1 A sequence u_1, u_2, u_3 , … is such that $u_n = 2u_{n-1} - 15n^2 + 60n + A$, where A is a constant, and $n \geq 2$. It is given that $u_1 = 2$ and $u_2 = 4$.

(i) Find A and u_3. [2]

It is known that the nth term of this sequence is given by $u_n = p(2^{n+1}) - qn^2 + r$.

(ii) Find p, q and r. [3]

2 It is known that the nth term of a sequence is given by $u_n = \ln\left(\frac{n}{n+1}\right)$, where $n \geq 1$.

(i) Determine if the sequence is convergent, stating your reason clearly. [1]

(ii) By using the method of differences or otherwise, find $\sum_{n=1}^{N} u_n$ in terms of N. [3]

3 A curve C has equation $y = 1 - \frac{1}{x}$, where $x \neq 0$. The line l is the tangent to C at $x = a$, where a is positive integer. Show that the equation of l may be expressed in the form $a^2 y - x = a^2 - 2a$. [3]

Find the value of a for which l is parallel to the line $9y - x = 1$. Hence find the equation of l in this case. [3]

4 The curve C has equation $y = \frac{x^2 + x + 3}{x - 2}$.

(i) Without using a calculator, find the set of values that y can take. [4]

(ii) Sketch the graph of C, stating clearly the equations of any asymptotes and the coordinates of any turning points and axial intercepts. [3]

(iii) By drawing a suitable graph on the same diagram in **(ii)**, find the range of values of k, where $k > 0$, such that the equation $(x+1)^2(x-2)^2 + \left(x^2 + x + 3\right)^2 = k^2\left(x-2\right)^2$ has at least one positive real root, giving your answer correct to 3 decimal places. [3]

5 The function f, with domain the set of all real values, is given by

$$\mathrm{f}(x)=\begin{cases}4-x^2 & \text{for } 0<x\le 2,\\ 2x-4 & \text{for } 2<x\le 4,\end{cases}$$

and that $\mathrm{f}(x)=\mathrm{f}(x+4)$.

(i) Find $\mathrm{f}(45)$. [1]

(ii) Sketch the graph of $y=\mathrm{f}(x)$ for $-4\le x\le 7$ and state the range of f. [3]

The function g is defined by $\mathrm{g}:x\mapsto(x-1)^2+2$ for $x\in\mathbb{R},\ 0<x<1$.

(iii) Explain why the composite function gf does not exist. [1]

(iv) Find an expression for $\mathrm{fg}(x)$ and hence, or otherwise, find $(\mathrm{fg})^{-1}(\frac{1}{2})$. [4]

6 The plane p has equation $x-2y+2z=12$. With reference to the origin O, the point A has position vector $2\mathbf{i}+\mathbf{j}-3\mathbf{k}$.

(i) Find the coordinates of the foot of perpendicular, N, from A to p. Hence determine the coordinates of the reflection of A in p. [5]

(ii) Find the cartesian equations of the planes such that the perpendicular distance from each plane to p is 15. [5]

7 **(a)** Find $\int \frac{2x^3}{1+x^4}\,\mathrm{d}x$. [2]

(b) Find $\int \cos 4x \sin 10x\,\mathrm{d}x$. [2]

(c) Show, by means of the substitution $x=\tan\theta$, that $\int_0^1 \frac{x^3}{\left(x^2+1\right)^3}\,\mathrm{d}x=\int_0^{\frac{\pi}{4}}\cos\theta\sin^3\theta\,\mathrm{d}\theta$. Hence find the exact value of $\int_0^1 \frac{x^3}{\left(x^2+1\right)^3}\,\mathrm{d}x$.

[5]

8 A marathon runner and a sprinter are running around in laps on a track. Each lap is 400 m. The sprinter runs his first lap in 56 seconds, and the time he takes for each subsequent lap is 6 seconds more than the previous lap. The marathon runner runs his first lap in 1 minute, and the time he takes for each subsequent lap is 4% more than the time taken for the previous lap.

[Turn over

(i) Show that the time the sprinter takes to complete n laps is $3n^2 + 53n$. [2]

(ii) Find the time the marathon runner takes to complete n laps, giving your answer in terms of n. [2]

(iii) How many complete laps does it take for the marathon runner to first exceed a total time of 40 minutes? [3]

The 2 runners decide to do a 12km race with the same starting point.

(iv) Determine which runner will be the first to complete the race. [3]

(v) Determine which lap the winner is on when he first overtakes the other runner. [3]

9 **(a)** The function f is defined as $\mathrm{f}(x) = x^4 - 4x^3 + ax^2 - 16x + b$, where a and b are real and non-zero.

(i) Given that two of the roots of $\mathrm{f}(x) = 0$ are of the form $k\mathrm{i}$, where k is real and non-zero, find these two roots and show that $16 - 4a + b = 0$. [4]

(ii) The graph of f meets the y-axis at (0, 20). Express $\mathrm{f}(x)$ as a product of two quadratic factors. [4]

(b) Find the modulus of the complex number

$$\frac{z\left(\mathrm{i} - z^*\right)}{3\mathrm{i}\left(z^2 + \mathrm{i}z\right)},$$

where z is a complex number. [3]

10 **(i)** On the same axes, sketch the graphs of $y = \sin 2x$ and $y = \cos x$ for $0 \le x \le \frac{\pi}{2}$, stating the exact coordinates of the points where the curves cross the axes. [2]

(ii) Solve exactly the inequality $\sin 2x > \cos x$, where $0 \le x \le \frac{\pi}{2}$. [2]

(iii) Hence, find $\int_0^{\frac{\pi}{2}} \left|\sin 2x - \cos x\right| \mathrm{d}x$ without using a calculator. [4]

(iv) The region bounded by the curves $y = \sin 2x$, $y = \cos x$ and the y-axis, where $x \ge 0$ is rotated through 2π radians about the x-axis. Find the exact volume of the solid obtained. [3]

11 [The volume of a cone of base radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$.]

A disposable cup for water dispenser, in the form of a cone, is made from a circular piece of paper of radius 8 cm with negligible thickness. A sector is cut off from the piece of paper as shown in Fig. 1. The rest of the paper is then folded to form a cone with slant height 8 cm, base radius r cm and height h cm as shown in Fig. 2.

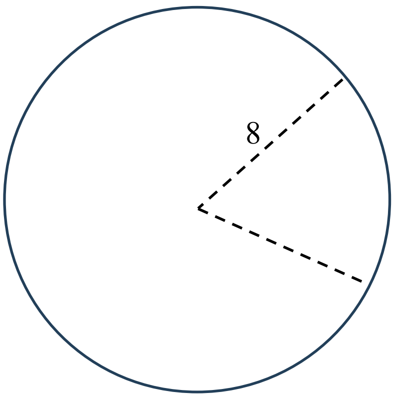

Fig. 1

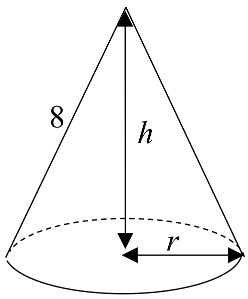

Fig. 2

(i) Find the maximum volume of the cone, giving your answer in exact form. [6]

(ii) Show that $r = \sqrt{2}h$ when the volume of the cone is a maximum. [2]

(iii) The cone with the volume found in **(i)** is then inverted and is held with its axis vertical and vertex downwards. Water from the dispenser flows into the cone at a rate of 3π cm^3 per second. At time t seconds after the start, the radius of the water surface is x cm as shown in Fig. 3. Find the rate of change of the height of the water in the cone after 6 seconds.

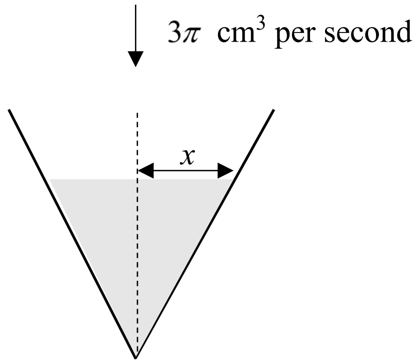

Fig. 3

[4]