ANGLO-CHINESE JUNIOR COLLEGE MATHEMATICS DEPARTMENT

MATHEMATICS Higher 2

9740 / 02

Paper 2

27 August 2012

JC 2 PRELIMINARY EXAMINATION

Time allowed: **3 hours**

Additional Materials: List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Index number, Form Class, graphic and/or scientific calculator model/s on the cover page. Write your Index number and full name on all the work you hand in.

Write in dark blue or black pen on your answer scripts.

You may use a soft pencil for any diagrams or graphs.

Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in the question, you are required to present the mathematical steps using mathematical notations and not calculator commands. You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question. At the end of the examination, fasten all your work securely together.

This document consists of 6 printed pages.



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ANGLO-CHINESE JUNIOR COLLEGE MATHEMATICS DEPARTMENT JC2 Preliminary Examination 2012						
MATHEMATICS 9740 Higher 2 Paper 2		/ 100				
Index No:	Form Class:					
Calculator model:						

Arrange your answers in the same numerical order.

Place this cover sheet on top of them and tie them together with the string provided.

Question No.	Marks
1	/4
2	/9
3	/11
4	/16
5	/3
6	/7
7	/9
8	/7
9	/9
10	/7
11	/9
12	/9

Section A: Pure Mathematics [40 marks]

1 The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix} 2.5k \\ k \\ 2k \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} 3 \\ 0.5k \\ 0.5k \end{pmatrix} + \mu \begin{pmatrix} 4 \\ 7 \\ 5 \end{pmatrix}, \ \lambda, \mu \in \mathbb{R}$$

respectively, where k is a constant.

- (i) Given that l_1 and l_2 intersect at a point, find the value of k. [3]
- (ii) Hence find the coordinates of the point of intersection. [1]

2 Given that
$$y = e^{\sin^{-1}2x}$$
, show that $(1-4x^2)\frac{d^2y}{dx^2} = 4y + 4x\frac{dy}{dx}$. [3]

By differentiating this result, or otherwise, find the series expansion of y in ascending powers of x, up to and including the term in x^3 . [4]

Hence find an approximate value of $e^{\frac{\pi}{6}}$ as a fraction in its simplest form. [2]

3 (i) Use the substitution
$$x = 1 + \sin u$$
 to find $\int \sqrt{2x - x^2} dx$. [5]
(ii) The finite region *R*, which includes the point (0,1), is bounded by the curves

(a) the exact area of
$$R$$
,
(b) the numerical value of the volume of revolution formed when R is

- (b) the numerical value of the volume of revolution formed when R is rotated completely about the x-axis, giving your answer to 3 decimal places. [2]
- 4 The equations of four planes p_1 , p_2 , p_3 and p_4 are

$$x + y + z = 2,$$

-x + ay + z = 1,
$$2x + 2y + 2z = b,$$

cx + 2y + 3z = d,

respectively, where a, b, c and d are constants. The planes p_1 and p_2 intersect in a

line *l* whose equation is
$$x-1 = \frac{y-1}{2} = -\frac{z}{3}$$
.
(i) Show that $a = 2$. [2]

(ii) Given that point A has coordinates (0,0,1), find the position vector of the foot

N of the perpendicular from A to the plane p_1 .

[3]

- (iii) Find the acute angle between planes p_1 and p_2 . [2]
- (iv) Given that the distance between p_1 and p_3 is $\sqrt{3}$, find the value of b where b > 0. [3]
- (v) Given that p_1 , p_2 and p_4 meet in a line, find the values of c and d. [4]
- (vi) Given instead that the planes p_1 , p_2 and p_4 have no point in common, what can be said about the values of c and d? [2]

Section B: Statistics [60 marks]

- 5 A company comprises 150 programmers, 15 secretaries and 15 section heads. The owner of the company wants to gather feedback from the entire company. Describe, in this context, how a sample of 12 could be chosen using stratified sampling and state an advantage of stratified sampling. [3]
- 6 Four boys and three girls are at a playground.
 -) In one of their games, all seven of them have to stand in a straight line.(i) If all the girls are separated, find the number of ways in which this can be
 - done. [2] (ii) If all the boys are to be together, find the number of ways in which this can
 - be done. [2]
 - (b) In another game, only six of them can play at a time and the six players have to stand in a circle with all the girls separated. Find the number of ways in which this can be done. [3]
- 7 In an experiment involving two chemicals x and y, a researcher recorded observations of values of y for controlled values of x as shown in the table.

x	6.3	12.3	8.1	7.5	4.4	5.1	10.6	11.7	9.4
у	6.3	4.5	5.3	5.6	3.2	2.7	4.6	4.5	4.8

- (i) Draw a scatter diagram to illustrate the data and calculate the value of the correlation coefficient. [2]
- (ii) Explain whether your answers in (i) suggest that a linear model is appropriate. [2]

The researcher realised that some of the observations came from contaminated materials. He then considered only the seven pairs of observations for which the values of x exceeded 6 and discarded the other observations. By considering the product moment correlation coefficient for these seven pairs of observations, identify which of the following is a better model:

(a)
$$y = ax^2 + b$$
, or

 $(\mathbf{b}) \quad y = a \ln x + b,$

where *a* and *b* are constants, giving a reason for your choice.

[2]

Hence using the better model, estimate the value of x when y = 6.1. For this model, comment on the validity of this estimated value. [3]

- 8 Cars arrive at a jetty randomly at an average rate of one every 5 minutes. From the jetty, a ferry transports the cars to an island. The ferry can transport at most 3 cars at a time. The ferry departs the jetty when it is full or half an hour after its arrival whichever is sooner.
 - (a) If the jetty is empty when the ferry arrives, find the probability that the ferry
 - (i) departs with 2 cars
 - (ii) departs when it is still not full.
 - (b) The ferry takes 5 minutes to travel from the jetty to the island and vice versa. The ferry leaves the jetty when it is empty and stays at the island for 10 minutes. The probability that more than k cars are waiting at the jetty when the ferry arrives is less than 0.1. Find the least value of k. [3]
- 9 (a) Each day a train travels from Town A to Town B and it is scheduled to arrive at Town B at 0800 hrs. During the first year of operation it leaves Town A at 0700 hrs and is found to be late arriving at Town B on an average of once every 4 days. In the second year it leaves Town A at 0650 hrs and is late once every 20 days. Assuming the time of journey has a normal distribution, find, to the nearest minute, the standard deviation of the time of journey. [4]
 - (b) Each day another train travels from Town B to Town C. The train is scheduled to arrive at Town C at 1200 hrs. The time taken for this journey is a normal random variable with mean 80 mins and variance 9. Find the last time of departure of the train (to the nearest minute) from Town B which ensures that its probability of arriving late at Town C does not exceed 0.02. [2]

The departure time of the train from Town B is now fixed at 1040 hrs and the arrival time of the train in Town C is observed over 20 days. Find the probability that the average arrival time of the train is within 2 minutes of the scheduled arrival time of 1200 hrs. [3]

10 A factory produces rectangular white marble floor tiles. The length of the floor tiles in centimetres can be regarded as a normal random variable with mean 18.9 and standard deviation 0.3. The breadth of the tiles in centimeters can be regarded as an independent normal random variable with mean 8.9 and standard deviation 0.1. Find the mean and standard deviation of the total perimeter of ten randomly chosen tiles. [3]

60% of the tiles produced by the factory have a red tint when laid. The tint appearing on each tile is independent of another tile. These tiles are sold in packaging containing 500 tiles each. The factory claims that each packaging has at least k redtinted tiles. Use a suitable approximation to find the greatest value of k such that the factory's claim is correct at least 95% of the time. [4]

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[2]

[2]

11 A random sample of 27 individuals from a population of young men aged 18 was chosen and the length of the right foot of each individual was measured to the nearest centimetre. The data is as given in the table below.

Foot Length (cm)	24	25	26	27	28	29	30
Number with this foot length	1	2	3	9	6	5	1

Use this data to test if the mean foot length of an 18 year old man is more than 27 cm at 2% level of significance, stating any assumptions you have made. [5]

A random sample of 60 individuals from a population of young women aged 18 have foot lengths summarised as $\overline{x} = 26.6$ and $\sum (x - \overline{x})^2 = 123.20$.

The mean foot length of an 18 year old women is claimed to be k. Find the range of values for k given that this claim is rejected at the 4% level of significance. [4]

- 12 Ai Ming plays a computer game with Shu Ting, where they each shoot at a target. For each shot, the probability that the shot hits the target is 0.25, independently of all other shots for both players and they each take the same number of attempts at the target. Find
 - (i) the most likely number of shots, out of 20 shots attempted, that hit the target for Ai Ming,[3]
 - (ii) the probability that, out of 20 shots attempted, Ai Ming hits the target at least five times given that she hits the target only at most ten times, [3]
 - (iii) the probability that Ai Ming and Shu Ting take the same number of shots to hit the target. [3]