Name	Index Number	Class

MARKING SCHEME

WOODGROVE SECONDARY SCHOOL

A COMMUNITY OF FUTURE-READY LEARNERS AND THOUGHTFUL LEADERS

O LEVEL PRELIMINARY EXAMINATION 2023

LEVEL & STREAM	**:**	SECONDARY 4 EXPRESS/ 5 NORMAL ACADEMIC
SUBJECT (CODE)	**:**	ADDITIONAL MATHEMATICS (4049)
PAPER NO	**:**	01
DATE (DAY)	**:**	11 SEPTEMBER 2023 (MONDAY)
DURATION	**:**	2 HOURS 15 MINUTES

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks in this paper is 90.

DO NOT TURN OVER THE QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Student's Signature		**Parent's Signature**	
Date		**Date**	

/ **90**

This document consists of **20** printed pages including this cover page.
Setter : Ms Nicole Ng

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

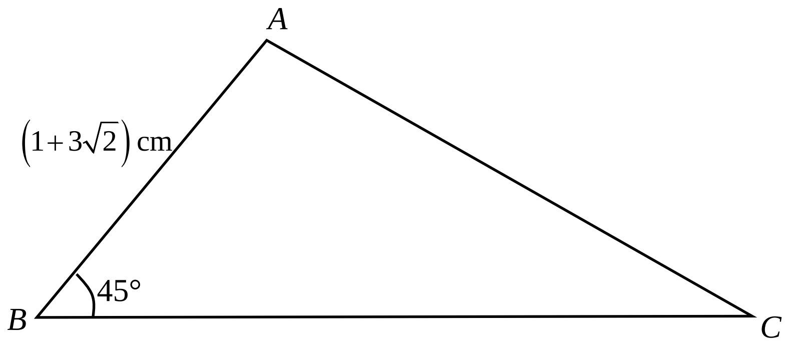
Formulae for Δ *ABC*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Triangle *ABC* is such that the length of side *AB* is $\left(1+3\sqrt{2}\right)$ cm, angle *ABC* is 45° and its area is $\left(7+4\sqrt{2}\right)\text{cm}^2$. Find, without using a calculator, the exact length of *BC*, in cm. Leave your answer in the form of $\left(a+b\sqrt{2}\right)$, where *a* and *b* are integers. [4]



$7+4\sqrt{2}=\frac{1}{2}\left(1+3\sqrt{2}\right)(BC)\sin 45°$ $M1$

$7+4\sqrt{2}=\frac{1}{2}\left(1+3\sqrt{2}\right)(BC)\left(\frac{1}{\sqrt{2}}\right)$ $M1$(for sin45°)

$$BC=\frac{2\sqrt{2}\left(7+4\sqrt{2}\right)}{1+3\sqrt{2}}\times\frac{1-3\sqrt{2}}{1-3\sqrt{2}} \qquad M1$$

$$=\frac{\left(14\sqrt{2}+16\right)\times\left(1-3\sqrt{2}\right)}{-17}$$

$$=\frac{14\sqrt{2}-84+16-48\sqrt{2}}{-17}$$

$$=\frac{-34\sqrt{2}-68}{-17}$$

$$=\left(4+2\sqrt{2}\right)\text{cm} \qquad A1$$

2 Given that $4^x \times 6^{2x+3} = 24^{2+x}$, find the value of 6^x without using a calculator. [4]

$4^x \times 6^{2x+3} = 24^{2+x}$

$2^{2x} \times 2^{2x+3} \times 3^{2x+3} = 8^{2+x} \times 3^{2+x}$

$2^{4x+3} \times 3^{2x+3} = 2^{6+3x} \times 3^{2+x}$ $M1$

$\frac{3^{2x+3}}{3^{2+x}} = \frac{2^{6+3x}}{2^{4x+3}}$

$3^{x+1} = 2^{-x+3}$ $M1$

$3^x \times 3^1 = 2^{-x} \times 2^3$

$\frac{3^x}{2^{-x}} = \frac{2^3}{3}$ $M1$

$6^x = \frac{8}{3}$ $A1$

3 When a polynomial $f(x)$ is divided by $(x+1)$ and $(x+2)$, the remainders are 3 and 5 respectively. Find the remainder when $f(x)$ is divided by $(x+1)(x+2)$. [4]

$f(x) = (x+1)(x+2)Q(x) + ax + b$

$f(-1) = 3$

$3 = -a + b$.........................(1) $M1$

$f(-2) = 5$

$5 = -2a + b$.......................(2) $M1$

$(1) - (2):$ $M1$

$-2 = a$

$b = 1$

$\therefore$ remainder = $-2x + 1$. $A1$

4 Given that $\int_{-1}^{2} \mathrm{f}(x)\,\mathrm{d}x = \int_{2}^{4} \mathrm{f}(x)\,\mathrm{d}x = 6$, find

(a) $\int_{-1}^{4} 2\mathrm{f}(x)\,\mathrm{d}x + \int_{4}^{2} \mathrm{f}(x)\,\mathrm{d}x$, [2]

$\int_{-1}^{4} 2\mathrm{f}(x)\,\mathrm{d}x + \int_{4}^{2} \mathrm{f}(x)\,\mathrm{d}x$

$= 2\left[\int_{-1}^{2} \mathrm{f}(x)\,\mathrm{d}x + \int_{2}^{4} \mathrm{f}(x)\,\mathrm{d}x\right] - \int_{2}^{4} \mathrm{f}(x)\,\mathrm{d}x$ $\quad M1$

$= 2(6+6) - 6$

$= 18$ $\quad A1$

(b) the value of k for which $\int_{-1}^{2} [\mathrm{f}(x) + kx]\,\mathrm{d}x = 9$. [3]

$\int_{-1}^{2} [\mathrm{f}(x) + kx]\,\mathrm{d}x = 9$

$\int_{-1}^{2} \mathrm{f}(x)\,\mathrm{d}x + \int_{-1}^{2} kx\,\mathrm{d}x = 9$

$6 + \left[\frac{kx^2}{2}\right]_{-1}^{2} = 9$ $\quad M1$

$\left[\frac{kx^2}{2}\right]_{-1}^{2} = 3$

$\left[\frac{k(2)^2}{2}\right] - \left[\frac{k(-1)^2}{2}\right] = 3$ $\quad M1$

$2k - \frac{k}{2} = 3$

$k = 2$ $\quad A1$

5 **(a)** Find the $\dfrac{1}{x}$ term in the expansion of $\left(x^2+\dfrac{2}{x}\right)^{10}$. [3]

$$T_{r+1}=\binom{10}{r}\left(x^2\right)^{10-r}\left(2x^{-1}\right)^r \qquad M1$$

$$=\binom{10}{r}2^r x^{20-3r}$$

$$20-3r=-1 \qquad M1$$

$$r=7$$

$$T_8=\binom{10}{7}2^7 x^{-1}=\frac{15360}{x} \qquad A1$$

(b) Hence, find the constant term in the expansion of $(1+3x)\left(x^2+\dfrac{2}{x}\right)^{10}$. [2]

$$(1+3x)\left(x^2+\frac{2}{x}\right)^{10}=(1)(0)+(3x)\left(\frac{15360}{x}\right) \qquad M1$$

$$=46080 \qquad A1$$

6 A spherical balloon expands at a constant rate of 8 cm^3/s. The balloon is initially empty.

(a) Find the rate of increase of its radius when the radius is 2.5 cm, leaving your answer in terms of π.

[The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.] [3]

$\frac{dV}{dr} = 4\pi r^2$ $M1$

$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$ *OR* $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$= \frac{1}{4\pi(2.5)^2} \times 8$ $M1$ $8 = 4\pi(2.5)^2 \times \frac{dr}{dt}$ $M1$

$= \frac{8}{25\pi}$ cm/s $A1$

(b) When the radius is beyond 5 cm, besides the expansion, air begins to leak out from the balloon at a rate of 2 cm^3/s. Find the rate of change of the radius when it is 8 cm. [2]

$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$ *OR* $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$= \frac{1}{4\pi(8)^2} \times 6$ $M1$(for $\frac{dV}{dt} = 6$) $6 = 4\pi(8)^2 \times \frac{dr}{dt}$

$= \frac{3}{128\pi}$ cm/s $A1$

Accept 0.00746 m (3 s.f)

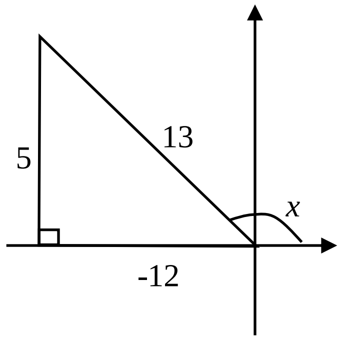
7 Given that $\sin x = \dfrac{5}{13}$ and x is obtuse, find the exact value of the following.

(a) $\sec(-x)$ [3]

$$\sec(-x) = \frac{1}{\cos(-x)} \qquad M1$$

$$= \frac{1}{\cos x} \qquad M1$$

$$= -\frac{13}{12} \qquad A1$$



(b) $\cos\dfrac{x}{2}$ [3]

$$\cos x = -\frac{12}{13}$$

$$-\frac{12}{13} = 2\cos^2\frac{x}{2} - 1 \qquad M1$$

$$\cos^2\frac{x}{2} = \frac{1}{26} \qquad M1$$

$$\cos\frac{x}{2} = \frac{\sqrt{26}}{26}\left(\text{accept } \frac{1}{\sqrt{26}}\right) \quad or \quad -\frac{\sqrt{26}}{26}\,(rej) \qquad A1$$

8 The number of ants, N, in a colony after t days can be modelled by $N = 1200e^{at}$, where a is a constant. There are 10 000 ants after 6 days.

(a) Find the initial number of ants in the colony. [1]

$N = 1200e^{a(0)} = 1200 \qquad B1$

(b) How many ants are there after 15 days? Give your answer correct to 2 significant figures. [3]

$10000 = 1200e^{6a} \qquad M1$

$e^{6a} = \dfrac{10000}{1200}$

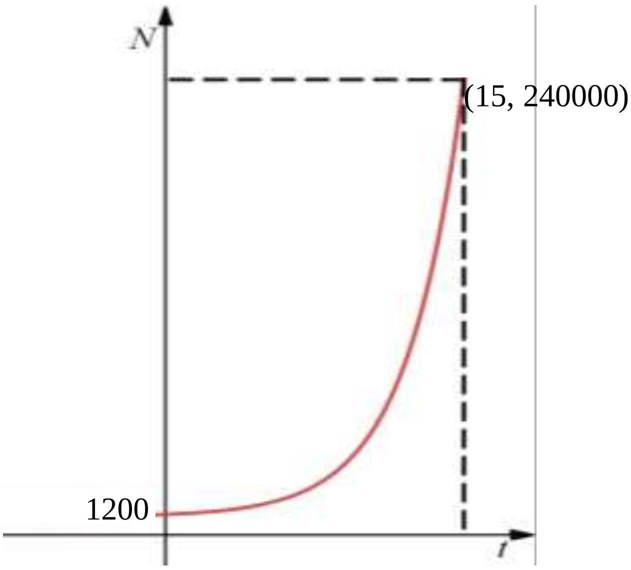
$6a = \ln\dfrac{10000}{1200} \qquad M1$

$a = 0.353377$

$N = 1200e^{(0.353377)(15)}$

$= 240000 \qquad A1$

(c) Sketch the graph of $N = 1200e^{at}$ for the first 15 days. [2]



B1 – for the shape of the graph

B1 – for the y-intercept at 1200 and the point at t = 15 days.

9 **(a)** Find the range of values of m for which the function $y=x^2-4mx+3-m$ is always positive for all real values of x. [3]

$b^2-4ac=(-4m)^2-4(1)(3-m)$ $M1$

$=16m^2+4m-12$

$16m^2+4m-12<0$ $M1$

$4m^2+m-3<0$

$(4m-3)(m+1)<0$

$-1<m<\frac{3}{4}$ $A1$

(b) Show that the line $y=4x+p$ intersects the curve $y=px^2-2p-6$ for all real values of x, where p is positive. [4]

$px^2-2p-6=4x+p$ $M1$

$px^2-4x-3p-6=0$

$b^2-4ac=(-4)^2-4(p)(-3p-6)$ $M1$

$=12p^2+24p+16$

Method 1

For $12p^2+24p+16$,

$b^2-4ac=(24)^2-4(12)(16)$

$=-192<0$ $M1$

$\therefore 12p^2+24p+16>0$

$\therefore$ will intersect. $A1$

Method 2

$b^2-4ac=12(p^2+2p)+16$

$=12(p+1)^2-12(1)^2+16$

$=12(p+1)^2+4$ $M1$

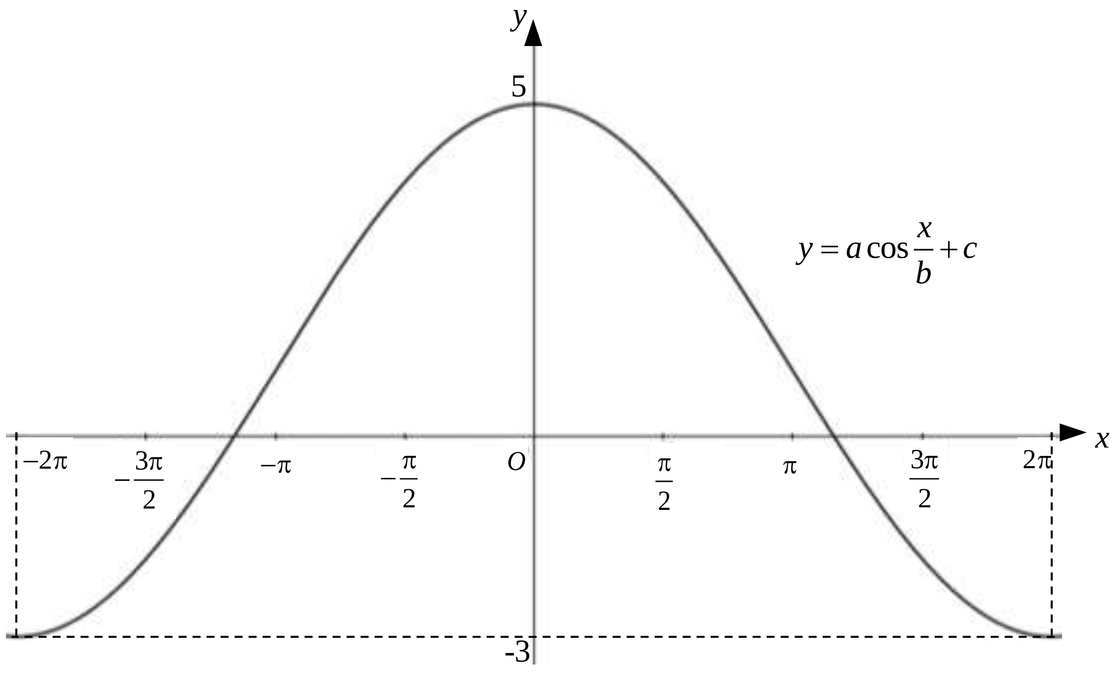
min value = 4>0, $\therefore b^2-4ac>0$

$\therefore$ will intersect. $A1$

10 **(a)** State the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$ in degrees. [1]

$-60°$ $B1$

(b) The diagram shows a sketch of the graph $y = a\cos\dfrac{x}{b} + c$, where a, b and c are integers. Find the values of a, b and c. [3]



$a = 4, b = 2, c = 1$ $B3$

(c) Given that $y = 8\cos^2 x - 2\sin^2 x$, express y in the form of $p\cos 2x + q$, stating the value of each of the integers p and q. Explain why y will never reach 10. [4]

$y = 8\cos^2 x - 2\sin^2 x$

$= 8\cos^2 x - 2\left(1 - \cos^2 x\right)$ $M1$

$= 10\cos^2 x - 2$

$= 5\left(2\cos^2 x - 1 + 1\right) - 2$

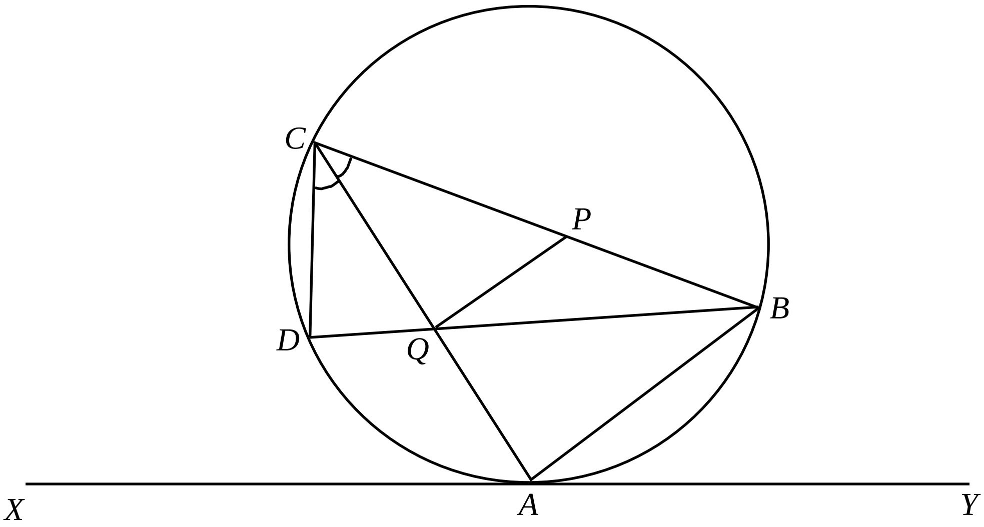
$= 5\left(\cos 2x + 1\right) - 2$ $M1$

$= 5\cos 2x + 3$

$p = 5, q = 3$ $A1$

Max value of $y = 5 + 3 = 8 < 10$ $B1$

11 The diagram below shows a circle with points A, B, C and D at its circumference where XY is a tangent to the circle at point A. P and Q are the midpoints of BC and AC respectively. BQD is a straight line and $\angle QCD = \angle QCP$.



(a) Prove that $\angle BAY = \angle QCD$. [2]

$\angle BAY = \angle QCP$ (angles in alternate segments or tangent chord thm) $M1$
$\angle QCP = \angle QCD$ (given)
$\therefore \angle BAY = \angle QCD$ (shown) $A1$

(b) **(i)** Show that ΔQCP is similar to ΔDCQ. [4]

In ΔQCP and ΔDCQ,
$\angle QCP = \angle DCQ$ (given)
$QP // AB$ (Midpoint Thm) $M1$
$\angle CQP = \angle CAB$ (corresponding angles) $M1$
$\angle CAB = \angle CDQ$ (angles in same segment) $M1$
$\therefore \angle CQP = \angle CDQ$
$\therefore$ ΔQCP and ΔDCQ and similar.(AA test) $A1$

(b) **(ii)** Show that $2QC \times DQ = AB \times DC$. [2]

From **(bi),**

$\dfrac{QC}{DC} = \dfrac{QP}{DQ}$ $M1$

$QC \times DQ = QP \times DC$

$QC \times DQ = \dfrac{1}{2} AB \times DC$ (Midpt Thm)

$\therefore 2QC \times DQ = AB \times DC$ $A1$

12 It is given that $y = \dfrac{2x^2+3}{x}$, $x \neq 0$.

(a) Prove that $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = y$. [4]

$y = \dfrac{2x^2+3}{x} = 2x + 3x^{-1}$

$\dfrac{dy}{dx} = 2 - 3x^{-2}$ $M1$

$\dfrac{d^2y}{dx^2} = 6x^{-3}$ $M1$

$x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = x^2\left(\dfrac{6}{x^3}\right) + x\left(2 - \dfrac{3}{x^2}\right)$ $M1$ Accept e.c.f

$= \dfrac{6}{x} + 2x - \dfrac{3}{x}$

$= \dfrac{3}{x} + 2x$

$= \dfrac{2x^2+3}{x} = y$ $A1$

(b) Find, in exact values, the x-coordinates of the turning points of y. [2]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 0$$

$$2 - \frac{3}{x^2} = 0 \qquad M1$$

$$x^2 = \frac{3}{2}$$

$$x = \pm\sqrt{\frac{3}{2}} \quad \text{OR} \quad \pm\frac{\sqrt{6}}{2} \qquad A1$$

(c) Determine the nature of each of the turning points. [2]

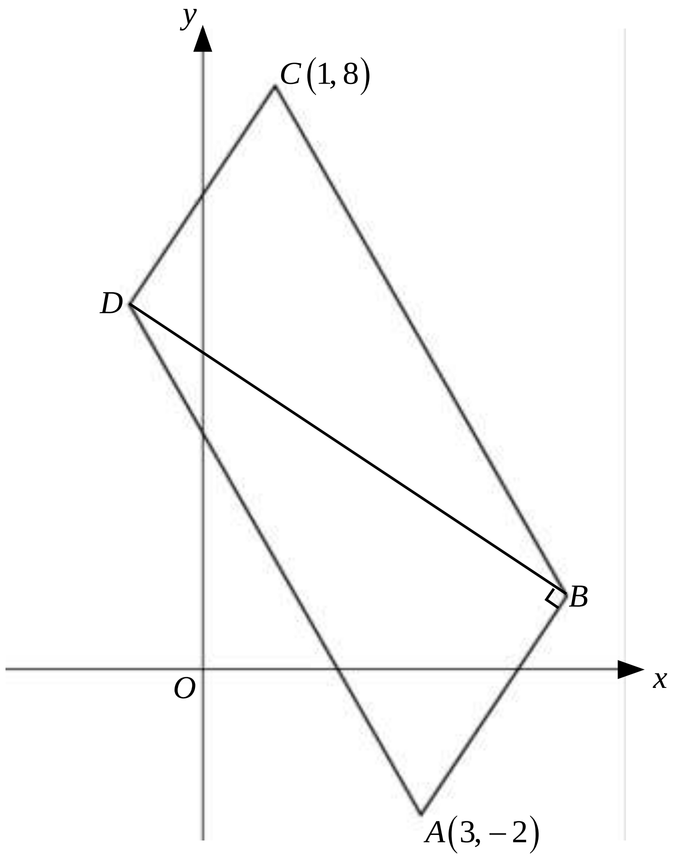
For $x = \frac{\sqrt{6}}{2}$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0, \therefore \text{min}.$ $B1$

For $x = -\frac{\sqrt{6}}{2}$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} < 0, \therefore \text{max}.$ $B1$

Accept e.c.f (full marks awarded if x values were wrong in previous parts.

13 Solutions to this question by accurate drawing will not be accepted.

The parallelogram *ABCD* is such that the points *A* and *C* are $(3,-2)$ and $(1,8)$ respectively. The line *BD* is parallel to the line $2x+3y=4$ and is perpendicular to *AB*.



(a) Show that the equation of *BD* is $2x+3y=13$. [4]

$m_{BD}=-\frac{2}{3}$ B1

Midpoint of $BD=(2,3)$ M1

$y=-\frac{2}{3}x+c$ M1 or $\frac{y-3}{x-2}=-\frac{2}{3}$

$3=-\frac{2}{3}(2)+c$

$c=\frac{13}{3}$

$y=-\frac{2}{3}x+\frac{13}{3}$

$2x+3y=13$ (shown) A1

(b) Calculate the coordinates of *B*. [4]

Find the equation of *AB*,

$m_{AB} = \frac{3}{2}$ $B1$

$y = \frac{3}{2}x + c$ OR $\frac{y+2}{x-3} = \frac{3}{2}$

$-2 = \frac{3}{2}(3) + c$

$c = -\frac{13}{2}$

$y = \frac{3}{2}x - \frac{13}{2}$ $A1$ OR $2y = 3x - 13$

$y = -\frac{2}{3}x + \frac{13}{3}$ ....(1)

$y = \frac{3}{2}x - \frac{13}{2}$ ....(2)

$-\frac{2}{3}x + \frac{13}{3} = \frac{3}{2}x - \frac{13}{2}$ $M1$ Accept e.c.f (if previous eqn of AB is wrong.

$-4x + 26 = 9x - 39$

$13x = 65$

$x = 5$

$B(5,1)$ $A1$

(c) Calculate the coordinates of *D*. [2]

Let *D* be (x, y).

$(2,3) = \left(\frac{x+5}{2}, \frac{y+1}{2}\right)$ $M1$

$\therefore D(-1, 5)$. $A1$

14 A particle starts from rest at a fixed point *O* and moves in a straight line such that its velocity $v\ \text{ms}^{-1}$ is given by $v = 4t - \frac{3}{2}t^2$, where *t* is the time in seconds after leaving *O*.

Calculate

(a) the velocity of the particle when its acceleration is zero, [3]

$a = \frac{\mathrm{d}v}{\mathrm{d}t} = 4 - 3t$ $M1$

$4 - 3t = 0$

$t = \frac{4}{3}s$ $M1$

$v = 4\left(\frac{4}{3}\right) - \frac{3}{2}\left(\frac{4}{3}\right)^2 = \frac{8}{3}\text{m/s}$ $A1$

(b) the time when the particle is instantaneously at rest again, [2]

$4t - \frac{3}{2}t^2 = 0$ $M1$

$t = 0s(\text{rej})$ *or* $4 - \frac{3}{2}t = 0$

$t = \frac{8}{3}s$ $A1$

Must rej t = 0 or show evidence like # symbol to show this is the final answer.

(c) the total distance travelled by the particle when it returns to O. [5]

$s = \int v \, dt$

$= \int 4t - \frac{3}{2}t^2 \, dt$ $M1$

$= 2t^2 - \frac{1}{2}t^3 + c$ $M1$

$t = 0, s = 0, c = 0$

$s = 2t^2 - \frac{1}{2}t^3$ $A1$

$t = \frac{8}{3},$

$s = 2\left(\frac{8}{3}\right)^2 - \frac{1}{2}\left(\frac{8}{3}\right)^3 = \frac{128}{27}$ $M1$

Accept e.c.f

Total distance = $\frac{128}{27} \times 2 = \frac{256}{27}$ m OR $9\frac{13}{27}$ m $A1$

Accept 9.48 m (3 s.f)

OR

$s = 2\int_0^{\frac{8}{3}} 4t - \frac{3}{2}t^2 \, dt$ $M1$(for 2)

$M1$(for integrating)

$= 2\left[2t^2 - \frac{1}{2}t^3\right]_0^{\frac{8}{3}}$ $M1$

$= 2\left(\frac{128}{27} - 0\right)$ $M1$

$= \frac{256}{27}$

Total distance = $\frac{256}{27}$ m $A1$

END OF PAPER