the **ratio** of the potential difference *across* the conductor to the current flowing through it.

Definition of the <u>unit</u> ohm (not in syllabus)

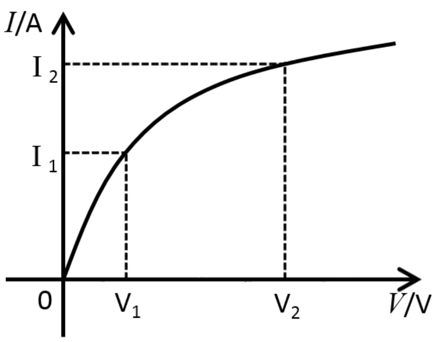
One ohm is the resistance of a conductor through which a current of one ampere is present when the potential difference across it is one volt.

Calculating Resistance

$$\boxed{\begin{array}{c} R = \dfrac{V}{I} \\ [R] = \Omega = \text{V A}^{-1} \end{array}}$$

where R is the resistance (in ohm),
V is the potential difference across the conductor (in volt),
I is the current through the conductor (in ampere)

Graphical Representation



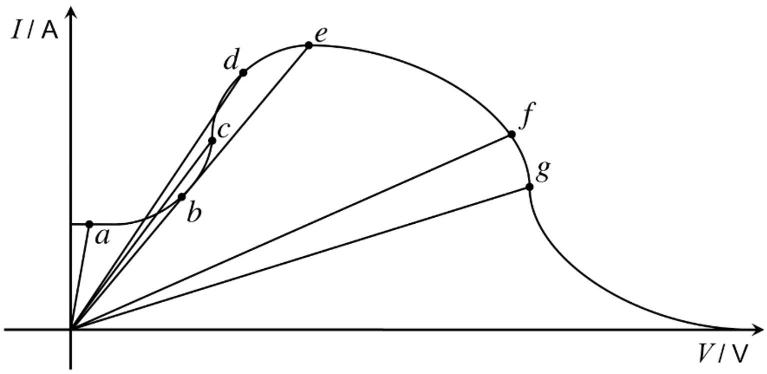
$$R_2 = \frac{V_2}{I_2}$$

$$R_1 = \frac{V_1}{I_1}$$

Note that the gradient of the tangent on an I–V graph **does not** have any physical meaning and it **does not** give the resistance of the conductor.

Example 4

The diagram below shows the I–V graph of an electrical component. List the points *a*–*g* in **descending** order of resistance.



Problem Solving Skills Set (PS³)

To determine R from I–V graph,

1. Draw a line **from the origin** to the point of interest on the I–V graph.
2. The gradient of this line (<u>not</u> the tangent at that point) gives the ratio $\frac{I}{V}$.
3. Calculate the inverse of $\frac{I}{V}$ to determine R.

The resistance of a conductor depends on many factors, and may not always be constant. One way to classify conductors is by how their resistance varies with increasing p.d. across it.

3.1 OHM'S LAW

Ohm's law is an empirical relationship between the current passing through a metallic conductor and the potential difference across it.

Ohm's law states that

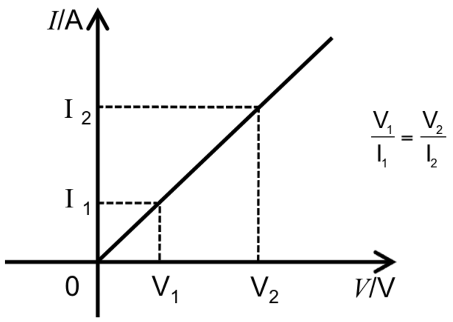
the current through a metallic conductor is proportional to the potential difference across its ends under constant physical conditions (e.g. temperature, mechanical stress).

Mathematical Representation of Ohm's Law

$$I \propto V$$

where I is the current through the conductor (in amperes),
V is the p.d. across the conductor (in volts)

Graphical Representation of Ohm's Law



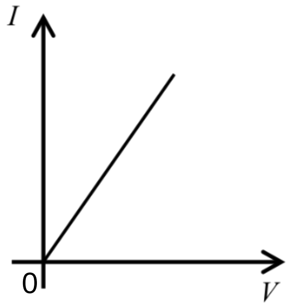
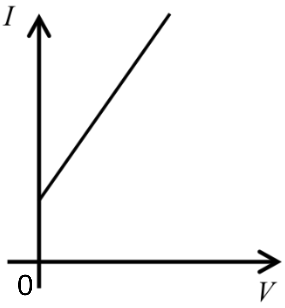
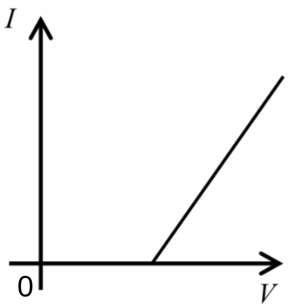
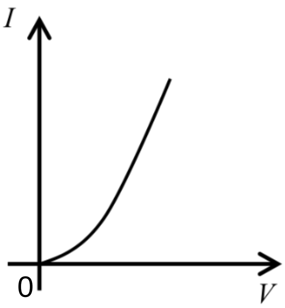
Since resistance $R = \frac{V}{I}$, for an ohmic conductor, $I = \frac{V}{R}$.

The **current** through an ohmic conductor **increases** with increasing p.d.

The **resistance** through an ohmic conductor **remains constant** with increasing p.d.

Example 5

For each of the following graphs, state **(a)** whether the conductor is ohmic, and **(b)** how resistance varies with increasing current.

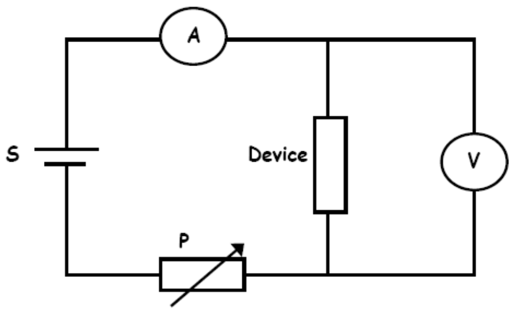
	**Answer:** (a) ____________ (b) ____________		**Answer:** (a) ____________ (b) ____________
	**Answer:** (a) ____________ (b) ____________		**Answer:** (a) ____________ (b) ____________

Ohmic vs Non-ohmic Conductors

Ohmic Conductors (particularly pure metals and alloys)	**Non-ohmic Conductors** (semiconductors, non-metals)
Since $I \propto V$, I–V graph is a straight line **passing through the origin**.	I–V graph can either be a straight line graph **or** passes through origin **but not both**.
Gradient of this straight line **is always** $\frac{1}{R}$.	Gradient of this graph is **not always** $\frac{1}{R}$.

The resistance of the component for each value of p.d. is calculated using the ratio $\frac{V}{I}$ (and not the inverse gradient of the tangent to the I–V graph).

3.2 I–V CHARACTERISTIC CURVES

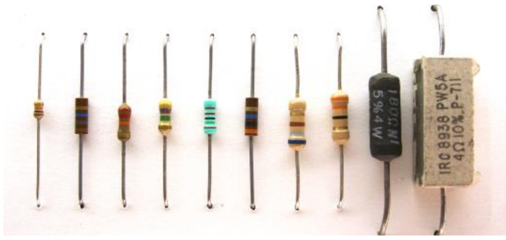


In order to investigate the resistance of different electrical components, the circuit below can be used. By applying a p.d. across the component under test, the current across it is recorded and its resistance deduced. Refer to Appendix B for the experiment procedure.

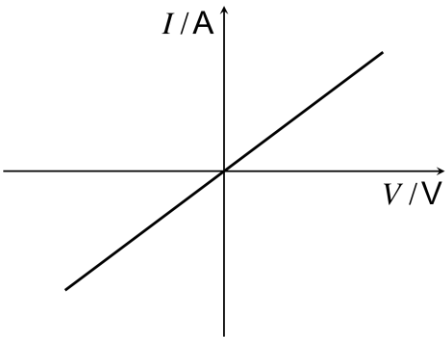
Certain electrical components possess unique electrical properties and hence display characteristic I–V curves which show how their resistance varies with increasing p.d. across them.

In the following sections, we will study the I–V curves of four devices: a **metallic conductor** at constant temperature, a **filament lamp**, a **semiconductor diode**, and an **NTC thermistor**.

3.2.1 METALLIC CONDUCTOR AT CONSTANT TEMPERATURE

Coils of metal wire:	Various types of resistors:
	
Circuit symbol for a wire:	Circuit symbol for a resistor: 

Metallic conductors and their circuit symbols



Characteristic I–V curve of metallic conductor

The I–V graph of a metallic conductor is a characteristic **straight line that passes through the origin**.

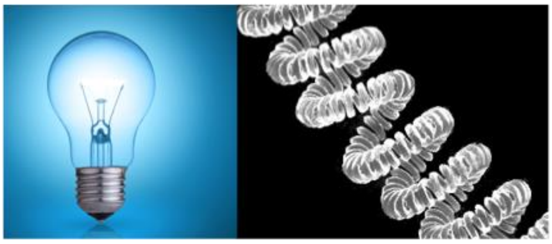
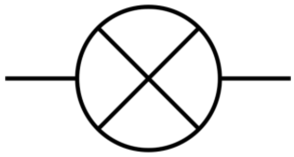
Since I increases proportionally with V, the ratio $\frac{V}{I}$ **is constant**. This shows that the **resistance is constant** and hence the conductor is ohmic.

Why is the I–V graph of a metallic conductor linear?

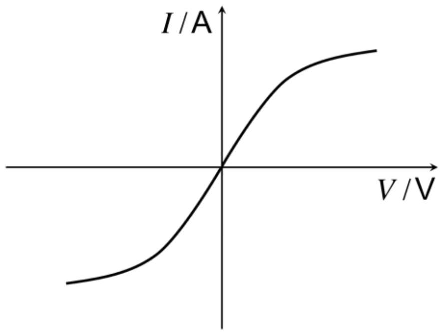
As the p.d. across the metallic conductor increases, the temperature of the metallic conductor remains constant. Since the mobility of charge carriers, which depends on temperature, remains constant, the resistance of the metallic conductor remains constant.

3.2.2 FILAMENT LAMP

The filament is a metallic, non-ohmic conductor that changes temperature significantly when the current through it varies. The change in temperature of the filament (a physical condition) results in a change in its resistance.

Filament bulb:	Circuit symbol for a filament lamp:
	

Filament bulb with a close-up shot of filament (left) and its circuit symbol (right).



Characteristic *I*–*V* curve of filament lamp

The *I*–*V* graph of a filament lamp is a characteristic **S-shape curve that passes through the origin**.

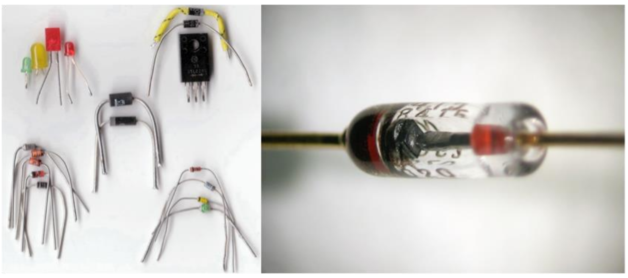
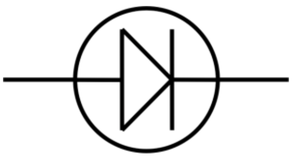
Since I increases at a decreasing rate with V, the ratio $\frac{V}{I}$ **increases**. This indicates that the **resistance increases with *V*** and hence the filament lamp is a non-ohmic conductor.

Why is the *I*–*V* graph of a filament lamp an S-curve?

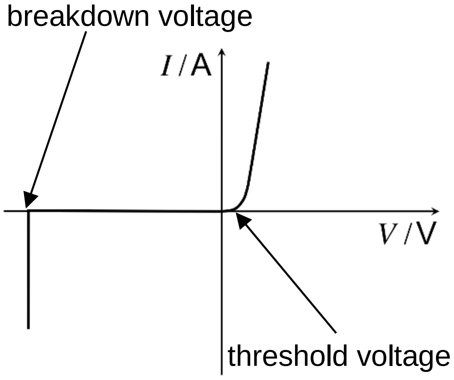
As the p.d. across the filament lamp increases, the steady-state temperature of the filament increases. An increase in temperature in the filament causes the conducting electrons to collide more frequently with the lattice ions in the wire, resulting in lower mobility of the electrons. Since the mobility of charge carriers decreases, the resistance of the filament increases.

3.2.3 SEMICONDUCTOR DIODE

A semiconductor diode is an electrical component that conducts electricity in only one direction. This direction, called the forward-bias direction, allows conduction much more easily than in the other direction which is called the reverse-bias direction.

Various types of diodes:	Circuit symbol for a diode (forward-bias to the right):
	

Diodes (left) and their generic circuit symbol (right).



Characteristic *I*–*V* curve of semiconductor diode

The *I*–*V* graph of a semiconductor diode is a characteristic **asymmetrical curve**.

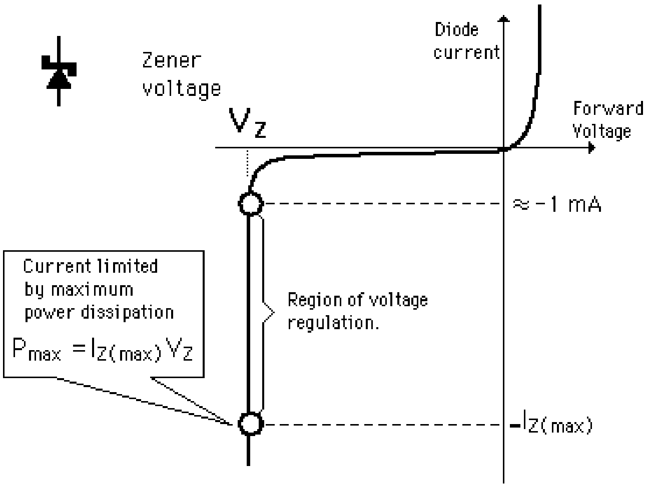
In the forward-bias direction, the graph is an **exponential curve** which begins shortly after the origin. Beyond the threshold voltage, since I increases exponentially with V, the diode's **resistance is insignificant**.

In the reverse-bias direction, the graph is a **horizontal line from the origin followed by a near-vertical line**. Since I is insignificant before the breakdown voltage, the diode's **resistance is very high**.

The ratio $\frac{V}{I}$ is not constant, hence the diode is a non-ohmic conductor.

Why does the I–V graph vary as such?

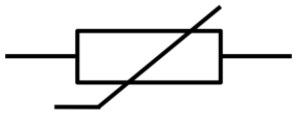
For some semiconductor diodes, there is a threshold potential difference beyond which the diode breaks down in the reverse-bias direction and conducts an infinitely large current. For a Zener diode, high current will flow in the reverse-bias direction when the p.d. applied is beyond the breakdown voltage of 3.2 V.



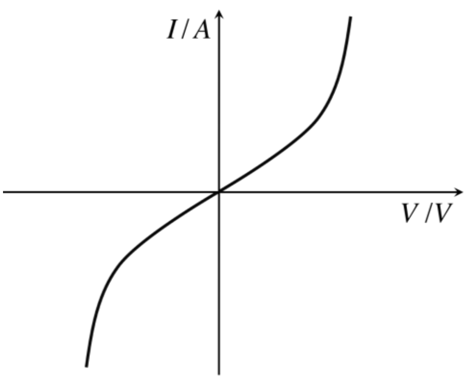
When the p.d. across a Zener diode exceeds the breakdown voltage Vz, the diode breaks down in the reverse-bias direction and conducts an infinitely large current.
(Anton Kruger, *Zener and Avalanche Diodes*).

3.2.4 THERMISTOR

A thermistor is a type of resistor (usually also made with a semiconductor) whose resistance varies significantly with its temperature, more so than standard resistors. The temperature change can be due to current flowing through it, or due to changes in ambient temperature.

Various types of thermistors:	Circuit symbol for a thermistor:
	

Thermistors (left) and their generic circuit symbol (right)



Characteristic I–V curve of thermistor

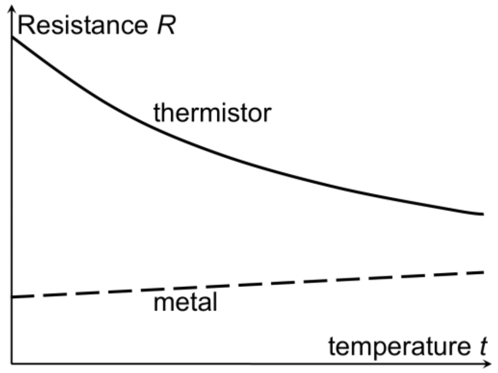
The I–V graph of a thermistor with negative temperature coefficient (NTC) is a characteristic **reverse-S-shape curve** that passes through the origin.

Since I increases at an increasing rate with V, the ratio $\frac{V}{I}$ decreases. This indicates that the **resistance decreases with V** and hence the thermistor is a non-ohmic conductor.

Why is the *I*–*V* graph of a thermistor an inverse S-curve?

As the temperature of the NTC thermistor rises, more charge carriers in the thermistor are liberated and become mobile (free to move). The effect of increased mobile charge carriers is greater than the effect of more frequent collisions with lattice ions, hence the electrical conduction of the thermistor improves and the resistance of the thermistor decreases.

Why is it called an NTC thermistor?



Unlike metals, a thermistor undergoes a decrease in resistance with increasing temperature, which means it has a negative temperature coefficient (NTC) of resistance.

Most metals undergo a slight increase in resistance with temperature, due to the effect of increased collisions with lattice ions at higher temperatures. This means they have a positive temperature coefficient (PTC). The temperature coefficients of some materials are found in Appendix C.

A material can also have different resistances along different dimensions, making it difficult to use resistance as a measure of how strongly a material opposes electric current. This property is known as the material's **resistivity**.

3.3 RESISTIVITY

Symbol: ρ
SI unit: Ohm metre (Ω m)

Definition of Resistivity

Resistivity is a measure of how strongly a material opposes electric current. It characterises the resistance of materials at a fixed temperature regardless of the materials' dimensions.

Relationship Between Resistivity and Resistance

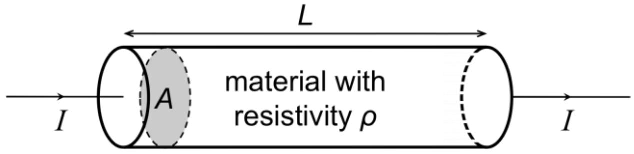
The resistance of an uniform conductor at a given temperature is found to be

a) dependent on its material,
b) directly proportional to its length L, and
c) inversely proportional to its cross-sectional area A.

i.e. $\boxed{R \propto \frac{L}{A}}$

The dependence of resistivity of a resistor on its material is represented by a proportionality constant, ρ, thus

$$\boxed{R = \rho \frac{L}{A}}$$

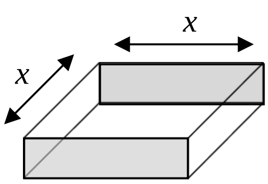


Example 6 (Self-Attempt)

Two wires *A* and *B* are made of the same material. *A* has twice the length and half the diameter of *B*. Determine the ratio $\frac{R_A}{R_B}$.

Example 7

A sample of resistive material is prepared in the form of a thin square slab with sides of length x. If its thickness remains constant, the resistance between opposite edge faces of the sample (shown shaded in figure) is:



(A) proportional to x^2

(B) independent of x

(C) inversely proportional to x^2

(D) proportional to x

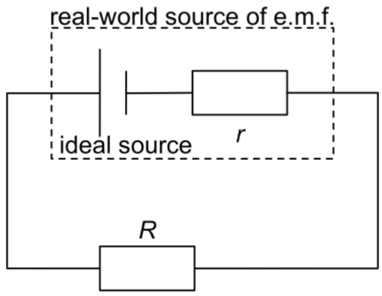
(E) inversely proportional to x

3.4 INTERNAL RESISTANCE OF E.M.F. SOURCE

In practice, not all the energy provided by the source is dissipated across the external loads because real-world sources are not perfectly efficient. In the example of an AA cell, some energy is lost as heat. A useful model of a real world cell is to treat it as being equivalent to an ideal e.m.f. source in series with an internal resistor.

Symbol: r
SI unit: Ohm (Ω)

Circuit Representation of Internal Resistance



A real-world e.m.f. source with internal resistance is represented as an ideal source of e.m.f. ε in series with an internal resistor (drawn inside a dashed-line box) and an external resistance R. The potential difference across the two ends of the real-world e.m.f. source is called the **terminal p.d., V_T**.

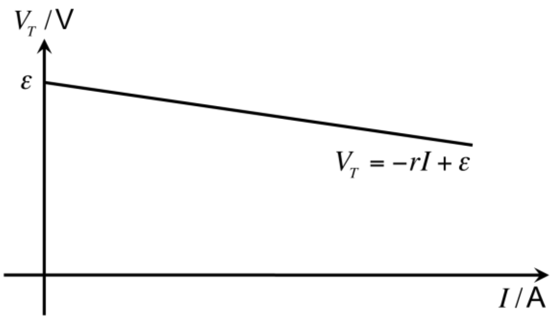
Equation Relating Terminal P.D. V_T, E.m.f. ε, and Internal Resistance r

When connected in a circuit in series with a resistor of resistance R, the total resistance of the circuit becomes $r + R$.

By conservation of energy,
Power supplied by e.m.f source = Power output in R + Power loss through r.

$$P_\varepsilon = P_R + P_r$$
$$I\varepsilon = I^2R + I^2r$$
$$\varepsilon = IR + Ir$$
$$\varepsilon = V_T + Ir \Rightarrow V_T = \varepsilon - rI$$

Graphical Representation of V_T vs I



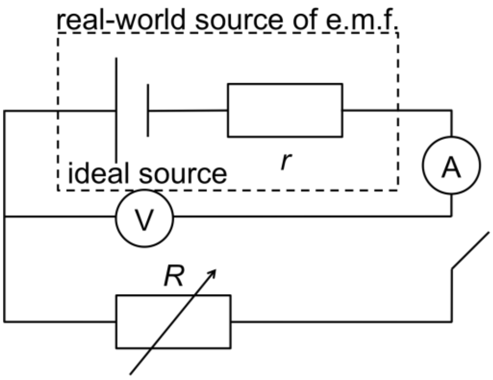
When V_T is plotted against I, we get the graph on the left.

The gradient of the best-fit-line gives the negative of the value of internal resistance r while the vertical intercept gives the value of the e.m.f. ε.

The symbol V_T represents **terminal p.d.** and denotes the potential difference between the two ends (terminals) of the source. The terminal p.d. V_T across the cell is less than the e.m.f. ε of the cell because there is a potential drop (Ir) due to current flowing through the internal resistance of the cell. The terminal p.d. of a cell is thus $V_T = \varepsilon - Ir$.

Experiment to Determine Internal Resistance

The internal resistance of an e.m.f. source *r* cannot be determined directly since the internal resistance is not physically separable from the source; it is not possible to measure *ε*. Hence, the circuit below is used instead to determine internal resistance *r*. Refer to Appendix D for the experiment procedure.



When the **switch is open**, the resistance in the circuit is infinite and the current in the circuit is zero. Hence, the reading on the voltmeter gives the e.m.f of the source.

When the **switch is closed**, the resistance in the external circuit is *R*+*r*. By varying *R*, readings on the voltmeter give the terminal p.d. V_T of the source and readings on the ammeter give the corresponding current *I* in the circuit.

A graph of V_T vs I allows the internal resistance *r* to be determined (see page 18).

Effects of the internal resistance of an e.m.f. source on the terminal p.d. and output power

The internal resistance of an e.m.f. source causes the terminal p.d. of the source to be lower than its e.m.f. and the actual power output to be lower than its theoretical power output.

A source of e.m.f. is said to be ideal if it has no internal resistance. The terminal p.d. will then be equal to its e.m.f.

Example 8

A battery is connected to a variable resistor and a voltmeter is connected across its terminals. When the variable resistor has resistance of 6.0 Ω, the voltmeter reading is 4.0 V. When the resistance is 10 Ω, the voltmeter reading is 4.4 V. Determine the e.m.f. and the internal resistance of the battery.

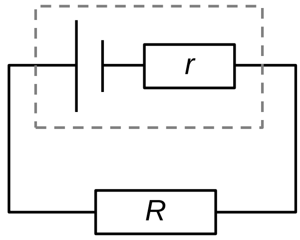
3.5 MAXIMUM POWER TRANSFER THEOREM

In practice, we want most of the power supplied by the e.m.f. source to be dissipated by the external load rather than wasted through internal resistance. Assuming that the power supplied by the source is constant, the power supplied to the external load depends on resistance of the external load.

For a circuit with external load of resistance R and internal resistance r,

$$\varepsilon = V_T + Ir$$
$$\varepsilon = IR + Ir$$
$$I = \frac{\varepsilon}{R+r}$$

Considering the power transferred to the external load,



Circuit diagram

$$P_R = I^2R$$
$$= \left(\frac{\varepsilon}{R+r}\right)^2 R$$
$$= \varepsilon^2 \frac{R}{(R+r)^2}$$

When the power transferred to the external load is the maximum,

Variation of power P_R dissipated in R with resistance R

$$\frac{dP_R}{dR} = 0$$
$$\frac{d}{dR}\left[\varepsilon^2\left(\frac{R}{(R+r)^2}\right)\right] = 0 \leftarrow (\varepsilon \text{ is constant})$$
$$\frac{d}{dR}\left[R(R+r)^{-2}\right] = 0$$

Applying product rule and simplifying,

$$(R+r)^{-2} - 2R(R+r)^{-3} = 0$$

Since $(R+r) > 0$, we can divide by $(R+r)^{-2}$:

$$2R(R+r)^{-1} = 1$$
$$2R = R + r$$
$$R = r$$

Hence, for the power transferred from the e.m.f. source to the external load to be maximum, the resistance of the load must be the same as the internal resistance. This is known as the **Maximum Power Transfer Theorem**.

APPENDIX A DERIVING THE EQUATION FOR ELECTRIC POWER

From the definition of power,

$$P = \frac{dW}{dt}$$

Where W is the amount of energy in units of joules converted **from electric potential energy to other forms of energy**.

Since the electrical device does work to move a quantity of charge Q across a potential difference V, from the definition of p.d. $V = \frac{W}{Q}$,

$$P = \frac{VQ}{t}$$

$$= V\left(\frac{Q}{t}\right) = VI$$

From the definition of electric current $I = \frac{Q}{t}$ at constant V,

$$P = VI$$

Since the p.d. V across the device and the current I in the corresponding section of the circuit is proportional to its resistance R, i.e. $R = \frac{V}{I}$,

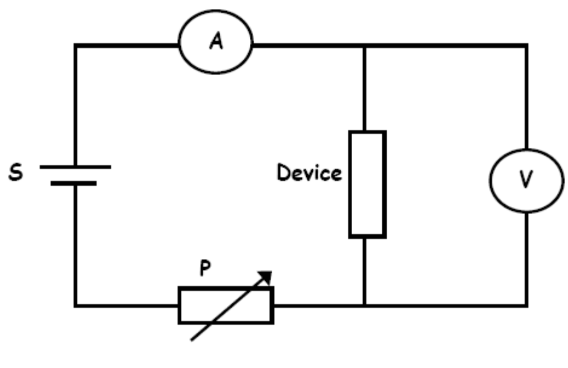
$$P = VI = (IR)I$$

$$\text{thus, } P = I^2R = \frac{V^2}{R}$$

APPENDIX B DETERMINING I–V CHARACTERISTIC CURVE OF AN ELECTRICAL COMPONENT

Experiment procedure:

1. Set up the apparatus as shown. Ensure that the contact of rheostat P is at the high-resistance end to minimise its resistance.
2. Measure and record the p.d. across the device V using the voltmeter.
3. Measure and record the current in the circuit I using the ammeter.
4. Repeat steps 2 and 3, varying the position of the contact to obtain at least six sets of data for V and I.
5. Plot a graph of I against V.



The resistance R of the device for each value of p.d. is calculated using the ratio V/I (and not the inverse of gradient).

APPENDIX C TABLE OF RESISTIVITY AND TEMPERATURE COEFFICIENT OF RESISTIVITY

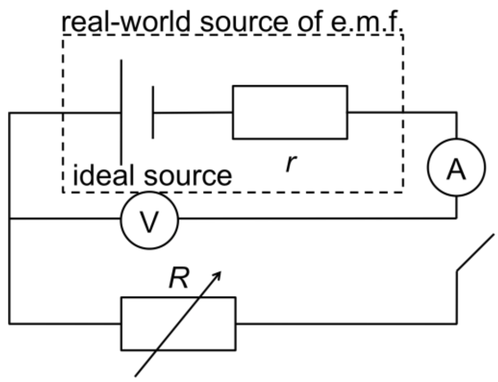
Material (Temperature = 20 °C)		Resistivity ρ / Ω m	Temperature coefficient of resistivity / Ω m K^{-1}
Conductor	Silver	1.59×10^{-8}	6.1×10^{-3}
	Copper	1.68×10^{-8}	6.8×10^{-3}
	Tungsten	5.60×10^{-8}	4.5×10^{-3}
	Mercury	9.80×10^{-7}	9.0×10^{-4}
Semiconductor	Carbon (Graphite)	$(3.00 \times 10^{-5}, 60.0 \times 10^{-5})$	-5.0×10^{-4}
	Germanium	$(1.00 \times 10^{-3}, 500 \times 10^{-3})$	-5.0×10^{-2}
	Silicon	$(1.00 \times 10^{-1}, 6.0 \times 10)$	-7.0×10^{-2}
Insulator	Glass	$(10^{9}, 10^{12})$	–
	Hard rubber	$(10^{13}, 10^{15})$	–

Giancoli, Douglas C., *Physics, 4th Ed*, Prentice Hall (1995).

APPENDIX D DETERMINING INTERNAL RESISTANCE OF SOURCE OF ELECTROMOTIVE FORCE

Experiment procedure:

1. Set up the apparatus as shown. The measured current on the ammeter should be 0 A.
2. Close the switch.
3. Measure and record the terminal p.d. across the e.m.f source V_T using a voltmeter.
4. Measure and record the current I using an ammeter.
5. Repeat steps 3 and 4, varying the contact position of the rheostat R to obtain at least 6 sets of data for V_T and I.
6. Plot a graph of p.d. across the device V_T against I.



Since $V_T = \varepsilon - Ir$, the gradient of the best-fit line gives the value of $-r$ and the vertical intercept gives the value of ε.

APPENDIX E HOUSEHOLD ELECTRICAL CONSUMPTION

Household electrical consumption is measured in terms of energy with the unit kilowatt-hour (kWh). One kWh is the energy delivered by one kilowatt of power in one hour. $1.0\text{ kWh} = 1000\text{ W} \times 3600\text{ s} = 3.6 \times 10^{6}\text{ J}$.

Tariffs for electricity are regulated by the Energy Market Authority and revised quarterly. For the fourth quarter of 2017 (Q4 2017), the tariff for electricity is 20.30 cents/kWh while the tariff for gas is 17.68 cents/kWh.

The national average household consumption for electricity is shown below. Households staying in high-rise apartments pay up to about $110 a month on average for their monthly electricity needs.

Premises Type	Jul '17	Jun '17	May '17	Apr '17	Mar '17	Feb '17	Jan '17
HDB 1-Room	149	150	141	132	118	133	132
HDB 2-Room	209	212	200	184	170	187	188
HDB 3-Room	302	310	284	265	239	261	263
HDB 4-Room	411	421	380	355	324	354	353
HDB 5-Room	477	487	437	412	378	409	404
HDB Executive	591	605	536	508	466	501	497
Apartment	589	615	563	523	471	510	509
Terrace	943	993	926	876	811	886	850
Semi-Detached	1251	1321	1216	1154	1087	1169	1139
Bungalow	2533	2667	2484	2355	2245	2393	2401

SPGroup. *National Average Household Consumption (kWh)*.
Retrieved October 04, 2017, from https://www.spgroup.com.sg/what-we-do/billing

APPENDIX F LITHIUM-ION CELLS (REAL-WORLD E.M.F. SOURCE)

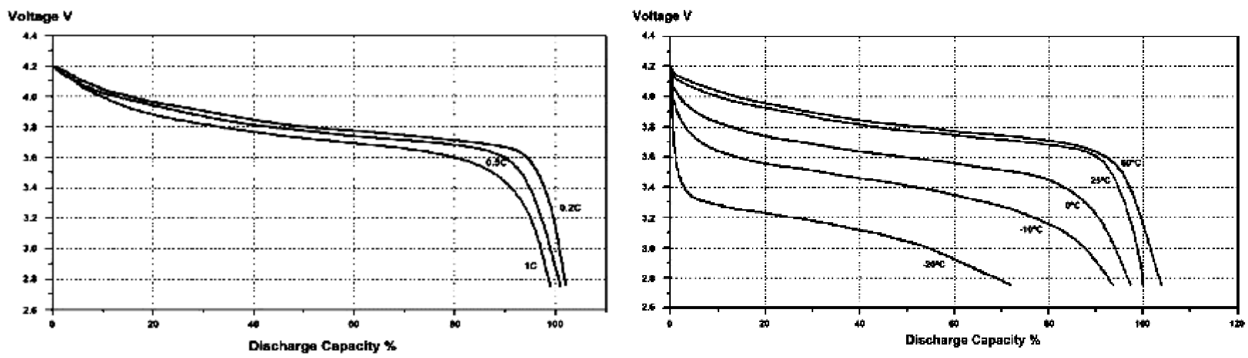
Similarly, battery capacity is often measured in terms of charge with the unit milliampere-hour (mAh). A battery with a capacity of 1000 mAh is able to deliver a current of 1.0 A over a period of one hour, or a current of 2.0 A over 0.5 hours, or a current of 0.5 A over 2 hours, and so on. $1.0\text{ Ah} = 1.0\text{ A} \times 3600\text{ s} = 3.6 \times 10^{3}\text{ C}$.

The most popular type of battery storage is the lithium ion (LI) battery. In a lithium ion cell, lithium ions move from the negative electrode to the positive electrode during discharge, and in the reverse direction during charging. These cells typically have a nominal voltage of 3.2 V or 3.6 V

A lithium ion battery comprises several cells as shown. Cells can be arranged in series to provide a higher terminal p.d., or in parallel to provide higher charge capacity.

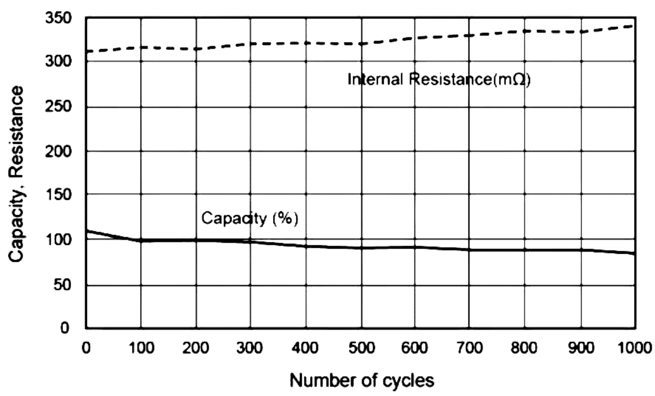
Laptop battery comprising 6 lithium-ion cells
Retrieved from http://jimlaurwilliams.org/wordpress/?p=3546

While the e.m.f. of an ideal source is assumed to be constant, a lithium-ion cell (and other kinds of chemical cells) produces an e.m.f. through a chemical reaction between lithium-cobalt oxide and graphite. The e.m.f. produced by the cell decreases as the concentration of reactants decreases. When the cell is no longer able to provide the required terminal p.d., it is considered to be depleted. The rate of decrease of e.m.f. depends on various factors, such as the current drawn through the cell, and the temperature of the cell, as shown in the graphs below.



Variation of e.m.f. with current (left) and with temperature (right).
Retrieved from http://www.ibt-power.com/Battery_packs/Li_Ion/Lithium_ion_tech.html

The internal resistance of a lithium-ion cell gradually increases as it is repeatedly charged and discharged. This leads to a decrease in capacity, as the cell's terminal p.d. drops more quickly, as shown below.



Variation of battery capacity and internal resistance with number of charging cycles.
Retrieved from https://www.planetofthevapes.co.uk/guides/advanced-vaping/guide-to-li-ion-batteries.html