Alternating Current

An **alternating current** is the current which <u>direction of flow of charge carriers reverses</u> <u>periodically with time</u>.

An example of an alternating current follows the harmonic function of time, as depicted below.



Graph of current vs time of a sinusoidal AC

We can describe the graph with the following relation:

$$I = I_0 \sin \omega t = I_0 \sin (2\pi f)t$$

<u>Note:</u>

Although the AC circuit may be different from the DC circuit in some ways, the AC circuit can still be described by the equations that apply to DC circuits.

Therefore, we can do the following analysis:

Using Ohm's Law,

$$V = IR = (I_0 \sin \omega t)R$$
$$= I_0 R \sin \omega t$$
$$= V_0 \sin \omega t$$

Therefore, we get the relation for the alternating potential difference (voltage) with respect to time:

$$V = V_0 \sin \omega t = V_0 \sin 2\pi f t$$

Quantifying AC: Root-Mean-Square Values

Root-Mean-Square value is the <u>effective value</u> of a <u>continuously varying quantity</u>, obtained from many samples taken at regular intervals during a cycle.

The **root-mean-square of an alternating current (or voltage)** is defined as the <u>value of steady</u> <u>direct current (or voltage)</u> that <u>dissipates energy at the same rate</u> as the alternating current (or voltage) in a given resistance.

$$I_{\rm rms} = \sqrt{\langle I_{\rm AC}^2 \rangle} = \sqrt{\rm mean\,value\,of\,I_{\rm AC}^2}$$

$$V_{\text{rms}} = \sqrt{\langle V_{\text{AC}}^2 \rangle} = \sqrt{\text{mean value of } V_{\text{AC}}^2}$$

For Sinusoidal AC

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

Graphical Derivation of Root-Mean-Square Values of AC

The calculation for r.m.s. values can be done graphically. In fact, in most cases, the graphical method is more direct. Here is how the method can be carried out.

To find I_{rms} :

Step (1): From the I-versus-t graph, sketch the I²-versus-t graph.

Step (2) : From the graph in step 1, deduce the mean value of I^2

(in other words, calculate the ratio: $\frac{\text{Area under the graph for 1T}}{1\text{T}}$).

Step (3) : Calculate the square-root of the mean value of I^2 to obtain the I_{rms} .

Employ the same method to calculate V_{rms} starting from the V-versus-t graph.

Relationship between r.m.s. values and Average Power <P>

$$\langle \mathsf{P} \rangle = \frac{1}{2} \mathsf{P}_0 = \frac{1}{2} \mathsf{I}_0 \mathsf{V}_0$$
$$= \left(\frac{\mathsf{I}_0}{\sqrt{2}}\right) \left(\frac{\mathsf{V}_0}{\sqrt{2}}\right) = \mathsf{I}_{\mathsf{rms}} \mathsf{V}_{\mathsf{rms}}$$
$$\langle \mathsf{P} \rangle = \mathsf{I}_{\mathsf{rms}} \mathsf{V}_{\mathsf{rms}}$$

In general for any AC,

$$\left< P \right> = \ I_{rms} V_{rms} \ = \ I_{rms}^2 R \ = \ \frac{V_{rms}^2}{R}$$

But for sinusoidal AC only,

$$\langle \mathsf{P} \rangle = \frac{1}{2} \mathsf{I}_0 \mathsf{V}_0 = \frac{1}{2} \mathsf{I}_0^2 \mathsf{R} = \frac{1}{2} \frac{\mathsf{V}_0^2}{\mathsf{R}}$$

Rectification

Rectification is the means by which <u>alternating current is converted into direct</u> <u>current.</u>

Diodes are used to achieve half-wave or full-wave rectification.



A simple single diode set up for half-wave rectification