Chapter 12 SUPERPOSITION

H2 PHYSICS 9749

Syllabus Content

- Principle of superposition
- Stationary waves
- Diffraction
- Two-source interference
- Single-slit and multiple-slit diffraction

Learning Outcomes

(a) explain and use the principle of superposition in simple applications.

(b) show an understanding of the terms interference, coherence, phase difference and path difference.

(c) show an understanding of experiments which demonstrate stationary waves using microwaves, stretched strings and air columns.

(d) explain the formation of a stationary wave using a graphical method, and identify nodes and antinodes.

(e) explain the meaning of the term diffraction.

(f) show an understanding of experiments which demonstrate diffraction including the diffraction of water waves in a ripple tank with both a wide gap and a narrow gap.

(g) show an understanding of experiments which demonstrate two-source interference using water waves, sound waves, light waves and microwaves.

(h) show an understanding of the conditions required for two-source interference fringes to be observed.

(i) recall and solve problems using the equation $\lambda = ax/D$ for double-slit interference.

(j) recall and use the equation $\sin\theta = \lambda/b$ to locate the position of the first minima for single slit diffraction.

(k) recall and use the Rayleigh criterion $\theta \approx \lambda/b$ for the resolving power of a single aperture.

(l) recall and use the equation $d \sin\theta = n\lambda$ to locate the positions of the principal maxima produced by a diffraction grating.

(m) describe the use of a diffraction grating to determine the wavelength of light (the structure and use of the spectrometer are not required).

12.1 Introduction

The Principle of Superposition

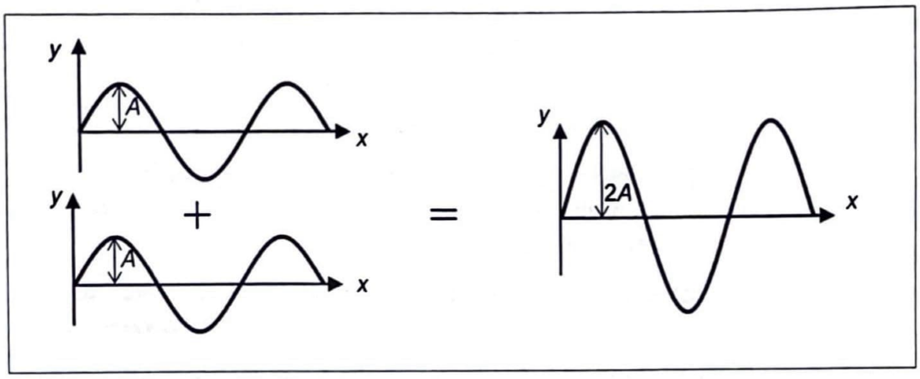
A wave is a disturbance that travels through a medium or vacuum. For mechanical waves, like sound and water waves, the disturbance refers to the displacement of the particles of the media from their equilibrium position. For electromagnetic waves, the disturbance refers to the varying electric and magnetic field.

What happens when two waves of the same kind (e.g. sound waves from two sources) meet at a point in space?

Definition

> The **principle of superposition** states that when two or more waves of the same kind meet at a point in space, the resultant displacement at that point is equal to the vector sum of the displacements of the individual waves at that point.

The diagrams below illustrate two waves meeting at an instant of time:



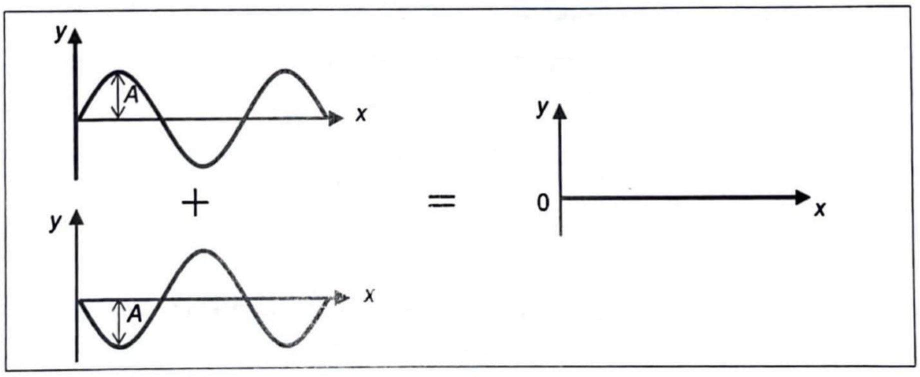


Fig. 1 Displacement of the resultant wave at any position is the vector sum of the displacements due to the two waves.

12.2 Stationary Waves

What is a Stationary Wave?

Definition

A stationary wave is the result of the superposition of two progressive waves of the same type, frequency, amplitude and speed, travelling along the same line but in opposite directions.

A stationary wave is also known as a standing wave.

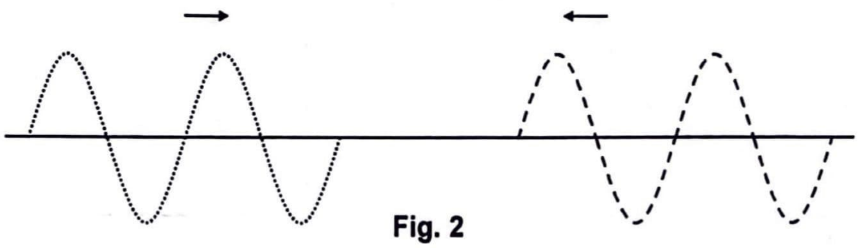


Fig. 2

The superposition of the two waves is observed along the line of propagation, giving rise to a resultant waveform.

The set of graphs in Fig. 3 represents two progressive waves of equal amplitude and frequency travelling in opposite directions.

At t = 0 s, superposition of the waves gives rise to a resultant wave which has twice the amplitude of either progressive wave.

At t = ¼ T, the waves have each moved a quarter wavelength in opposite directions. The resultant displacement is zero everywhere.

At t = ½ T, the resultant wave has twice the amplitude of either progressive wave once again.

This continues to t = ¾ T, and resultant displacement is zero everywhere. Finally, at exactly one period, when t = T, the resultant wave has twice the amplitude of either progressive wave once again.

Definition

An **antinode** is a point in a stationary wave where the amplitude is the maximum.

The component waves always arrive in phase at the antinodes. Their positions are marked by A in Fig. 3.

A **node** is a point in a stationary wave where the amplitude is zero.

The component waves always arrive anti-phase at the nodes. Their positions are marked by N in Fig. 3.

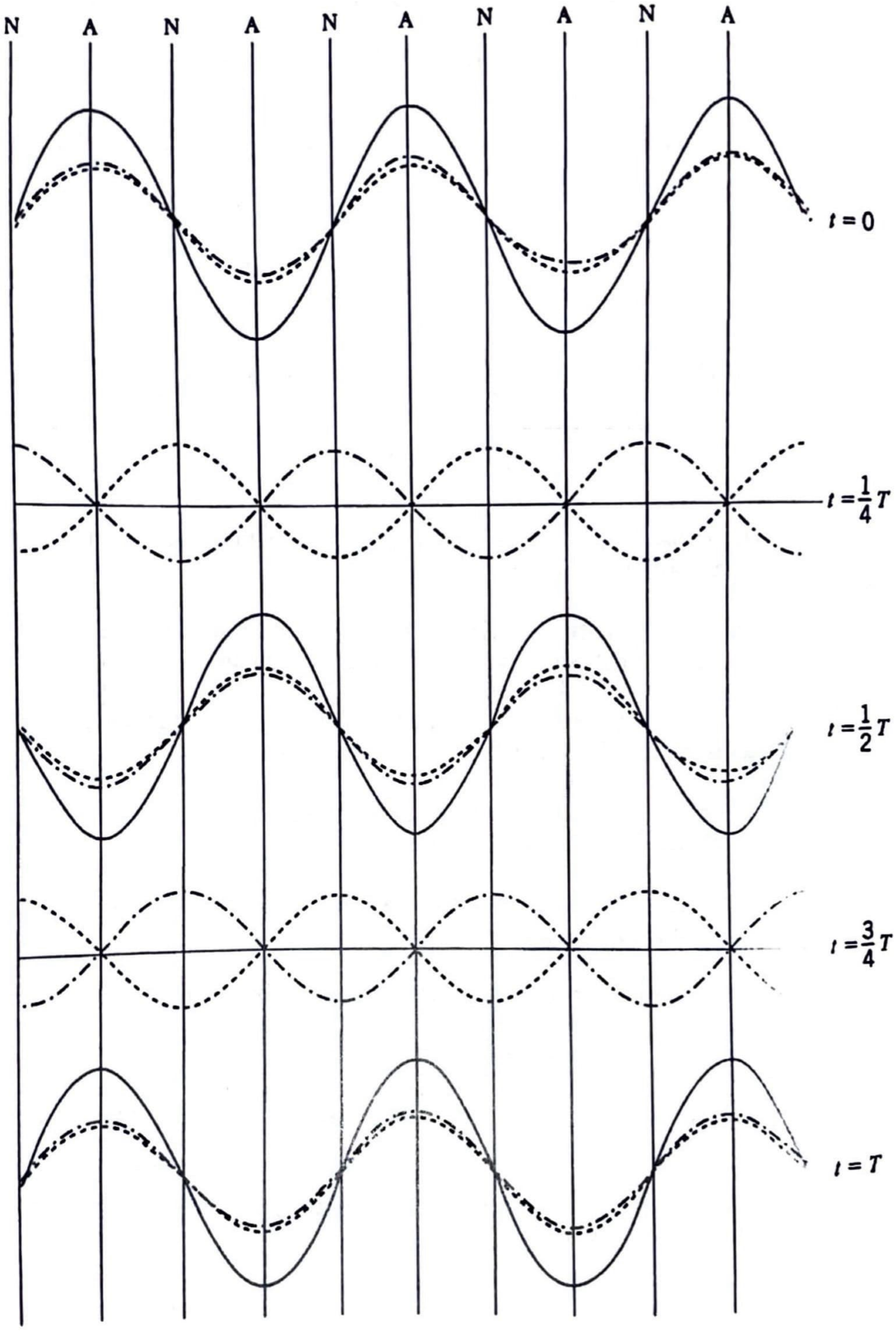


Fig. 3. Formation of stationary wave by superposition of two progressive waves travelling in opposite directions.

Properties of a Stationary Wave

The resultant wave has the following features:

1. The wave profile does not propagate. As such, the resultant wave is known as a **stationary wave** or **standing wave**.
2. The particles of the wave oscillate (except those at the nodes) about their respective equilibrium positions with the **same frequency**, but **different amplitudes**. The frequency is the same as that of the two component waves.
3. An **antinode** is a point in a stationary wave where the amplitude is the maximum. The component waves always arrive in phase at the antinodes. These are labelled 'A' on Fig. 4.
4. A **node** is a point in a stationary wave where the amplitude is zero. The component waves always arrive anti-phase at the nodes. They are labelled 'N' on Fig. 4.
5. Between two adjacent nodes, all particles oscillate **in phase**, i.e., they reach their respective maxima, minima and equilibrium positions at the same instant. Note that these particles do not have the same amplitude.
6. Distance between two adjacent nodes (or antinodes) is $\frac{1}{2}\lambda$.
7. Particles in neighbouring segments vibrate 180° (or π rad) out of phase with each other.

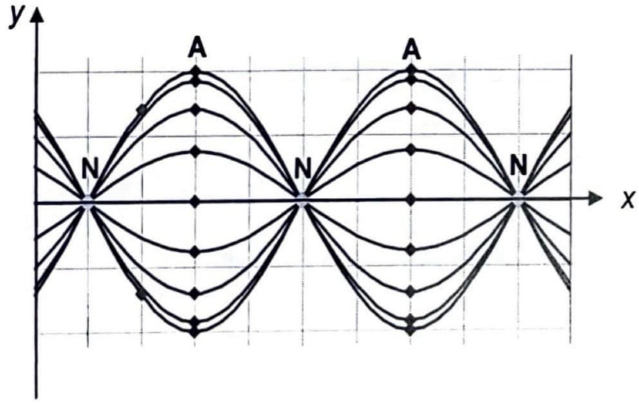


Fig. 4a: The waveform of a stationary wave at equal time interval of $T/16$ (on displacement-distance axes).

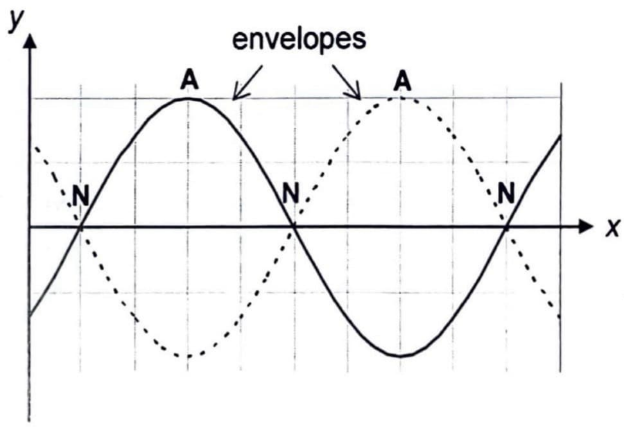


Fig. 4b: A **graphical representation** of a stationary wave (where the maximum displacement is drawn on displacement-distance axes)

Note: An **envelope** of a rapidly varying signal is a curve outlining its amplitudes.

Comparing Stationary Waves and Progressive Waves

Important Note

The table below compares the properties of a progressive wave with those of a stationary wave.

Property	Progressive Wave	Stationary Wave
Waveform	Propagates with the velocity of the wave.	Does not propagate.
Energy	Transports energy.	Does not transport energy.
Amplitude	Every point oscillates with the same amplitude.	Amplitude varies from 0 at the nodes to the maximum at the antinodes.
Phase	All particles within one wavelength have different phases.	All particles between two adjacent nodes have the same phase. Particles in adjacent segments have a phase difference of π rad.
Frequency	All points vibrate in s.h.m. with the frequency of the wave.	Except for the nodes which are at rest, all points vibrate in s.h.m. with the same frequency as the progressive waves that give rise to it.
Wavelength	Equals to the distance between adjacent points which have the same phase.	Equals to twice the distance between a pair of adjacent nodes or antinodes.

Example 1

(J90/P1/13)

Progressive waves of frequency 300 Hz are superposed to produce a system of stationary waves in which adjacent nodes are 1.5 m apart. What is the speed of the progressive waves?

Solution

$$\lambda = 1.5 \times 2 = 3.0 \text{ m}$$

$$v = f\lambda = 300 \times 3.0 = 900 \text{ m s}^{-1}$$

12.3 Stationary Waves in Strings and Pipes

Stationary Waves in Stretched Strings

If a stretched string is plucked and allowed to vibrate freely, standing waves of various frequencies are set up. At these frequencies, the amplitude of vibration is large, and the string is said to be undergoing **resonance**.

The frequency of sound produced is equal to the frequency of the stationary wave set up in the string, or a combination of various harmonics.

Consider a string that is stretched and fixed at the two ends (Fig. 5). **The fixed ends ensure that nodes will always be produced at these points.**

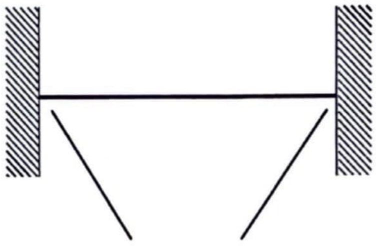


Fig. 5

When one end of the string of length L is fixed to a mechanical oscillator driven by a signal generator, different standing wave patterns are formed as the frequency of the signal generator is changed.

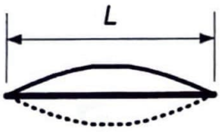


Fig. 6

The simplest way which the string can vibrate is shown in Fig. 6, where the wave pattern is a single loop. This is called the **fundamental mode of vibration** and the frequency at which the standing wave is vibrating at is known as the **fundamental frequency**.

Fig. 7

The frequency of the signal generator is gradually increased. At a particular frequency, the string exhibits its **second mode of vibration**, whereby the wave pattern has two loops (Fig. 7). This is also known as the 1st overtone or 2nd harmonic.

As the frequency of the signal generator continues to increase, the string will continue to exhibit higher modes of vibration.

Fig. 8 shows a summary of the first few modes of vibration of a stretched string as well as expressions to deduce the corresponding frequencies of the stationary wave on a string.

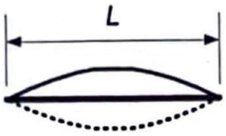
Modes of Vibration	Graphical Representation	Wavelength, λ	Frequency, f	Also known as...
Fundamental mode		$L = 1\left(\frac{\lambda_1}{2}\right)$ $\Rightarrow \lambda_1 = 2L$	$f_1 = \frac{v}{2L}$	1st harmonic
1st overtone		$L = 2\left(\frac{\lambda_2}{2}\right)$ $\Rightarrow \lambda_2 = L$	$f_2 = 2\left(\frac{v}{2L}\right)$	2nd harmonic
2nd overtone		$L = 3\left(\frac{\lambda_3}{2}\right)$ $\Rightarrow \lambda_3 = \frac{2L}{3}$	$f_3 = 3\left(\frac{v}{2L}\right)$	3rd harmonic
$(n-1)^{th}$ overtone		$L = n\left(\frac{\lambda_n}{2}\right)$ $\Rightarrow \lambda_n = \frac{2L}{n}$	$f_n = n\left(\frac{v}{2L}\right)$	n^{th} harmonic

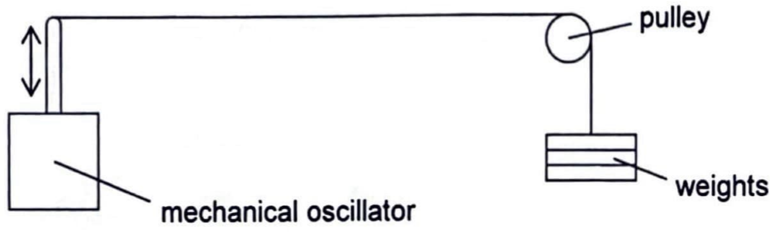
Fig. 8 Stationary Waves on Strings

Take Note

- Recall that $f = v/\lambda$, where v is the speed of the progressive waves in the string or medium.
- At the fixed ends, there must be nodes (since the string cannot vibrate).
- In reality, multiple harmonics give the timbre or characteristics of an instrument.

Example 2 (N95/P3/3(b))

In order to investigate stationary waves on a stretched string, a student sets up the apparatus shown below.



(i) Explain why it is necessary to adjust either the length of the string or the frequency of the oscillator in order to obtain observable stationary waves on the string.

(ii) What is meant by a node? Explain why a node must exist at the pulley.

Solution

(i) Since the tension of the string is constant, the speed of the wave on the string is constant. Stationary waves will only be formed if the length of the string is equal to certain multiples of half-wavelength of the wave (i.e. when resonance occurs).

$$L = n \times \tfrac{1}{2}\lambda = n \times \tfrac{1}{2}\frac{v}{f} \; ; \qquad n = 1, 2, 3.....$$

We can achieve this condition for a fixed frequency f by adjusting the length L, or for a fixed length L by adjusting the frequency f.

(ii) A node is a point on the stationary wave where the particle is always at rest. A node must exist at the pulley because the pulley, and hence the string, is fixed in position.

Note:

- The end of the string that is attached to the oscillator is also considered as a node because its amplitude of oscillations is small compared to that of the antinode.
- The speed v of a wave in a string is given by $v = \sqrt{T/\mu}$, where T = tension, μ = mass per unit length.

Example 3 (J83/P2/12)

A taut wire is clamped at two points 1.0 m apart. It is plucked near one end. Which are the three longest wavelengths present on the vibrating wire?

Solution

Fundamental $\lambda_1 = \dfrac{2(1.0)}{1} = 2.0$ m

2nd harmonic $\lambda_2 = \dfrac{2(1.0)}{2} = 1.0$ m

3rd harmonic $\lambda_3 = \dfrac{2(1.0)}{3} = 0.67$ m

Stationary Waves in Air Columns or Pipes

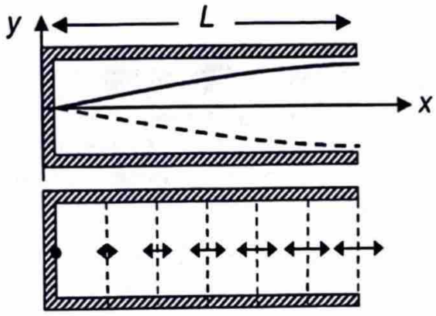
It is also possible to set up stationary sound waves in air columns or pipes. A pipe is termed *closed* if *one* end of the pipe is closed while the other is open, and termed *opened* if *both* ends of the pipe are open.

Fig. 9a closed pipe

Fig. 9b open pipe

Closed Pipes

When a sound wave is sent into a closed pipe, the wave propagates to the end of the pipe and is reflected. The reflected wave superposes with the incident wave and a stationary wave is formed. A *displacement node* is formed at the closed end of the pipe, while a *displacement antinode* is formed at the open end:



Graphical representation of a stationary wave in closed pipe – it shows the amplitude of vibration of the air molecules, but it does not mean that sound wave is a transverse wave!

Arrows indicate the amplitude of vibration of air molecules of the stationary wave.

Fig. 10

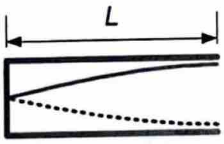
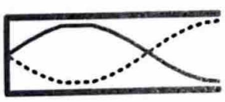
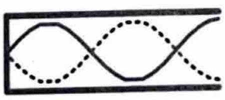
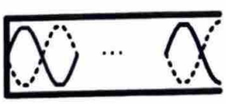
Modes of Vibration	Graphical Representation	Wavelength	Frequency	Also known as...
Fundamental mode		$L = 1\left(\frac{\lambda_1}{4}\right)$ $\Rightarrow \lambda_1 = 4L$	$f_1 = \frac{v}{4L}$	1st harmonic
1st overtone		$L = 3\left(\frac{\lambda_3}{4}\right)$ $\Rightarrow \lambda_3 = \frac{4L}{3}$	$f_3 = 3\left(\frac{v}{4L}\right)$	3rd harmonic
2nd overtone		$L = 5\left(\frac{\lambda_5}{4}\right)$ $\Rightarrow \lambda_5 = \frac{4L}{5}$	$f_5 = 5\left(\frac{v}{4L}\right)$	5th harmonic
$(n-1)^{th}$ overtone		$L = (2n-1)\left(\frac{\lambda_{2n-1}}{4}\right)$ $\Rightarrow \lambda_{2n-1} = \frac{4L}{(2n-1)}$	$f_{2n-1} = (2n-1)\left(\frac{v}{4L}\right)$	$(2n-1)^{th}$ harmonic

Fig. 11 Stationary Waves in Closed Tubes (no even harmonics)

Open Pipes

When a sound wave is sent into an open pipe, a stationary wave is formed as shown in Fig. 12. As the air molecules are able to move freely at both ends, displacement antinodes are formed at both ends.

Note:
As a sound wave travels down an open pipe and reaches the other end, part of the wave is reflected. The reflected and incident waves superpose to produce a **pressure node at the open end** so that the pressure there is atmospheric. Hence, a **displacement antinode occurs at the open end**.

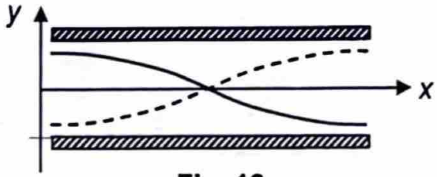


Fig. 12

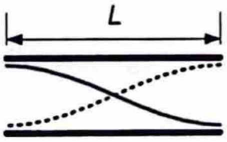
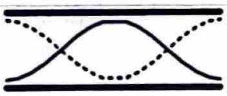
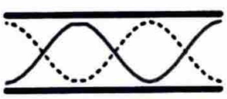
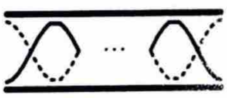
Modes of Vibration	Graphical Representation	Wavelength	Frequency	Also known as...
Fundamental frequency		$L = 1\left(\frac{\lambda_1}{2}\right)$ $\Rightarrow \lambda_1 = 2L$	$f_1 = \frac{v}{2L}$	1st harmonic
1st overtone		$L = 2\left(\frac{\lambda_2}{2}\right)$ $\Rightarrow \lambda_2 = L$	$f_2 = 2\left(\frac{v}{2L}\right)$	2nd harmonic
2nd overtone		$L = 3\left(\frac{\lambda_3}{2}\right)$ $\Rightarrow \lambda_3 = \frac{2L}{3}$	$f_3 = 3\left(\frac{v}{2L}\right)$	3rd harmonic
$(n-1)^{th}$ overtone		$L = n\left(\frac{\lambda_n}{2}\right)$ $\Rightarrow \lambda_n = \frac{2L}{n}$	$f_n = n\left(\frac{v}{2L}\right)$	n^{th} harmonic

Fig. 13 Stationary Waves in Open Tubes

End Correction

The displacement antinode at the open ends of the pipes are actually located slightly outside the pipe as shown below. This results in a small *end correction c* to be included in the calculation of the wavelength.

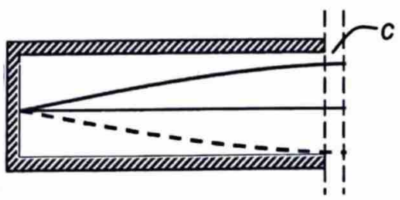


Fig. 14

Example 4 (N76/P2/3)

An organ pipe, 0.33 m long, is open at one end and closed at the other. The speed of sound in air is 330 m s^{-1}. Assuming that end corrections are negligible, calculate

(a) the frequencies of the fundamental and the first overtone,

(b) the length of a pipe which is open at both ends and which has a fundamental frequency equal to the difference of those calculated in (a).

Solution

(a) Fundamental mode:

$$L = \frac{\lambda}{4} \Rightarrow \lambda = 4(0.33) = 1.32 \text{ m}$$

$$f_1 = \frac{v}{\lambda} = \frac{330}{1.32} = 250 \text{ Hz}$$

1st overtone

$$f_3 = 3f_1 = 750 \text{ Hz}$$

(b) $f = f_3 - f_1 = 750 - 250 = 500 \text{ Hz}$

$$\lambda = \frac{v}{f} = \frac{330}{500} = 0.66 \text{ m}$$

$$L = \frac{1}{2}\lambda = \frac{1}{2}(0.66) = 0.33 \text{ m}$$

Example 5

A source of sound of frequency 250 Hz is used with a resonance tube, closed at one end, to measure the speed of sound in air. Strong resonance is first obtained at tube lengths of 0.30 m and then 0.96 m. Calculate

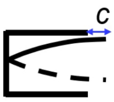
(a) the speed of sound in air, and

(b) the end correction of the tube.

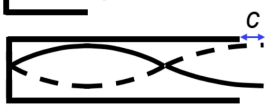
Solution

(a)

$$\frac{\lambda}{4} = 0.30 + c \quad \text{-----} \ (1)$$



$$\frac{3}{4}\lambda = 0.96 + c \quad \text{-----} \ (2)$$



Solving $(2)-(1)$: $\frac{2}{4}\lambda = 0.96$

$$\lambda = 1.32 \text{ m}$$

$$v = f\lambda = 250 \times 1.32 = 330 \text{ ms}^{-1}$$

(b)

$$\frac{\lambda}{4} = 0.30 + c \quad \text{-----} \ (1)$$

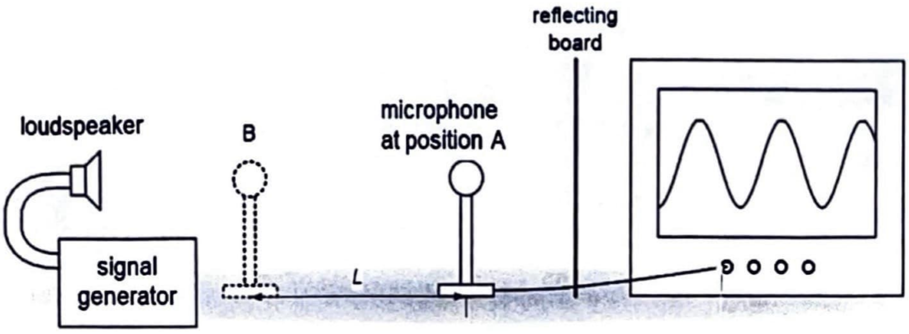
$$\frac{3}{4}\lambda = 0.96 + c \quad \text{-----} \ (2)$$

From (1): $c = \frac{\lambda}{4} - 0.30 = 0.03 \text{ m}$

12.4 * Using Stationary Waves to Determine Wavelength and Speed of Sound

** Learning Outcome (k) of Chap 11.*

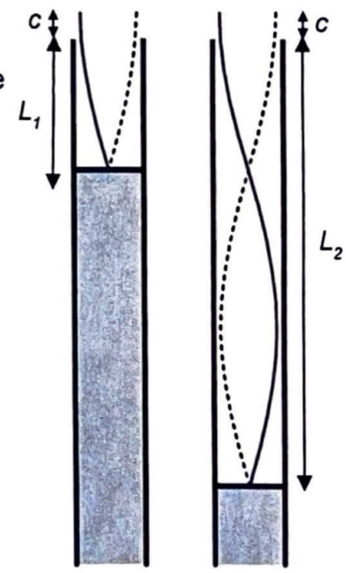
Using Cathode-Ray Oscilloscope



1. A small microphone, connected to a CRO, is positioned between a reflecting board and a loudspeaker connected to a signal generator.
2. Sound wave of constant frequency travels from the loudspeaker towards the reflecting board. Interference between the incident and reflected sound waves produces a stationary wave.
3. As the microphone is moved slowly towards the loudspeaker, the amplitude of the waveform on the CRO increases to a maximum (pressure antinode) at position A and then the next maximum at position B.
4. Since the distance L between two successive pressure antinodes (or displacement nodes) is $L = \frac{1}{2}\lambda$ and the frequency f can be determined from the CRO, the speed of the sound can be calculated using

$$v = f\lambda = 2fL$$

Using Resonance Tube



1. A tuning fork of frequency f is struck and held over the top of a tube filled with water.
2. The water level is gradually lowered or drained until the note is at its loudest as shown in (a). Note the length of the air column L_1. The air column is said to be resonating with the frequency of the tuning fork: $L_1 + c = \frac{1}{4}\lambda$ --- (a)
3. Repeat the above steps while lowering the water level further until a second weaker resonance is heard, as shown in (b). Note the new length L_2:

   $L_2 + c = \frac{3}{4}\lambda$ --- (b)
4. As shown in the diagram, equations (b) – (a):

   $\frac{1}{2}\lambda = L_2 - L_1$
5. Since the frequency f of the tuning fork is known, speed of sound in air can be calculated using

$$v = f\lambda = 2f(L_2 - L_1)$$

12.5 Diffraction

What is Diffraction?

Definition

Diffraction is the bending of waves after passing through an aperture or round an obstacle.

Due to the effects of diffraction, waves bend from a straight path and enter a region that would otherwise be shadowed.

For example, when you are in a room, you can hear someone along the corridor through the open door of your room, even if you cannot see them. Why?

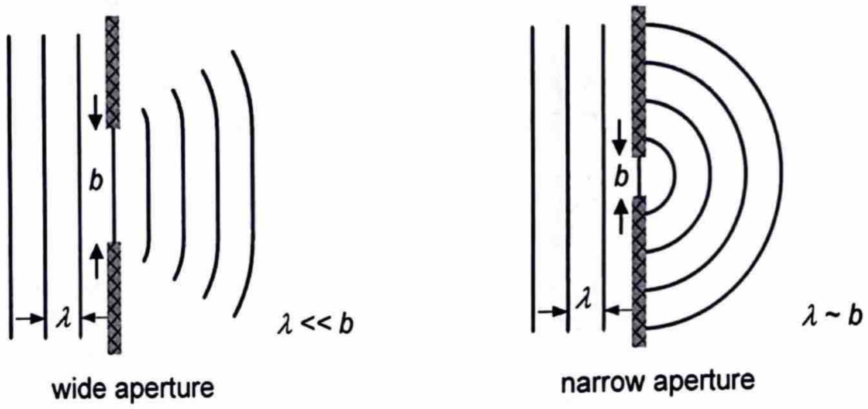


Fig. 15

The degree of diffraction depends on the relative size of the wavelength and the aperture.

Generally, **diffraction is pronounced when the wavelength of the wave is of the same order of magnitude as the width of the aperture or obstacle.** (Diffraction is explained using Huygen's principle in Sect 12.10 on pg27.)

This is the reason why under normal circumstances, we do not observe any diffraction of light because the holes and apertures that we come across everyday are much larger than the wavelengths of light.

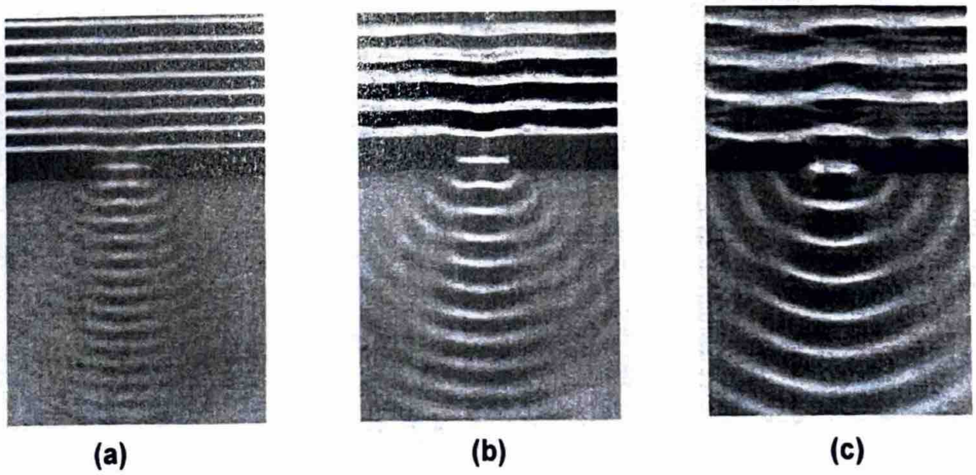


Fig. 16 Ripple tank images of water waves emerging from an opening of fixed width. As the wavelength is increased from **(a)** to **(c)**, the effect of diffraction becomes more pronounced.

12.6 Interference

Coherence

Definition

Waves are **coherent** when they have a **constant phase difference**.

Waves exhibit **coherence** when they have a **constant phase difference**.

This implies that coherent waves must have the same frequency, but the reverse is not true. Velocities, and hence wavelengths, of the waves are assumed to be identical.

The terms *coherent* and *coherence* can also be applied to **sources** that have a constant phase difference between them.

Examples of coherent sources:

- diffracted laser beams through two slits incident by a single laser beam
- two speakers fed by the same source

These are not coherent sources:

- two filament lamps

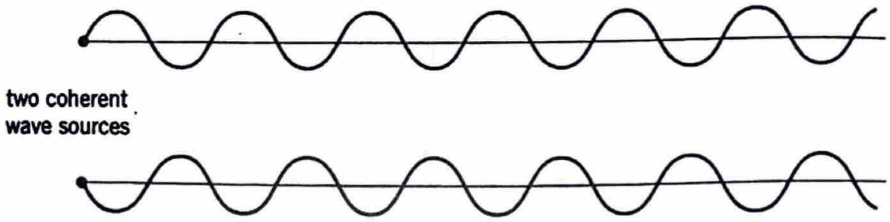


Fig. 17a Coherent wave trains: Both waves maintain a constant phase relationship.

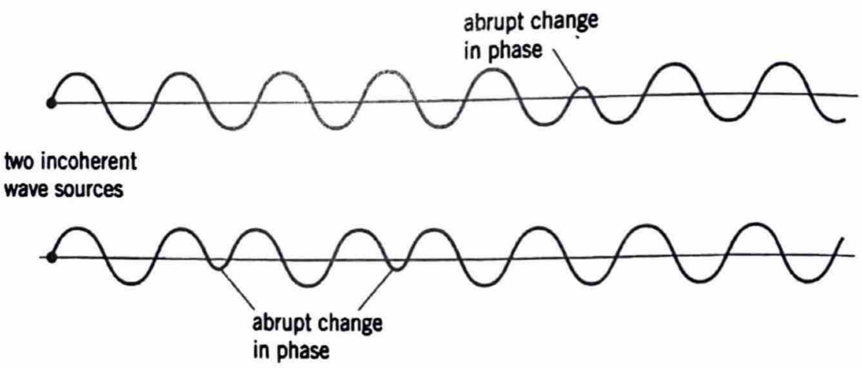


Fig. 17b Incoherent wave trains: Light is emitted in pulses. In between each pulse there is an abrupt change in phase. Waves from two sources may be in phase at one instant, but out of phase the next. The human eye cannot cope with the rapid changes, so the pattern is not observable.

Interference

Definition

Interference is the **superposition** of two or more waves to give a resultant wave whose resultant amplitude is given by the **principle of superposition**.

When two coherent waves interfere, they can give rise to **constructive interference** and **destructive interference**.

Constructive Interference

Definition

Constructive interference occurs when two coherent waves arrive at the same point with a **phase difference of zero** to produce a maximum.

Amplitude at that point is fixed, but displacement is varying.

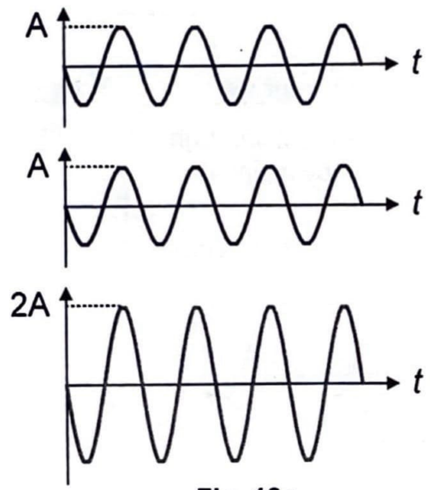


Fig. 18a

Destructive Interference

Definition

Destructive interference occurs when two coherent waves arrive at the same point with a **phase difference of π rad** to produce a minimum.

Amplitude at that point is zero, only if both waves have the same amplitude.

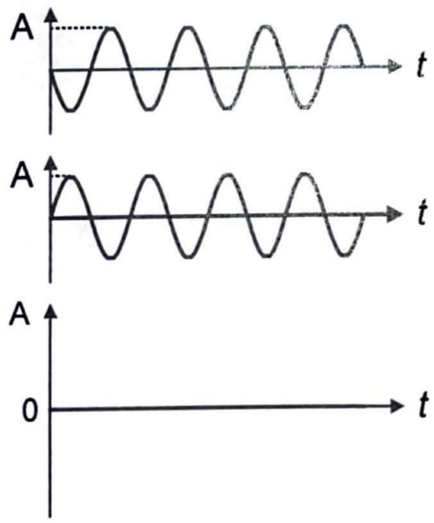


Fig. 18b

12.7 Two-source Interference

Conditions for Observable Interference Patterm

Important Note

1. The waves or sources must be **coherent** (i.e. they have the same frequency and a constant phase difference).
2. The waves must have **similar amplitude** (for a better contrast).
3. The waves must **overlap** and be of the **same type** (to produce regions of constructive and destructive interference).
4. For electromagnetic waves, they must be **unpolarised** or **polarised** in the same plane.

The Ripple Tank Experiment

Two ball-ended dippers S_1 and S_2, attached to a mechanical oscillator (and hence are coherent sources), send out two sets of circular wavefronts (let's say crests). These waves interfere when they overlap as shown in Fig. 19.

By the principle of superposition, **constructive interference** takes place along the **anti-nodal lines (solid lines)** when the two waves arrive **in phase**.

In between the anti-nodal lines, **destructive interference** takes place along the **nodal lines (dotted lines)** when the two waves arrive **π rad out of phase**.

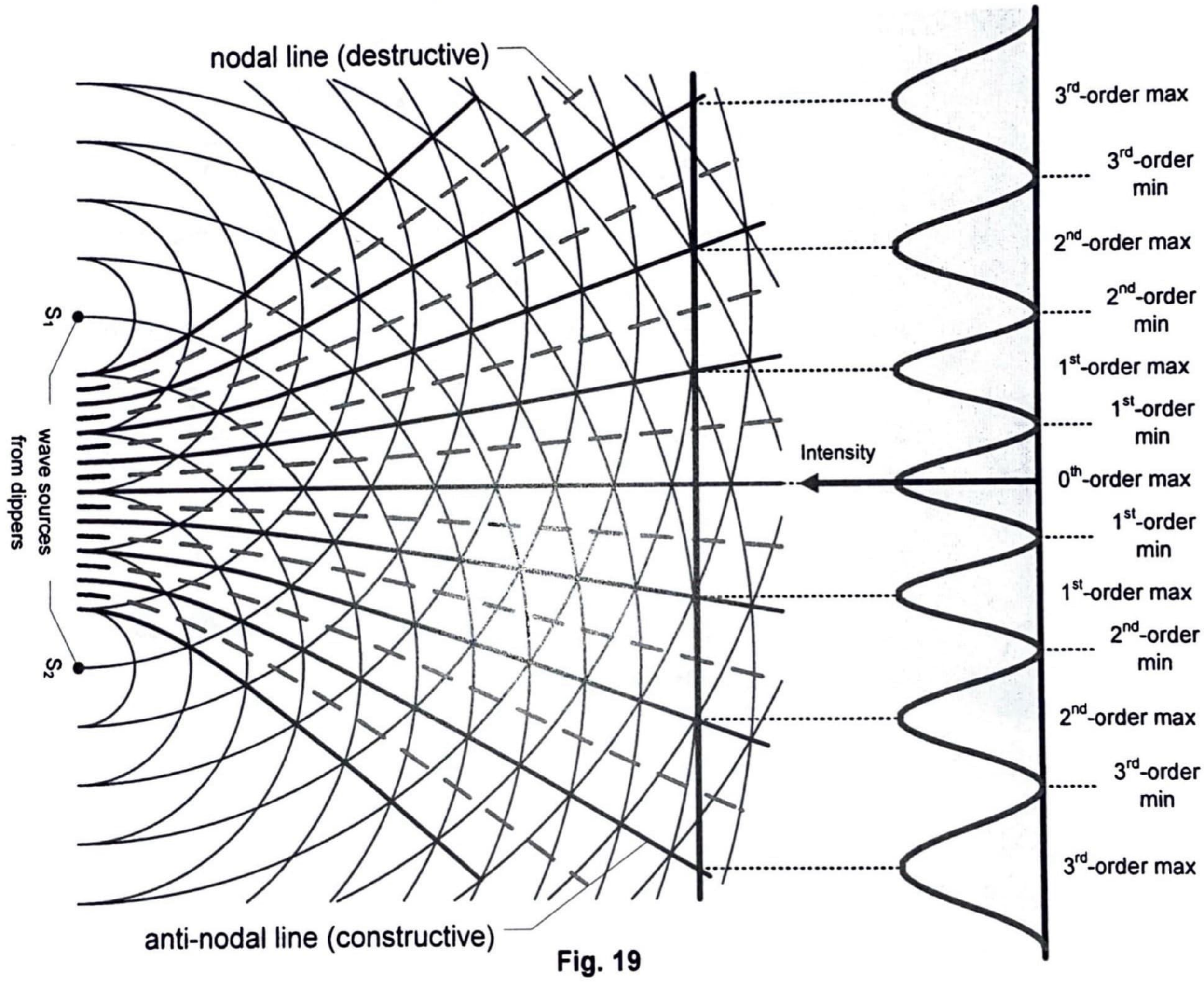


Fig. 19

Path Difference

Definition

The path difference between two waves is the difference in the distance that each wave travels from its source to the point where they meet.

We make use of path difference to analyse whether two waves meet in phase or out of phase, and hence determine whether constructive or destructive interference results.

Consider sources S_1 and S_2, and points P & Q where two waves meet:

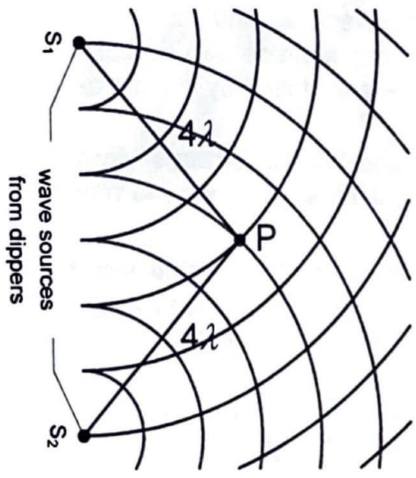


Fig. 20a

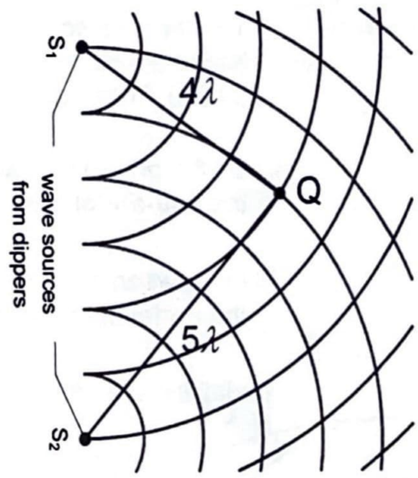


Fig. 20b

At point P, the two waves meet in phase (i.e. constructive interference),

$$\text{Path difference} = S_2P - S_1P = 0$$

At point Q, the two waves also meet in phase (i.e. constructive interference),

$$\text{Path difference} = S_2Q - S_1Q = \lambda$$

Hence, when two sources are **in phase**, for **constructive interference** to occur,

Path difference = $n\lambda$ (where n = 0, 1, 2, ...)

Important Note

The **n^{th} order maximum** (e.g. bright fringe or loud sound) occurs at positions where the **path difference is $n\lambda$**.

Now, consider points R & T where the two waves meet:

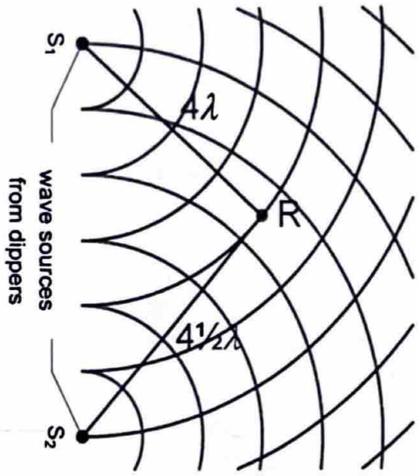


Fig. 21a

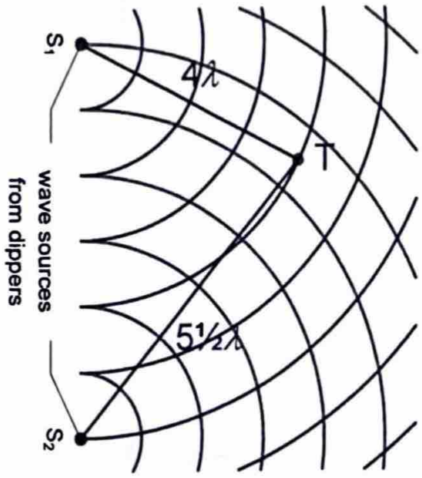


Fig. 21b

At point R, the two waves meet π rad out of phase (i.e. destructive interference),

$$\text{Path difference} = S_2R - S_1R = \tfrac{1}{2}\lambda$$

At point T, the two waves also meet π rad out of phase (i.e. destructive interference),

$$\text{Path difference} = S_2T - S_1T = 1\tfrac{1}{2}\lambda$$

Hence, when two sources are **in phase**, for **destructive interference** to occur,

$$\textbf{Path difference} = (n + \tfrac{1}{2})\lambda \text{ (where n = 0, 1, 2, ...)}$$

Question: What happens at P, Q, R and T if the two **sources** are out of phase by π rad?

Answer:

Waves arrive at P & Q π rad out of phase ⇒ destructive interference at P & Q.

Waves arrive at R & T in phase ⇒ constructive interference at R & T.

Take Note

In summary, the following table can be used to determine whether constructive or destructive interference occurs at a certain point where two waves meet.

1. Determine whether the two sources are in phase or π rad out of phase.
2. Determine the path difference in terms of λ.

You do not need to rely the table once you are familiar with the concept.

Important Note

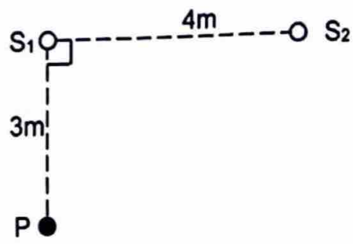
Path difference Δ	2 sources in phase	2 sources π rad out of phase
Constructive interference (maxima)	$\Delta = n\lambda$	$\Delta = (n + \tfrac{1}{2})\lambda$
Destructive interference (minima)	$\Delta = (n + \tfrac{1}{2})\lambda$	$\Delta = n\lambda$

Example 6

(J80/P2/10; N88/P1/8)
Two wave-generators S_1 and S_2 produce water waves of wavelength 1 m. They are placed 4 m apart in a water tank and a detector P is placed on the water surface 3 m from S_1 as shown in the diagram.

When operated alone, each generator produces a wave at P which has amplitude A.

When the generators are operating together and in phase, what is the resultant amplitude at P?



Solution

$S_2P = \sqrt{3^2 + 4^2} = 5$ m

path difference $= S_2P - S_1P = 5 - 3 = 2$ m

Since path difference = 2 m = 2λ (an integral number of wavelengths), constructive interference occurs at P as sources are in phase.

Resultant amplitude = $A + A = 2A$

12.8 Young's Double-Slit Experiment

Experimental Set-Up

An experimental set-up for viewing two-source interference pattern with light is the Young's double-slit experiment shown below.

A monochromatic light source is placed behind a single slit to create a small, well-defined source of light. Light from this source is diffracted at the single slit, producing two light sources at the double slits. Because light from the two slits originate from the same single slit, they are coherent and create a sustained and observable interference pattern.

At points of constructive interference (maxima), bright fringes are observed. At points of destructive interference (minima), dark fringes are observed.

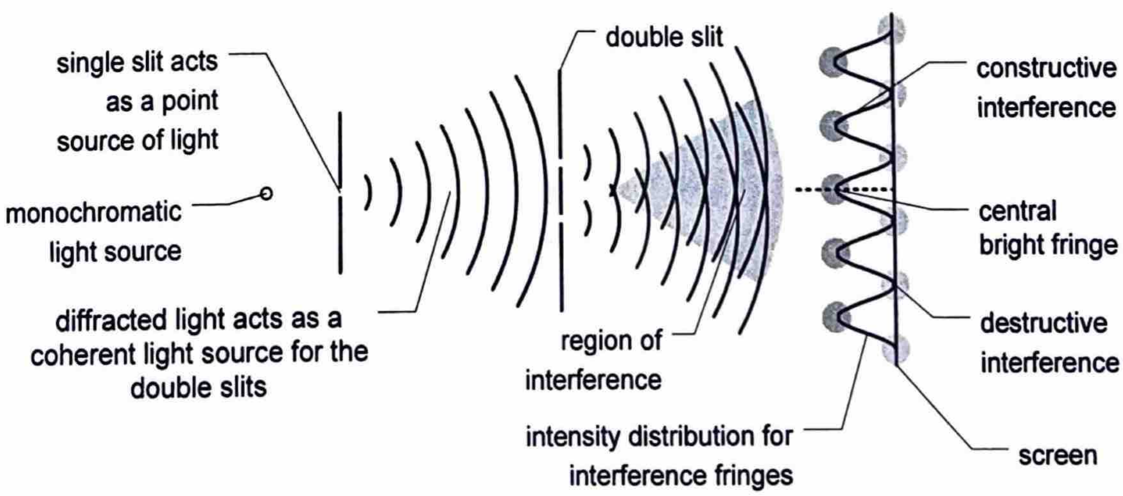


Fig. 22

Derivation of $x = \lambda D / a$

Let us now derive an equation for the fringe separation, i.e. the spacing between two successive bright (or dark) fringes.

Notations: x_n = distance of n^{th} bright fringe from central fringe
a = slit separation, D = distance between slits and screen

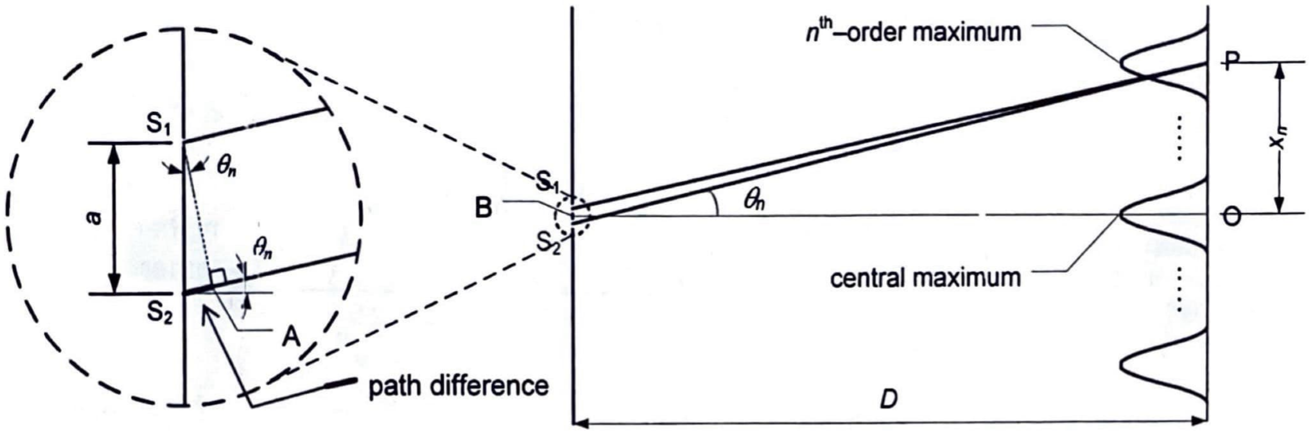


Fig. 23a **Fig. 23b**

Consider position P where the n^{th} order bright fringe (maximum) lies. If $a \ll D$, the rays from S_1 and S_2 to P are almost parallel and $\angle A \approx 90°$.

$$\therefore \text{ path difference } = S_2P - S_1P = S_2A \approx a\sin\theta_n \text{ (Fig. 23a)}$$

Since the waves passing through slits S_1 and S_2 are in phase, for constructive interference,

$$a\sin\theta_n = n\lambda$$

$$\sin\theta_n = \frac{n\lambda}{a} \qquad \text{where } n = 0, 1, 2, \ldots$$

From ΔOBP (Fig. 18b),

$$\tan\theta_n = \frac{x_n}{D}$$

When $a \gg \lambda$, angle θ is small, using small angle approximation (i.e. $\theta < 6°$),

$$\tan\theta_n \approx \sin\theta_n$$

$$\frac{x_n}{D} \approx \frac{n\lambda}{a}$$

Hence, constructive interference takes place at these positions,

$$x_n = \frac{n\lambda D}{a} \qquad \text{where } n = 0, 1, 2, \ldots$$

Therefore, spacing between successive bright fringes,

$$x = x_n - x_{n-1} = \frac{n\lambda D}{a} - \frac{(n-1)\lambda D}{a} = \frac{\lambda D}{a}.$$

Fringe Separation

Formula

$$x = \frac{\lambda D}{a}$$

Intensity Distribution of Interference Fringes

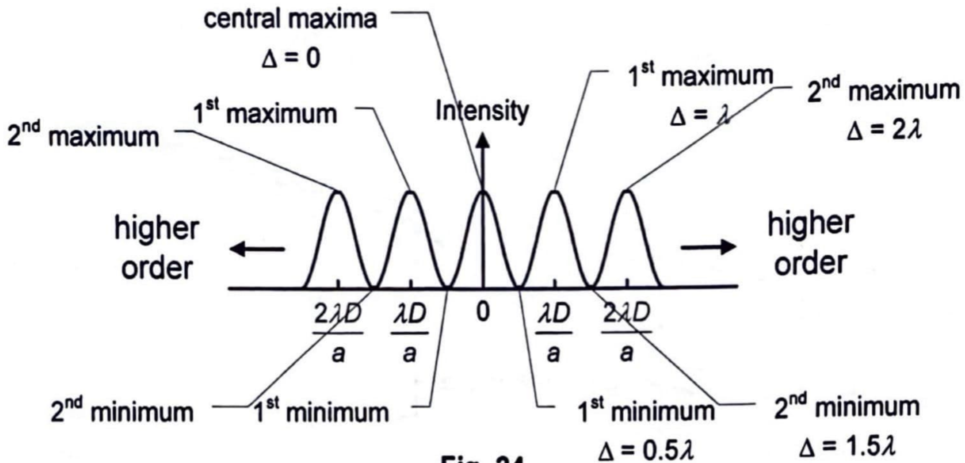


Fig. 24

Take Note

Important Note

- The formula $x = \frac{\lambda D}{a}$ is applicable only if
  - the rays are almost parallel (so that $a \sin\theta \approx n\lambda$, or $a << D$)
  - the angle θ of the fringes from the normal is small (so that $\sin\theta \approx \tan\theta$, $a >> \lambda$)

- The fringe separation x is only approximately constant for small angle θ. For larger θ, fringe separation actually increases.

- For interference of light, the typical values for slit width $w \sim 0.2$ mm, slit separation $a \sim 0.5$ mm, slit-screen distance $D \sim 1$ m, wavelength of light $\lambda \sim 500$ nm.

- The sources must be more than one wavelength apart to produce an observable interference pattern.

- The single slit acts as a point source of light to ensure that waves incident on and exiting the double slits are coherent.

- If the two coherent sources have a phase difference of π rad, the positions of constructive and destructive interference would be interchanged.

- The intensity of the fringes is often illustrated to be constant. In reality, the intensity is limited by the single slit diffraction envelope and will decrease with successive order. (Refer to Sect 12.10 on pg 29.)

Example 7

In a Young's double-slit experiment, the separation between the first and the fifth bright fringe is 2.5 mm when the wavelength used is 620 nm. If the distance from the slit to the screen is 0.80 m, calculate the separation of the two slits.

Solution

$$x = \frac{2.5}{4} = 0.625 \text{ mm}$$

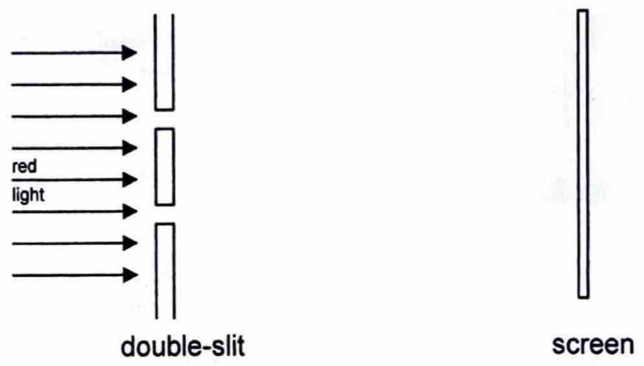
$$x = \frac{\lambda D}{a}$$

$$0.625 \times 10^{-3} = \frac{(620 \times 10^{-9})(0.80)}{a}$$

$$a = 7.94 \times 10^{-4} \text{ m}$$

Example 8

(N99/P2/3)
A student sets up the apparatus shown below to observe two-source interference fringes.



State and explain what change, if any, occurs in the separation of the fringes and in the contrast between bright and dark fringes observed on the screen, when each of the following changes is made separately:

(a) increasing the intensity of light incident on the double-slit.

(b) increasing the distance between the double-slit and the screen.

(c) reducing the intensity of light incident on one of the double-slit

Solution

(a) Fringe separation remains the same based on $x=\lambda D/a$. However, contrast increases because intensity of bright fringes increases.

(b) Fringe separation increases based on $x=\lambda D/a$. However, contrast decreases because intensity of bright fringes decreases due to increased distance of screen.

(c) Fringe separation remains the same based on $x=\lambda D/a$. However, contrast decreases because the minima are now not completely dark and the maxima are not as bright as before.

12.9 Diffraction Grating

Diffraction Grating

A diffraction grating consists of a large number of parallel, equally spaced lines (or slits) of equal width. A diffraction grating typically consists of 100 lines to 1000 lines per mm.

When a narrow beam of monochromatic light is incident on a diffraction grating, sharp maxima are obtained at various angular positions θ_n as shown in Fig. 25b.

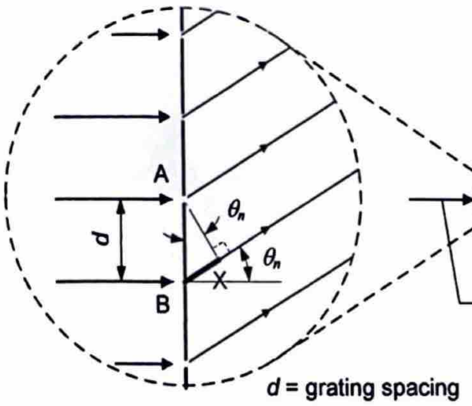


Fig. 25a

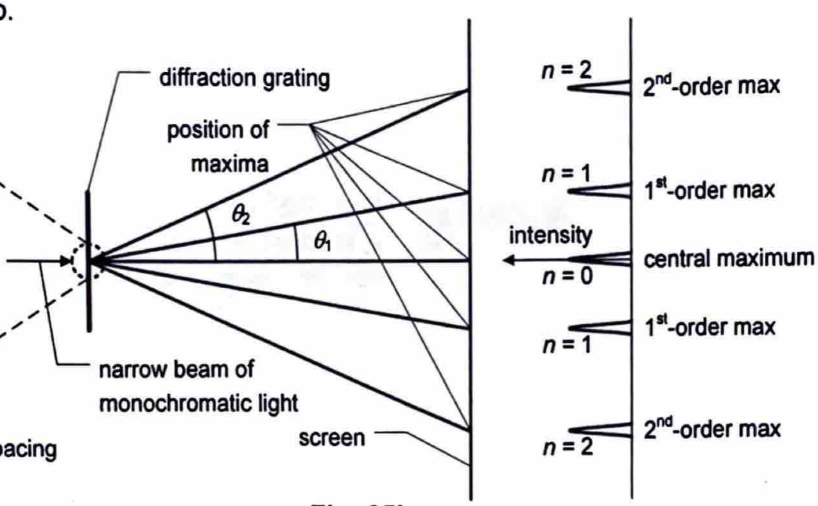


Fig. 25b

Derivation of $d \sin \theta = n\lambda$

Consider a narrow beam of monochromatic light incident on a diffraction grating of N lines per metre. At angular position θ_n, n^{th} order bright fringe (maxima) is obtained. The path difference between adjacent rays (Fig. 25a) is

$$\text{path difference} = BX = d\sin\theta_n, \quad \text{where } d = \text{ slit separation} = \frac{1}{N} \text{ in metre.}$$

Since the waves passing through all the slits are in phase, for constructive interference,

Formula

$$\boxed{d\sin\theta_n = n\lambda} \qquad \text{where } n = 0, 1, 2, \ldots$$

Take Note

1. Since N is typically 100 to 1000 lines per mm, d is of the order of 10^{-6} m. Since d is very small, in fact only a few times more than the wavelength of visible light (about 0.4×10^{-6} to 0.7×10^{-6} m), the angle of diffraction of even the first order ($n = 1$) is quite big. Hence, $x=\lambda D/a$ may not be applicable to diffraction gratings, unless it has a small N or large d.

2. Since

$$\theta_n < 90^\circ$$
$$\sin\theta_n < 1$$
$$\frac{n\lambda}{d} < 1$$

Formula

$$\boxed{n < \frac{d}{\lambda}}$$

Hence, we can determine the maximum order of the bright fringes.

3. An important use of the diffraction grating is in a spectrometer, which is an apparatus for investigating spectra. By measuring the angle at which a diffracted image appears, the wavelength of the light producing that image maybe determined. Refer to Appendix D for more details.

Example 9 Light of wavelength 656 nm is incident normally on a diffraction grating which has 400 lines per mm. Determine the angular positions of the first, second and third-order maxima.

Solution

Slit separation (or line spacing), $d = \dfrac{1\times10^{-3}}{400} = 2.50\times10^{-6}$ m

For n = 1, $\sin\theta_1 = \dfrac{\lambda}{d} = \dfrac{656\times10^{-9}}{2.50\times10^{-6}} = 0.2624$

$\theta_1 = 15.2°$

For n = 2, $\sin\theta_2 = \dfrac{2\lambda}{d} = \dfrac{2\times656\times10^{-9}}{2.50\times10^{-6}} = 0.5248$

$\theta_2 = 31.7°$

For n = 3, $\sin\theta_3 = \dfrac{3\lambda}{d} = \dfrac{3\times656\times10^{-9}}{2.50\times10^{-6}} = 0.7872$

$\theta_3 = 51.9°$

Example 10 How many bright fringes can we observe using a diffraction grating of 600 lines per mm illuminated normally with light of wavelength 633 nm?

Solution

$$\sin\theta_n < 1 \Rightarrow \frac{n\lambda}{d} < 1 \Rightarrow n < \frac{d}{\lambda} = \frac{\left(\frac{1\times10^{-3}}{600}\right)}{633\times10^{-9}} = 2.63$$

Copy!

Since n must be an integer, the highest order bright fringe observable is the 2nd order.

Therefore, the total number of bright fringes is 5 (the central bright fringe, plus 2 on either side).

Diffraction of White Light

When white light is incident on a diffraction grating, each wavelength is diffracted by a different amount, according to the equation $d \sin \theta = n\lambda$ .Hence, white light is split into its component colours to produce a continuous spectrum from the first order maximum onwards.

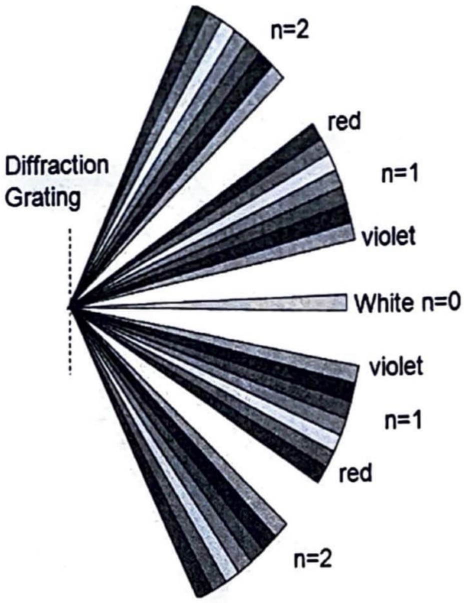


Fig. 26

The central (or zeroth-order) maximum is white because all component colours converge with zero path difference at this point.

Within the same-order spectrum, the longest wavelength deviates the most from the central axis.

There may be some overlapping of different orders, depending on the grating spacing. For example, the red component of the 2nd-order spectrum may overlap with the blue component of the 3rd-order spectrum.

Example 11

Describe, with the aid of a labelled diagram, the appearance of the 1st order spectra when white light having wavelengths from 380 nm (violet) to 780 nm (red) is incident normally on a diffraction grating of 500 lines per mm.

Solution

Slit separation, $d = \dfrac{1\times10^{-3}}{500} = 2.0\times10^{-6}$ m

For violet light,

$$\sin\theta_{1V} = \frac{\lambda}{d} = \frac{380\times10^{-9}}{2.0\times10^{-6}} = 0.19$$

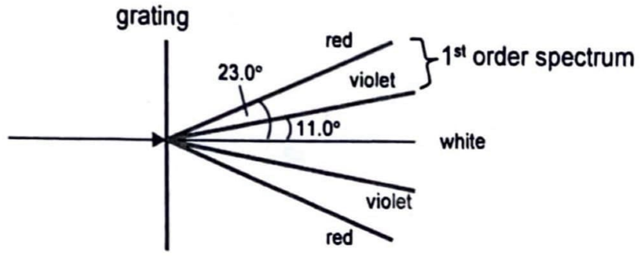
$$\theta_{1V} = 11.0°$$

For red light,

$$\sin\theta_{1R} = \frac{\lambda}{d} = \frac{780\times10^{-9}}{2.0\times10^{-6}} = 0.39$$

$$\theta_{1R} = 23.0°$$

The longest wavelength deviates the most from the central fringe. The central fringe remains white as all colours fall on it.



12.10 Single-Slit Diffraction

Huygens' Principle

If a single drop of water falls into a pond, it will create a circular wavefront that spreads outwards. Dutch physicist Christian Huygens, a contemporary of Newton, put forward a wave theory of light which was based on the way in which circular wavefronts advance.

He proposed that at any instant, all points on a wavefront could be regarded as secondary sources giving rise to their own outward spreading circular wavelets. The envelope (or tangent curve) of the wavefronts produced by these secondary sources gives the new position of the original wavefront.

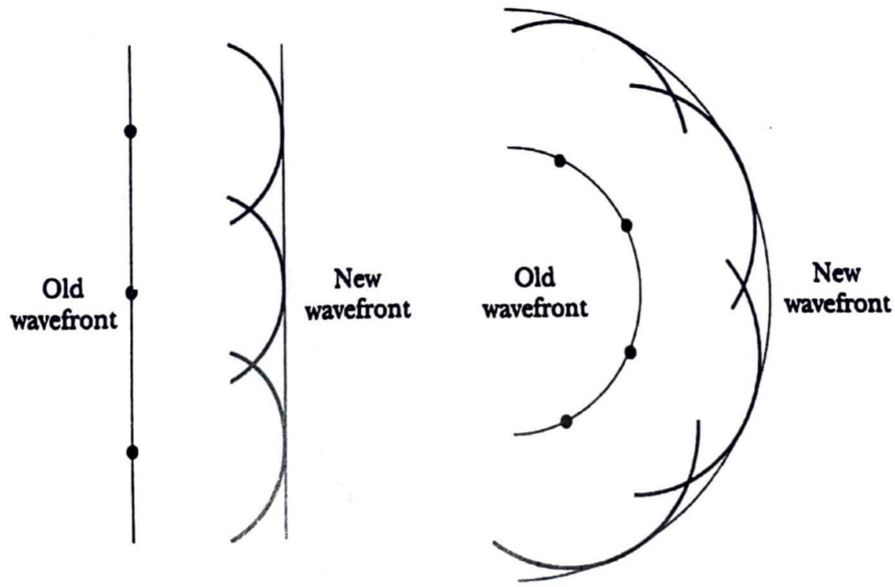


Fig. 27: Huygen's construction for a plane wave and spherical wave propagating to the right.

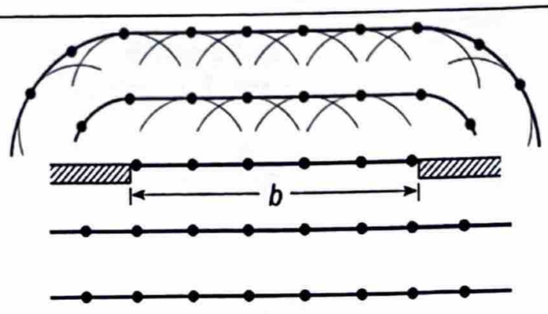


Fig. 28

If a plane (or straight) wavefront is restricted by an obstacle as shown in Fig. 28, some of the wavelets making up the wavefront are removed, causing the edges of the wavefront to be curved. If the aperture is comparable to the wavelength of the incident wave, the bending becomes pronounced and the transmitted wavefront looks circular. (See Fig.15 on pg 14.) The wavelength of diffracted waves is unchanged.

Single-Slit Diffraction Pattern

We have seen the interference patterns set up by waves passing through double slits and diffraction gratings.

When monochromatic light is passed through a single slit, an interference pattern is also observed. This is shown in the first interference pattern in Fig. 29.

But how can a single-slit produce an interference pattern? The explanation that follows is based on Huygen's principle. It was explained in greater detail by French scientist Augustin Fresnel two centuries later.

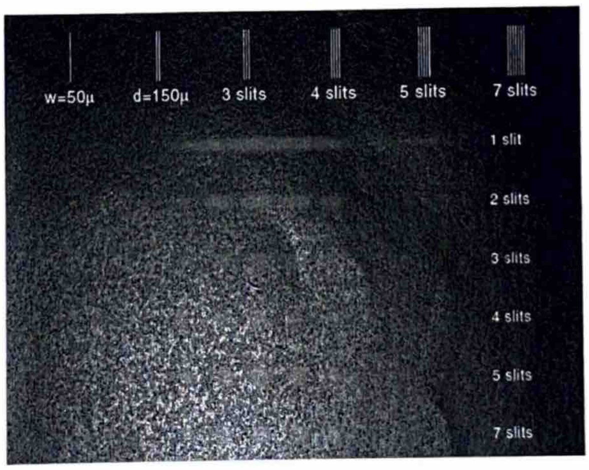


Fig. 29 Slits of same width and slit separation

Single-Slit Minima

Fig. 30 shows plane wavefronts incident on a single-slit of width *b*.

According to Huygen's principle, each point on the wavefront passing through the slit acts as a secondary source of wavelets. All the waves that originate from the slit are in phase.

To analyse the diffraction pattern, we can divide the slit into two halves, and consider a source of waves at the top and another one at the mid-point a distance *b*/2 below.

Waves from these 2 points behave like sources in a two-source interference experiment. The wavelets spreading out from these points overlap and create an interference pattern.

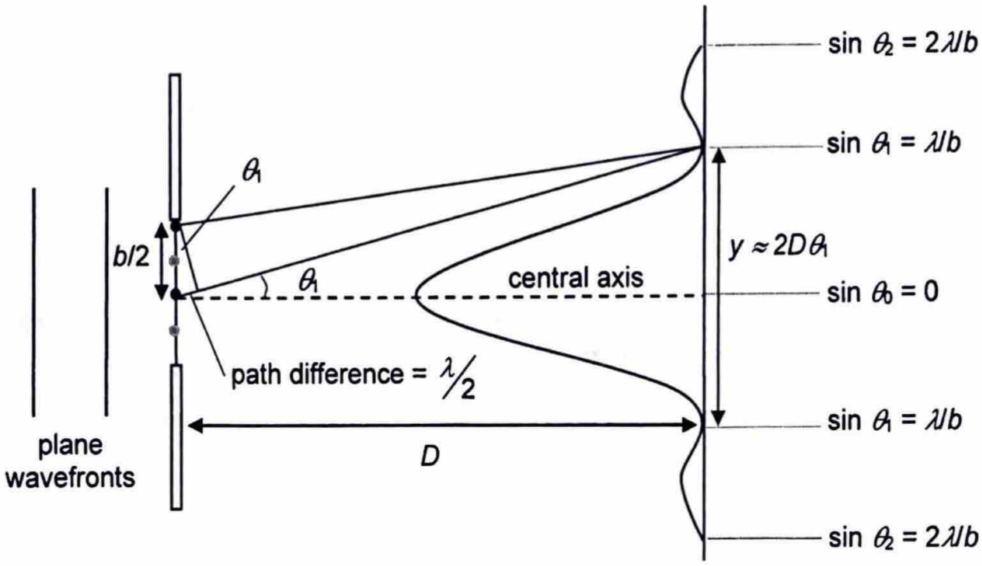


Fig. 30

At the position of the 1st dark fringe or 1st minimum, the path difference between the waves from the two secondary sources must be half a wavelength ($\lambda/2$) in order for destructive interference to occur. This is true for any two waves that originate at points separated by half the slit width.

Thus, at the **1st** order minimum,

path differenceis $\quad \frac{b}{2}\sin\theta_1 = \frac{\lambda}{2}$,

$$\sin\theta_1 = \frac{\lambda}{b}$$

If we divide the wavefront into four equal parts and apply the same argument to determine the positions of destructive interference, we now find that the **2nd** order minimum occurs where

$$\frac{b}{4}\sin\theta_2 = \frac{\lambda}{2}$$

$$\sin\theta_2 = \frac{2\lambda}{b}$$

Dividing the wavefront into six equal parts, we can find the **3rd order** occurs minimum where

$$\frac{b}{6}\sin\theta_3 = \frac{\lambda}{2}$$
$$\sin\theta_3 = \frac{3\lambda}{b}$$

Therefore, for single-slit diffraction pattern, the m^{th} dark fringe (or minimum or destructive interference) occurs at angle θ according to:

$$\sin\theta = \frac{m\lambda}{b}, \text{ where } m \text{ is an integer.}$$

Hence, the location of **the first minimum for single slit diffraction** can be obtained using the following equation

Formula

$$\boxed{\sin\theta = \frac{\lambda}{b}}$$

where θ is the angle from the central axis, λ is the wavelength of light and b is the **width** of the single slit.

For θ **< 6°**, the width y of the central bright fringe on a screen a distance D away is (approximating the width y using arc length formula $s = r\phi$)

$$\boxed{y \approx D(2\theta) \approx \frac{2D\lambda}{b},} \quad \text{where } \theta \approx \frac{\lambda}{b}.$$

Take Note

Important Note

- The equation $y \approx \frac{2D\lambda}{b}$ is only valid when the distance D of the screen from the single-slit is much larger than the width b of the single-slit. This is because the rays used in the derivation are assumed to be parallel.

- To determine the positions of the minima, the wavefront along the slit can be divided into any number of parts. However, for ease of illustration, even number of parts are used to demonstrate destructive interference.

Example 12

Light of wavelength 580 nm is incident on a slit of width 0.300 mm. A screen is 2.00 m away from the slit. Determine the positions of the first dark fringes from the central axis and the width of the central bright fringe on the screen.

Solution

At the positions of the first dark fringes, $\sin\theta = \dfrac{\lambda}{b}$

$$\theta = \frac{580\times10^{-9}}{0.300\times10^{-3}} = 1.93\times10^{-3}\text{ rad}$$

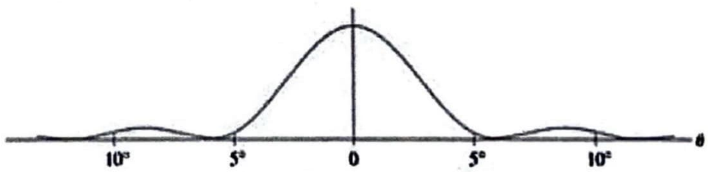
Since the angle is assumed to be small, $y = D(2\theta)$

$$= 2.00\times2\times1.93\times10^{-3}$$
$$= 7.72\times10^{-3}\text{ m}$$

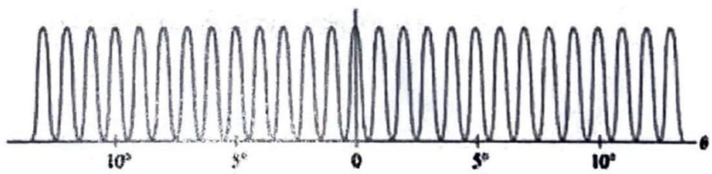
Positions of the first dark fringes from the central axis is $\frac{1}{2}y = 3.86\times10^{-3}$ m

Linking Single-Slit Pattern to Double-Slit and Diffraction Grating Patterns

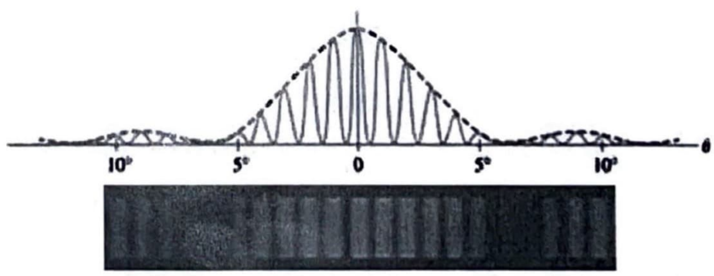
Fig. 29 on pg 28 shows the different interference patterns for various numbers of slits, where the slits have the same width and separation. Fig. 31 and Fig. 32 provide further illustrations of the various interference patterns.



Single-slit diffraction pattern



Double-slit pattern without considering single slit diffraction



Double-slit pattern taking into account single slit diffraction

Fig. 31

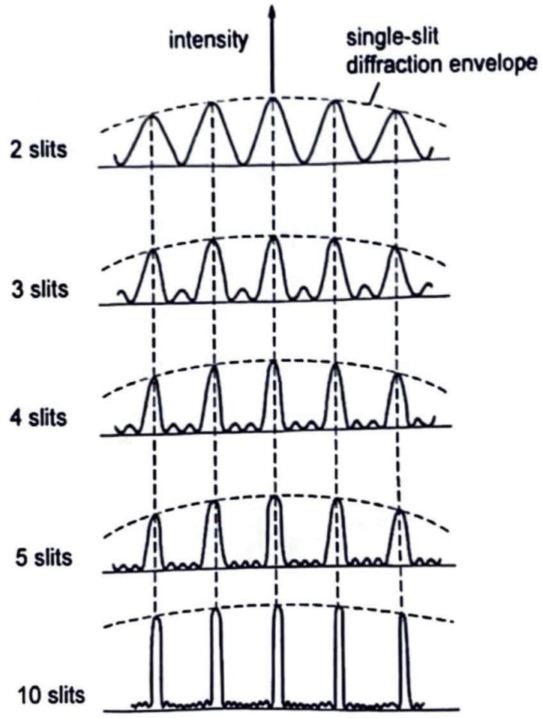


Fig. 32

The following are some observations when comparing the interference patterns of single-, double- and multiple-slit gratings.

Type	Appearance	Intensity
Single-slit	Broad central bright fringe with narrow, dark fringes on either side. Beyond these is a further succession of bright and dark fringes.	The intensity peaks in the middle of the central bright fringe, while tapering gradually towards the first minimum. Successive bright fringes are much dimmer and narrower than the central one. The envelope of light intensity of the central bright fringe will set the limiting intensities for multiple-slit arrangements.
Double-slit	Wide fringes that are equally spaced (near the central region).	Intensity of the fringes is limited by the envelope of the single slit diffraction pattern.
Multiple-slit	Narrow fringes that are not equally spaced (angular separation increases with order).	Intensity of the fringes is limited by the envelope of the single slit diffraction pattern.

Example 13

(a) Why are the higher-order maxima dimmer in the Young's double-slit and diffraction grating interference pattern?

(b) Why are some orders missing from the Young's double-slit and diffraction grating interference pattern?

(c) In Fig. 31 on pg 31, the 6th-order bright fringe is missing from the double-slit interference pattern. Determine the ratio of the slit separation *a* to the slit width *b*.

Solution

(a) This is because the intensity of the double-slit and diffraction grating interference pattern is restricted by that of the single-slit diffraction pattern, which decreases with order.

(b) This is because the positions of the maxima of the double-slit and diffraction grating interference pattern coincide with the minima of the single-slit diffraction pattern.

(c) The 6th-order maximum of the double-slit pattern is missing because it falls at the same angle as the 1st-order minimum of single-slit pattern.

To locate the double-slit maxima: $a \sin\theta = n\lambda$, where $n = 6$
To locate the single-slit minima: $b \sin\theta = m\lambda$, where $m = 1$

Since the angle θ and the wavelength λ are the same in both equations, we have

$$\frac{\sin\theta}{\lambda} = \frac{n}{a} = \frac{m}{b} \qquad \therefore \frac{a}{b} = \frac{n}{m} = \frac{6}{1} = 6$$

12.11 Rayleigh Criterion

Resolving Power of A Single-Slit

The resolving power of a single slit is its ability to distinguish between closely spaced objects. This resolving power is limited by the wavelength of light.

Fig.33 shows two light sources S_1 and S_2 situated in front of a narrow slit of width *b*. Because of diffraction, what is observed on the screen is the sum of the diffraction patterns of each source.

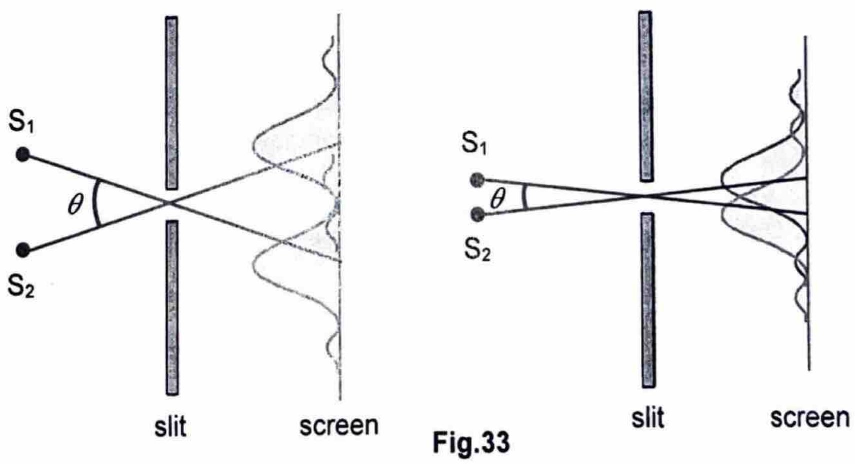


Fig.33

If the sources are well separated such that their images through a single-slit are distinguishable, we say that the images are well resolved. However, when the sources are too close to each other or too far from the single-slit, the images are unresolved as they overlap and become indistinguishable.

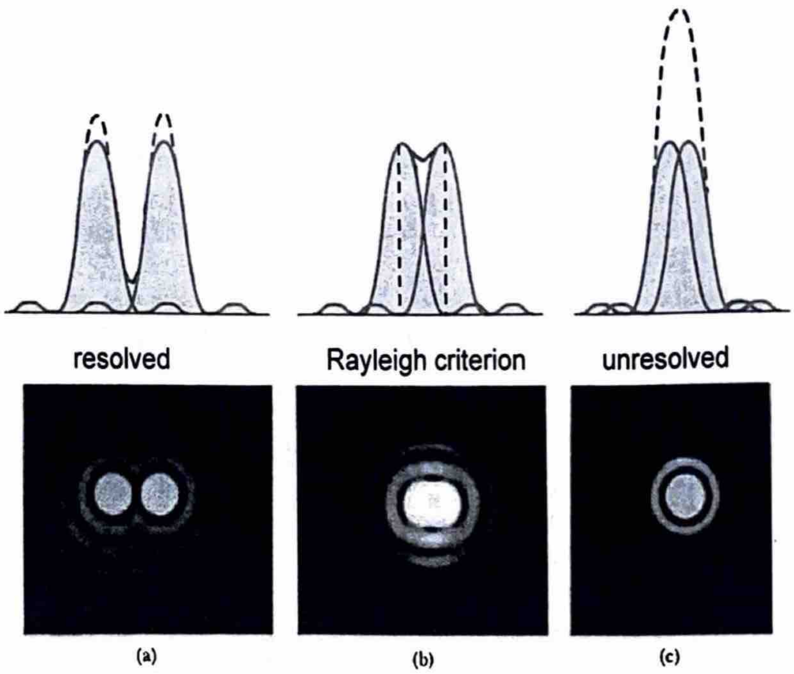


Fig. 34

Rayleigh Criterion

Definition

Rayleigh criterion states that two images are just resolved by an aperture when the central maximum of the diffraction pattern of one image falls on the first minimum of the diffraction pattern of the other image.

From our study of the single-slit diffraction pattern, the angle θ between the central maximum and the first minimum is

$$\sin\theta = \frac{\lambda}{b}$$

Hence, Rayleigh criterion is satisfied when the minimum angular separation θ_{min} (or the limiting angle of resolution) between the two image is

Formula

$$\theta_{min} \approx \frac{\lambda}{b}$$

where λ is the wavelength of the light radiating or reflected from the two objects and b is the width of the rectangular slit.

Therefore, for two images to be well resolved, the angular separation of the images, and hence the objects, must be greater than λ/b.

Take Note

- The expression for the resolving power of a single slit is obtained by applying the approximation $\sin\theta \approx \theta$ to determine the position of the first minimum from the central maximum of the single-slit diffraction pattern.
- In reality, most optical systems use circular apertures rather than rectangular slits. For a circular aperture, $\theta_{min} \approx 1.22\frac{\lambda}{D}$, where D is the diameter of the aperture.
- If the minimum angular separation between images is to be reduced to allow finer details to be observed (i.e. increasing the resolving power), light of shorter wavelength, such as UV light, can be used.
- It is impossible to see an object as small as an atom using visible light with an optical microscope. To 'see' an object, the wavelength of 'light' used must be comparable to the size of the object. Hence, electron microscopes, capable of producing radiation of short wavelengths, are used to observe very small structures.

Example 14

Assume that the resolution of a human eye of pupil diameter 2.0 mm is only limited by diffraction. For a wavelength of 500 nm, calculate

(a) the limiting angle of resolution for the eye, and

(b) the minimum separation between two point sources of light that the eye can distinguish, if the sources are a distance of 25 cm from the eye .

Solution

a.

$$\theta_{min} = 1.22\frac{\lambda}{D}$$

$$= 1.22\frac{(500\times10^{-9})}{(2.00\times10^{-3})}$$

$$= 3.05\times10^{-4}\text{ rad} \quad \text{or} \quad 0.0175°$$

b.

- Since the angle θ_{min} is small, we can approximate the separation y_{min} using

$$y_{min} \approx L\theta_{min} = (25\times10^{-2})(3.05\times10^{-4}) = 76\times10^{-6}\text{ m},$$

where L is the distance of the sources to the eye.

- This is approximately the width of a strand of human hair.

Appendix A Stationary Waves – A Mathematical Approach

An analytical treatment of the production of stationary waves from the superposition of two progressive waves having the same frequency and amplitude is given below.

The two progressive waves may be represented by the equations

$$y_1 = a\sin\left(\frac{2\pi x}{\lambda} - \omega t\right) \text{ and } y_2 = a\sin\left(\frac{2\pi x}{\lambda} + \omega t\right)$$

where y_1 and y_2 are travelling toward the right and left respectively.

Using the principle of superposition of waves, the resultant wave can be represented by the equation

$$\begin{aligned} y &= y_1 + y_2 \\ &= a\sin\left(\frac{2\pi x}{\lambda} - \omega t\right) + a\sin\left(\frac{2\pi x}{\lambda} + \omega t\right) \\ &= 2a\sin\frac{2\pi x}{\lambda}\cos\omega t \\ &= \left(2a\sin\frac{2\pi x}{\lambda}\right)\cos\omega t \\ &= A\cos\omega t \end{aligned}$$

where $A = 2a\sin\frac{2\pi x}{\lambda}$ is the amplitude of the resultant stationary wave for a point at a distance x from a reference point.

At the nodes, the amplitude is always zero and

$$\frac{2\pi x}{\lambda} = 0, \pi, 2\pi, \ldots$$

$$x = 0, \frac{\lambda}{2}, \lambda, \ldots$$

At the antinodes, the amplitude is maximum and equals $2a$. This occurs when

$$\frac{2\pi x}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$$

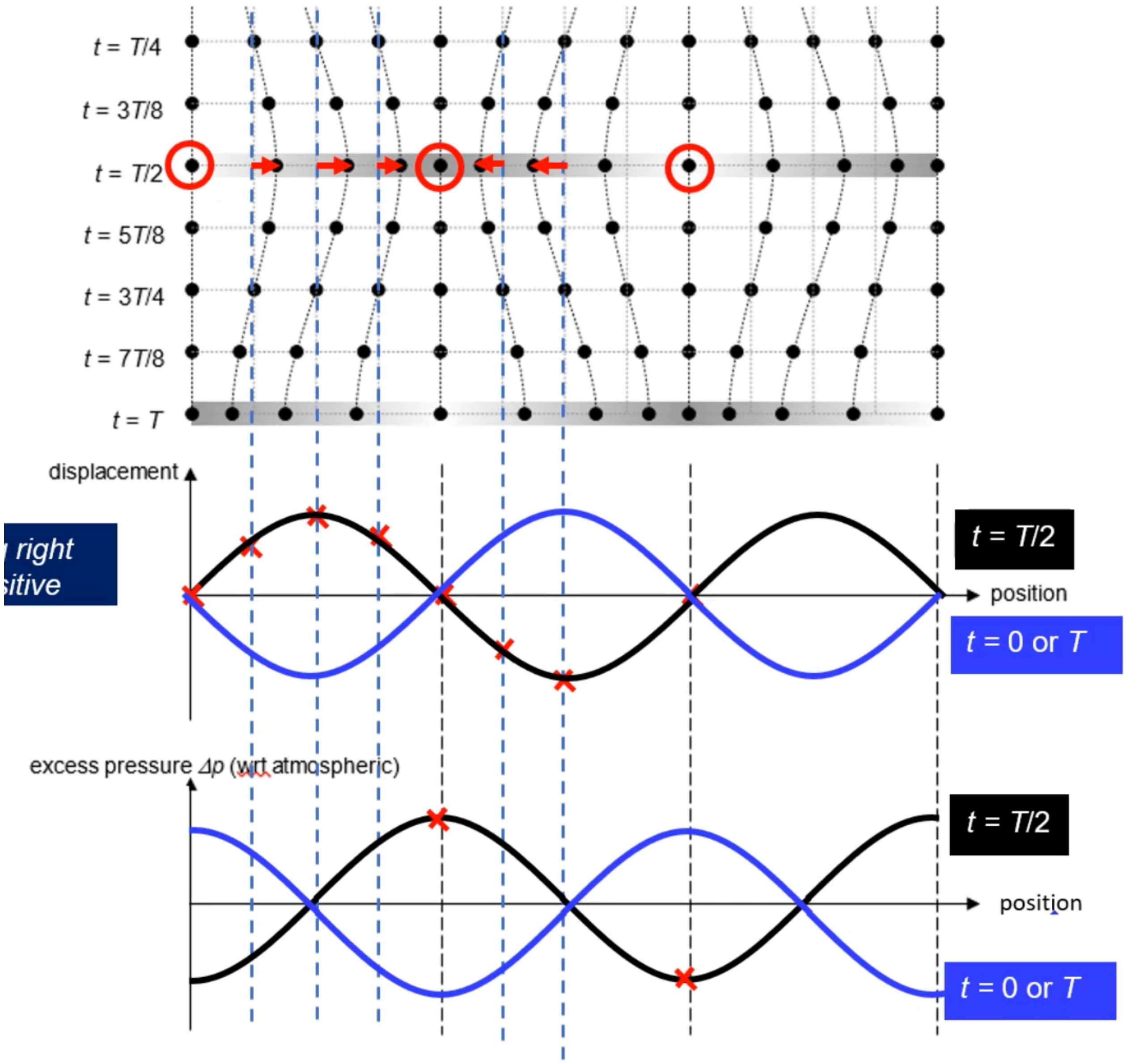
We find that the distance between two successive nodes or antinodes is always $\frac{\lambda}{2}$.

Note: $\sin P + \sin Q = 2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}$

Appendix B Visualizing a Longitudinal Stationary Wave

The following diagram shows the actual positions of the particles in a longitudinal stationary wave. Dark regions represent areas of **compression (high pressure)**, while light regions represents areas of **rarefaction (low pressure)**.

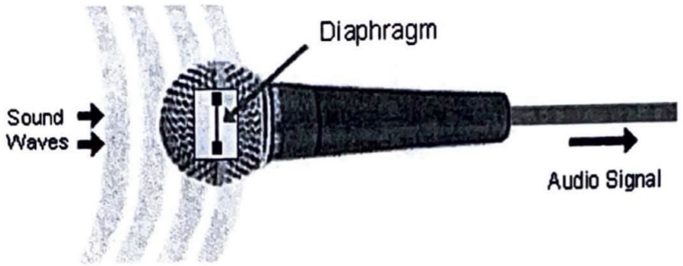
Notice that both the **maximum compressions** and **maximum rarefactions** occur at the displacement nodes. Hence, **displacement nodes** are also **pressure antinodes**, and vice versa.



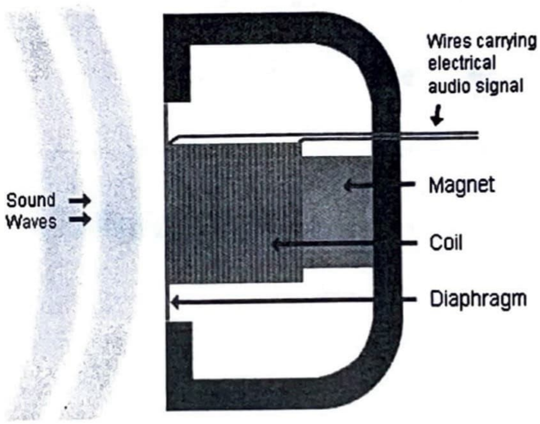
Appendix C How Do Microphones Work?

Microphones are a type of *transducer* - a device which converts energy from one form to another. Microphones convert acoustical energy (sound waves) into electrical energy (audio signal).

Different types of microphone have different ways of converting energy, but they all share one thing in common: the *diaphragm*. This is a thin piece of material (such as aluminium) which vibrates when it is struck by sound waves. In a typical hand-held microphone like the one below, the diaphragm is located in the head of the microphone.

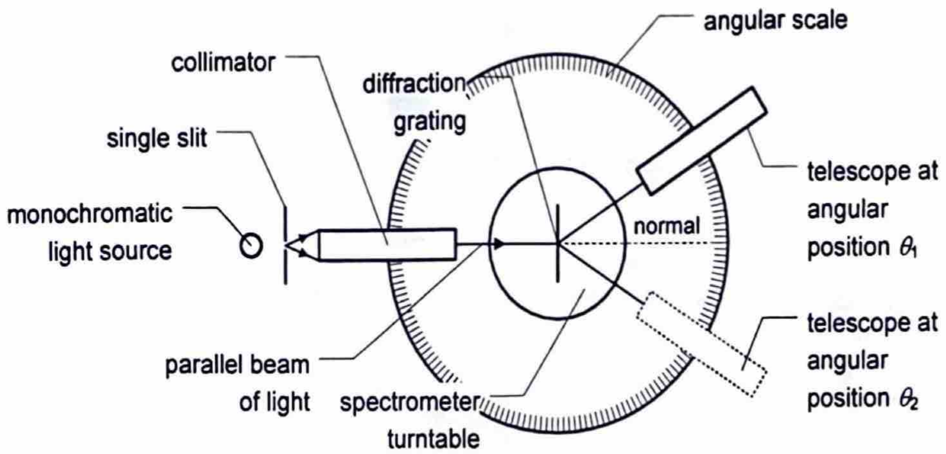


Cross-Section of Dynamic Microphone



When the diaphragm vibrates, it causes other components in the microphone to vibrate. These vibrations produce an induced emf which is amplified and sent to a loudspeaker.

Appendix D Determination of Wavelength of Light using Diffraction Grating



A parallel, narrow beam of monochromatic light, after emerging from the collimator, is incident normally on the diffraction grating. As a result, bright fringes are produced at various angular positions.

The telescope is first rotated such that the n^{th} order bright fringe is at the centre of the crosswire in the telescope and its angular position θ_1 is noted. It is then positioned on the opposite side of the normal to the grating and the angular position θ_2 of the same order bright fringe is noted.

The angular position θ_n of the n^{th} order bright fringe from the normal is

$$\theta_n = \frac{1}{2}(\theta_2 - \theta_1).$$

Hence,

$$\lambda = \frac{d}{n}\sin\theta_n.$$

Appendix E Simulation Applets

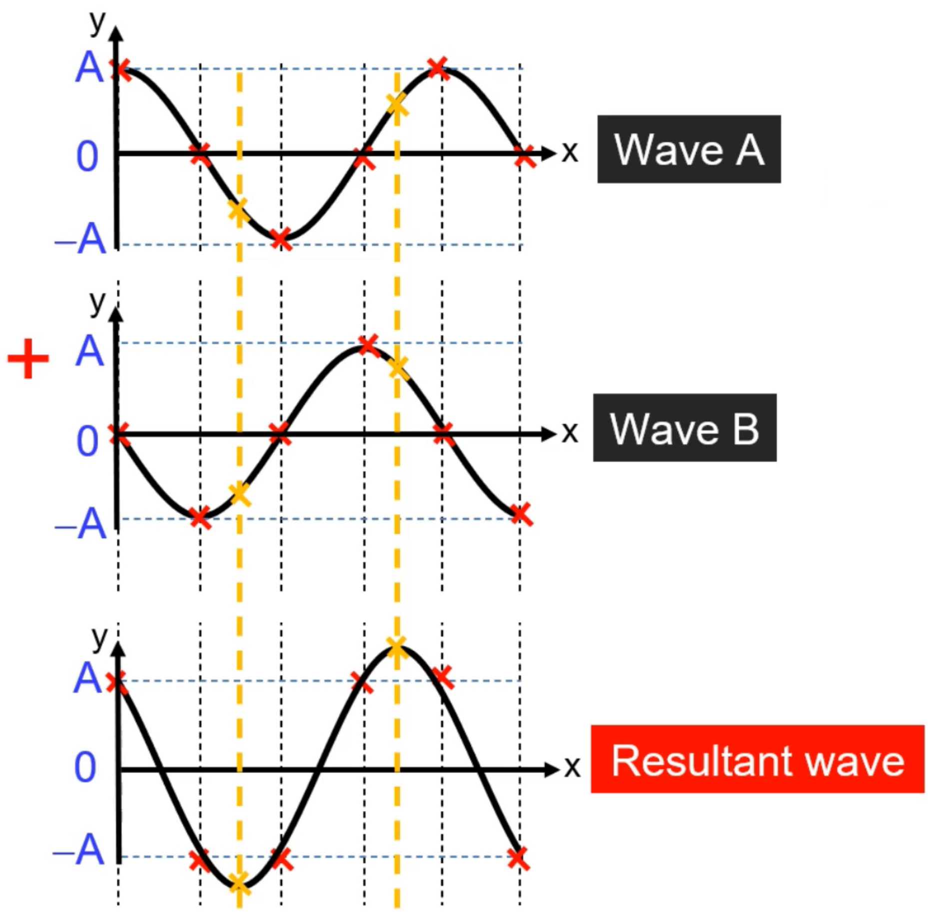
Standing wave in a violin:
http://zonalandeducation.com/mstm/physics/waves/standingWaves/standingWaves1/StandingWaves1.html

Single slit pattern:
https://micro.magnet.fsu.edu/primer/java/diffraction/basicdiffraction/index.html

Double slit pattern:
http://vsg.quasihome.com/interfer.htm
http://zonalandeducation.com/mstm/physics/waves/interference/twoSource/TwoSourceInterference1.html

Simulation
https://academo.org/physics/
http://physics.bu.edu/~duffy/HTML5/
http://www.falstad.com/mathphysics.html
http://www.cabrillo.edu/~jmccullough/Physics/index.html
http://zonalandeducation.com/mstm/physics/physics.html
https://fnoschese.wordpress.com/physics-applets-animations/
https://www.walter-fendt.de/html5/phen/standingwavereflection_en.htm
https://www.walter-fendt.de/html5/phen/standinglongitudinalwaves_en.htm

Quick Check ! Sketch the resultant wave.



RAFFLES INSTITUTION
YEAR 5-6 PHYSICS DEPARTMENT

Tutorial 12 SUPERPOSITION

H2 PHYSICS 9749

Self - Check Questions

S1 What is the principle of superposition?

S2 How is a stationary wave formed?

S3 Compare the properties of a stationary wave and a progressive wave.

S4 What is the longest possible wavelength of a stationary wave set up in a string fixed at both ends? What about that of a stationary sound wave set up in a pipe with one end closed and with both ends open?

S5 What is diffraction? Is the effect of diffraction more pronounced when the size of the aperture is reduced or when it is enlarged?

S6 Why is diffraction of light not easily observed? (It was thought that light travels in a straight line.)

S7 What do you understand by the terms *interference*, *constructive* and *destructive interference*, *phase difference*, *path difference* and *coherence*?

S8 What are the necessary conditions for a two-source interference pattern to be observable?

S9 Explain why light from two monochromatic sources do not produce an observable interference pattern, but sound from two loudspeakers can.

S10 In a Young's double-slit experiment, what is the relation between the fringe separation x, the wavelength λ, the slit separation a and the slit to screen distance D? Under what conditions will the relationship be applicable for any two-source interference experiment?

S11 What is the equation to determine the positions of the principal maxima of a diffraction grating? Define the symbols used and state the assumption used in its derivation.

S12 How do you calculate the maximum number of orders that can be observed when a beam of monochromatic light passes through a diffraction grating?

S13 State the equation to locate the first minima for single slit diffraction. How is the width of the central bright fringe related to the width of the single slit aperture?

S14 State the Rayleigh criterion for the resolving power of a single aperture. Explain your symbols clearly.

S15 Sketch the intensity patterns of single slit, double slit and multiple slit diffraction.

Self - Practice Questions

SP1 A string of length L, fixed at both ends, is plucked at its midpoint and emits its fundamental note of frequency f_1. When the string is plucked at a different point, the first overtone frequency f_2 is also produced.

(a) What is the ratio f_2/f_1?

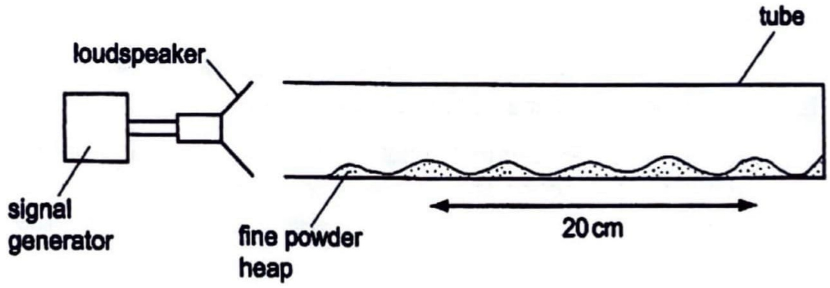
(b) What is the speed of the transverse waves in the string in terms of f_2 and L?

[J86/1/12]

SP2 An organ pipe of effective length 0.60 m is closed at one end. Given that the speed of sound in air is 300 m s^{-1}, determine the two lowest resonant frequencies.

SP3 A microwave transmitter emits waves which are reflected from a metal plate. The reflected wave and the incident wave superpose to form a standing wave. When a microwave detector is traversed along the standing wave, successive points at which the detector detected zero intensity were located 1.5 cm apart. What is the frequency of the waves?

SP4 A long horizontal tube, containing a fine powder, is closed at one end. A loudspeaker, connected to a signal generator, is positioned at the other end.



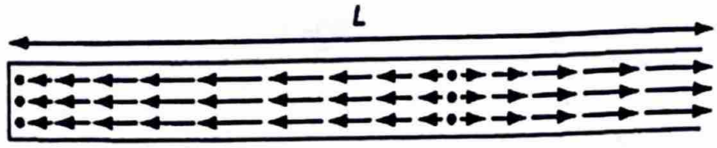
At a particular frequency, a stationary wave is set up inside the tube and the powder forms heaps at the nodes. The speed of sound is 330 m s^{-1}.

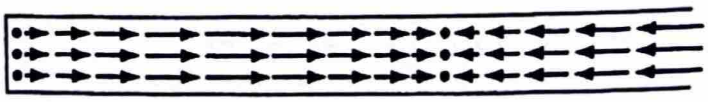
What type of wave is the stationary wave and what is its frequency?

	type of wave	frequency / kHz
A	longitudinal	3.3
B	transverse	3.3
C	longitudinal	6.6
D	transverse	6.6

(2007 P1 Q21)

SP5 The diagrams show particle movement in an air column when a stationary wave exists in the column.





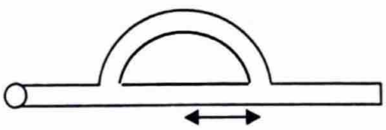
The first diagram shows the displacement of some particles at one instant and the second diagram shows the displacement of some particles half a cycle later.

What is the length L of the column in terms of the wavelength λ, and at which position within the column does the pressure change by the largest amount?

	length L	maximum pressure change at
A	$\frac{3}{4}\lambda$	node
B	$\frac{3}{4}\lambda$	antinode
C	$\frac{3}{2}\lambda$	node
D	$\frac{3}{2}\lambda$	antinode

(2008 P1 Q22)

SP6 Sound waves of wavelength 0.42 m enter a tube where a semi-circular pipe of radius r is linked to a straight pipe as shown below.



What is the smallest radius r such that a minimum will be heard at the detector?

SP7 Radio waves of wavelength 50 mm are emitted from two aerials and create a fringe pattern 1.0 km from the aerials. Calculate the distance between the aerials if the fringe spacing is 80 cm.

SP8 In a Young's double-slit experiment, the distance between slits is 0.50 mm and the slits are 1.0 m from the screen. Two interference patterns can be seen on the screen when lights of wavelengths 480 nm and 600 nm are incident perpendicularly to the slits. What is the separation between the third-order interference fringes on the screen?

SP9 A diffraction grating has 400 lines per mm and is illuminated normally by monochromatic light of wavelength 600 nm. Calculate

(a) the grating spacing,

(b) the angle to the normal at which the first order maximum is seen,

(c) the number of diffraction maxima observed.

SP10 When a cadmium light is viewed through a diffraction grating having 500 lines per mm, the first order spectral lines were observed at the stated angles.

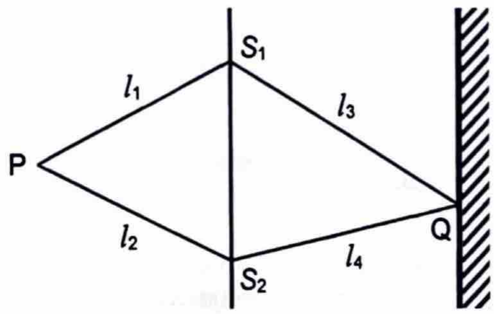
colour	angle / °
red	18.78
green	14.74
light blue	13.89
dark blue	13.53

(a) Find the wavelengths of each of these lines.

(b) What is the maximum order of each colour?

SP11 In the spectrum of white light obtained by using a certain diffraction grating, the second and third orders partially overlap. What wavelength in the third-order spectrum will appear at the angle corresponding to a wavelength of 650 nm in the second-order spectrum?

SP12 Two identical narrow slits S_1 and S_2 are illuminated by light of wavelength λ from a point P.

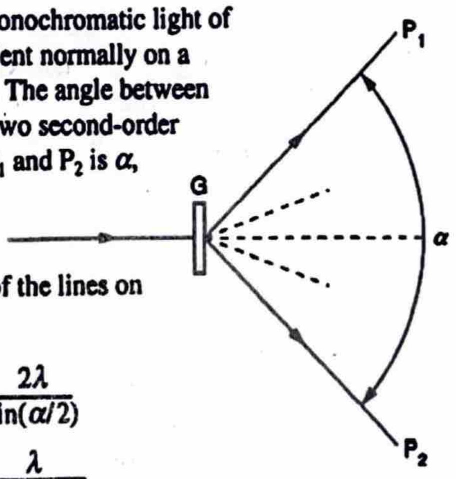


If, as shown in the diagram above, the light is then allowed to fall on a screen, and if m is a positive integer, the condition for destructive interference at Q is that

A $(l_1 - l_2) = (2m + 1)\ \lambda/2$
B $(l_3 - l_4) = (2m + 1)\ \lambda/2$
C $(l_3 - l_4) = m\lambda$
D $(l_1 + l_3) - (l_2 + l_4) = (2m + 1)\ \lambda/2$
E $(l_1 + l_3) - (l_2 + l_4) = m\lambda$

[N80/II/9]

SP13 A parallel beam of monochromatic light of wavelength λ is incident normally on a diffraction grating G. The angle between the directions of the two second-order diffracted beams at P_1 and P_2 is α, as shown.



What is the spacing of the lines on the grating?

A $\frac{2\lambda}{\sin\alpha}$ **C** $\frac{2\lambda}{\sin(\alpha/2)}$

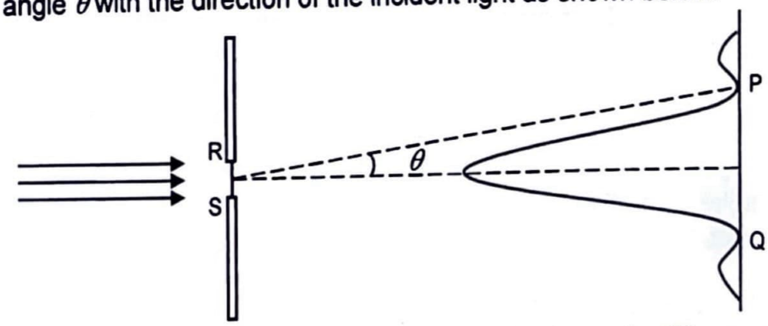
B $\frac{\lambda}{\sin\alpha}$ **D** $\frac{\lambda}{\sin(\alpha/2)}$

J86/I/II; N99/I/11

SP14 A parallel beam of white light (range of wavelengths 4.5×10^{-7} m to 7.5×10^{-7} m) is incident normally on a diffraction grating. The most deviated wavelength in the second order spectrum is diffracted through an angle of 60° from the direction of the incident beam. How many lines per metre are there on the grating?

[J77/II/14]

SP15 Monochromatic light of wavelength λ is incident normally on a single slit RS of width a. The diffraction pattern is formed on a screen PQ. The first minimum of this pattern makes an angle θ with the direction of the incident light as shown below.



Which of the following gives the correct expressions for the path difference SP–RP and for sin θ?

	SP–RP	sin θ
A	$\lambda/2$	$\lambda/2a$
B	$\lambda/2$	λ/a
C	λ	$\lambda/2a$
D	λ	λ/a

SP16 With reference to SP15, when the width of the slit is reduced,

A the intensities of all the peaks decreased and the width of the pattern remained unchanged.

B the intensities of all the peaks decreased and the width of the pattern decreased.

C the intensities of all the peaks remained unchanged and the width of the pattern increased.

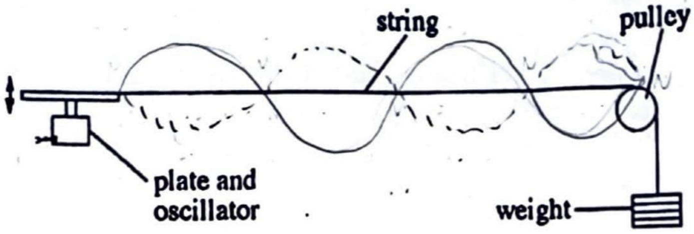
D the intensities of all the peaks decreased and the width of the pattern increased.

SP17 Calculate the angle between the centre of the diffraction pattern and the first minimum when a sound wave of wavelength 1.0 m passes through a door 1.2 m wide.

Discussion Questions

Stationary Waves

D1 One end of a horizontal string is attached to an oscillating plate. The string passes over a pulley and the string is kept under tension by means of a weight as shown below.



The frequency of oscillation of the plate is increased and at certain frequencies, stationary waves are produced on the string.

(a) Draw the stationary wave on the string when the frequency is such that the distance between the plate and the pulley corresponds to two wavelengths of the wave on the string. [1]

(b) On your diagram, label the positions of the nodes N on the string. [1]

(c) Explain why a stationary wave is observed on the string only at particular frequencies of vibration of the plate. [2]

[J99/P3/Q4(part)]

D2 A 60.0 cm long, vertical hollow tube is initially submerged just below the surface of water. As it is raised, the tube first resonates to a vibrating tuning-fork of frequency 512 Hz when the air column above the water level is 14.8 cm. The next resonance is observed when the length of air column is 48.0 cm. Determine

(a) the end-correction, and [2]

(b) the lowest frequency at which the tube will resonate when it is open at both ends. [3]

D3 A loudspeaker S, which is emitting sound of frequency 600 Hz, is placed in front of a reflector R to produce a stationary wave. A sensitive microphone M is moved between R and S, and the signal from M is monitored on a CRO.

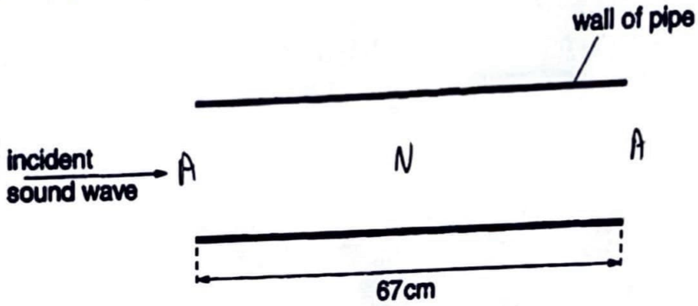
CRO signals show that the first peak-signal is detected when M is located at the 130 mm mark on a ruler placed between R and S. The third peak-signal is detected when M is at the 698 mm mark.

Calculate

(a) the speed of sound in air, and [2]

(b) the positions of the two minima that lie between these two peak-signals. [2]

D4 **(a)** A sound wave passes into a pipe that is open at both ends, as shown in the figure below.



The frequency of the sound wave is gradually increased from a low value. A loud sound is heard in the pipe for the first time at a frequency of 250 Hz. The length of the pipe is 67 cm.

(i) On the figure, mark all the positions of
1. the displacement antinodes (use the letter A),
2. the displacement nodes (use the letter N). [1]

(ii) Calculate a value for the speed of sound in the pipe. [2]

(b) An alternative, more reliable, method of measuring the speed of sound shows that the value in **(a)(ii)** is an underestimate.

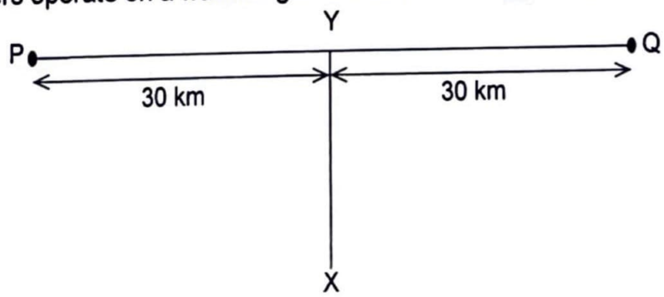
This underestimate cannot be attributed to the uncertainty in the measurement of either the frequency or the length of the pipe.

State and explain what can be deduced about the positions of either the nodes or the antinodes of the stationary wave in the pipe. [2]

[N2010/P3/Q7(part)]

Two-Source Interference

D5 A ship at X is equidistant from two shore-based radio transmitters P and Q as shown below. Both transmitters operate on a wavelength of 300 m and radiate signals of equal amplitude.



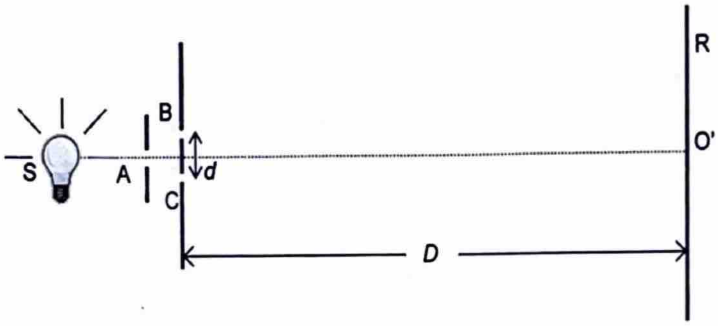
(a) In the figure, the ship at X detects no signal. What information does this give about the signals from P and Q? [1]

(b) The ship moves in a straight line from X to Y. Throughout the journey the amplitude of the signal detected by the ship is zero. Explain this. [2]

(c) The ship moves in the direction YQ until the signal detected has amplitude twice that from either transmitter alone. How far has the ship moved? [1]

(d) When the ship sails from Y to the harbor alongside transmitter Q, the detected signal rises and falls in amplitude. Calculate how many dips in intensity will be detected between Y and Q, inclusive. [2]

D6 The figure below illustrates the Young's double-slit experiment. A source of light S illuminates a narrow slit A, which acts as a source for the narrow slits B and C, and produces fringes on the screen. With light of wavelength λ, bright fringes of separation s are formed on the screen.



(a) State the relation between λ, s, d, and D. [1]

(b) Suggest suitable values for d and D. [1]

(c) Describe and explain what happens to the fringes if

(i) both slits B and C are made narrower whilst keeping d constant, [2]

(ii) the light emerging from slit B is reduced in intensity to half that from slit C, [3]

(iii) a thin sheet of transparent plastic is inserted between slit B and slit A, [3]

(iv) slits B and C are both covered with sheets of polaroid with their polarising axis aligned, and that in front of B is slowly rotated. [2]

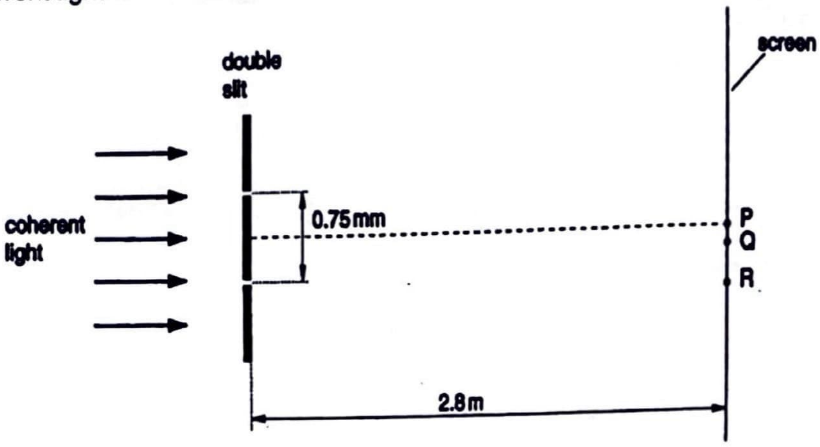
(d) When the wavelength of the source is 500 nm, the center of the 120th dark fringe counting from O' lies at R.

Upon replacement of the source by one of unknown wavelength, R is found to be the location of the 90th bright fringe (take O' as the 0th order maximum).

Find the wavelength of the unknown source. [2]

[J85/P3/Q8(modified)]

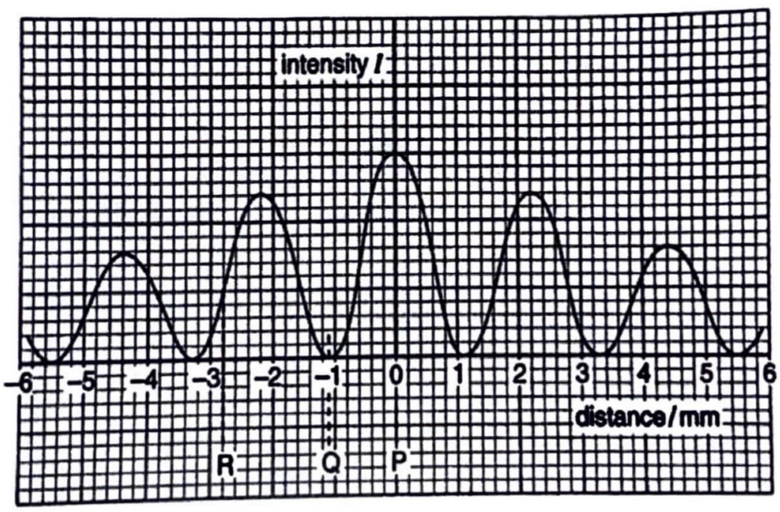
D7 Coherent light is incident normally on a double-slit, as shown in the figure below.



The separation of the slits in the double-slit arrangement is 0.75 mm.

A screen is placed parallel to and at a distance of 2.8 m from the double slit. P is a point on the screen that is equidistant from the two slits.

The figure below shows the variation with distance from P of the intensity I of the light on the screen.



(a) Calculate the wavelength, in nm, of the coherent light. [4]

(b) Points Q and R are points on the screen. Their positions are indicated in the graph of intensity variation. Determine the phase angle between the waves from the double slit when the waves meet at

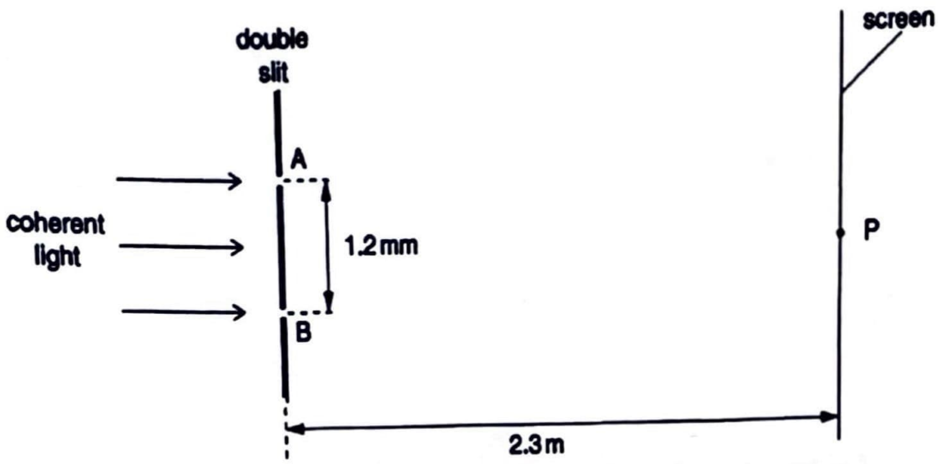
(i) Point Q [1]

(ii) Point R [2]

(c) Suggest why the maxima on the figure are not all of the same intensity [2]

[N2006/P2/Q6]

D8 Coherent light of wavelength 590 nm is incident normally on a double slit, as shown in the figure below.



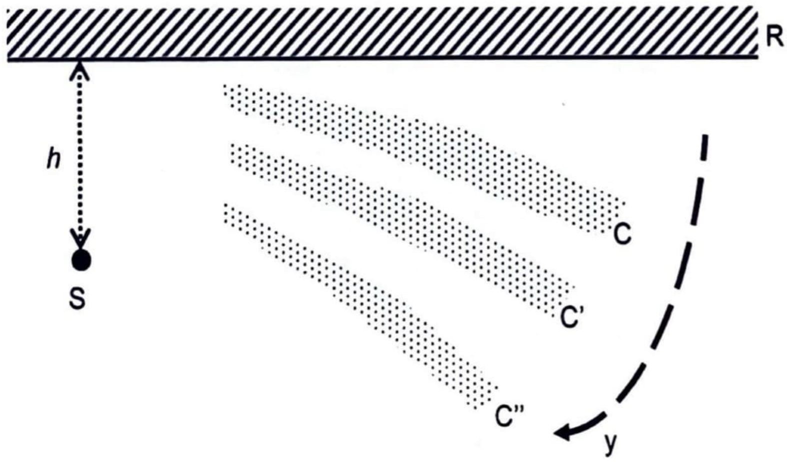
The separation of the slits in the double-slit arrangement is 1.2 mm. Interference fringes are observed on a screen placed parallel to the plane of the double slit and 2.3 m from it. Assume that, for the fringes near point P on the screen, the light reaching the screen from slit A alone has intensity I and that from slit B alone has intensity $\frac{1}{3}I$.

(a) Determine the separation of the bright fringes [3]

(b) Point P on the screen is equidistant from the two slits A and B. Determine the intensity, in terms of I, of a dark fringe near P. [3]

[N2011/P3/Q3(part)]

D9



(a) A source S of continuous waves a distance h from a plane reflector R produces regions of high intensity such as C, C' and C" as shown above. Account for this. [2]

(b) When the frequency of S is changed slowly, the regions C, C' and C" move in the direction y as shown.

(i) Account for this, and [2]

(ii) deduce whether the frequency has been increased or decreased. Assume that no phase change occurs upon reflection with R. [2]

(c) In the Appleton's experiment, S was a radio transmitter on the Earth's surface and R was the Heaviside layer – a reflecting layer in the atmosphere 80 km above the ground. When the transmitted wavelength is slowly changed from 200 m to 180 m, a receiver on the ground 120 km away from S observed fluctuations in the received signal strength.

Calculate the number of signal maxima observed during this change of frequency. [5]

[J79/P3/Q1]

Diffraction Grating

D10 **(a)** A beam of red light from a laser is shone normally on to a diffraction grating. Bright light is seen emerging at certain angles as shown in Fig. 10(a).

Use the principle of superposition to suggest a qualitative explanation of this effect. [2]

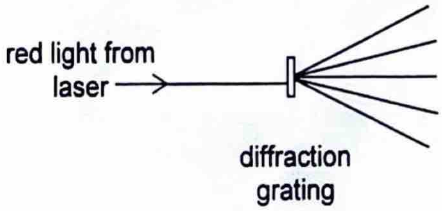


Fig. 10(a)

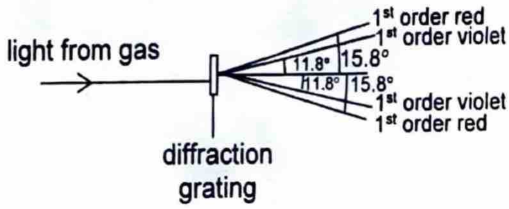


Fig. 10(b)

(b) A diffraction grating with a grating spacing of 2.20×10^{-6} m is used to examine the light from a glowing gas. It is found that the first order violet light emerges at an angle 11.8° and the first order red light at an angle 15.8° as shown in Fig. 10 (b)

(i) Calculate the wavelengths of these two colours. [3]

(ii) Describe and explain what will be observed at an angle of 54.8°. [3]

(iii) Without making any further calculations, draw a sketch similar to Fig. 10(b) showing the whole pattern observed. [2]

[J93/P3/Q2(part)]

D11 A narrow beam of coherent light of wavelength 589 nm is incident normally on a diffraction grating having 4.00×10^5 lines per metre.

(a) Determine the number of orders of diffracted light that are visible on each side of zero order. 4 [3]

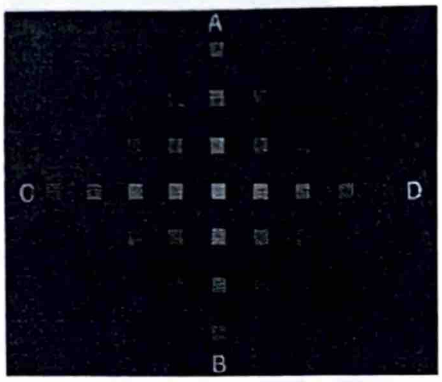
(b) A student suspects that there are two wavelengths of light in the incident beam, one at 589.0 nm and the other at 589.6 nm.

(i) State the order of diffracted light at which the two wavelengths are most likely to be distinguished.

(ii) The minimum angular separation of the diffracted light for which two wavelengths may be distinguished is 0.10 °. Make calculations to determine whether the student can observe the two wavelengths as separate images. [4]

[N2003/P2/Q5(part)]

D12 When a distant streetlight, which is behaving as a point source of monochromatic light of wavelength 5.90×10^{-7} m, passes through a nylon net curtain, the pattern of light incident on a screen is shown in the diagram below.



The main feature of this pattern is the two lines (AB and CD) of bright images.

(a) State which line of AB or CD is produced by the horizontal nylon threads. [1]

(b) Determine the number of threads per millimetre in the nylon net curtain if the angle as viewed between the central image and the first adjacent image is 0.60 °? [4]

[N94/P3/Q2 & N2000/P2/Q4 (modified)]

Single-Slit Diffraction and Rayleigh Criterion

D13 Parallel light of wavelength 590 nm is incident on a rectangular slit of width 0.60 mm, as shown in Fig. 13.1

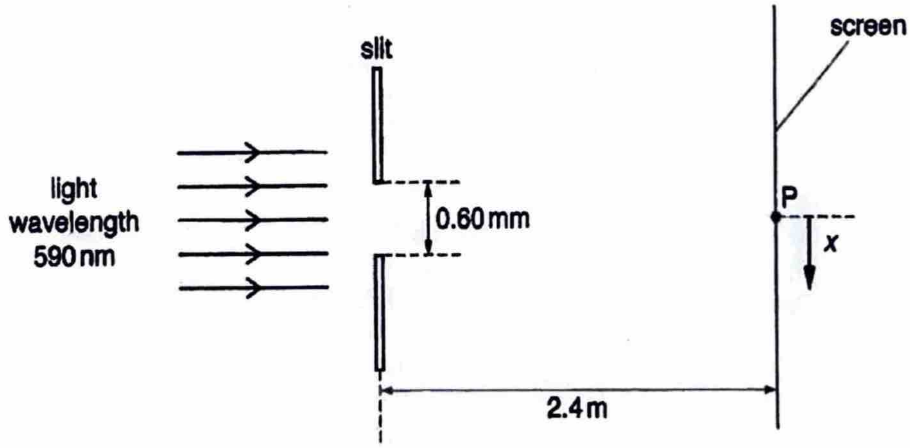


Fig. 13.1 (not to scale)

Light passing through the slit is incident on a screen that is 2.4 m from the plane of the slit.

The centre of the interference pattern formed on the screen is at P.

(a) **(i)** Calculate the width of the central fringe, as observed on the screen. [3]

(ii) On Fig. 13.2, sketch a graph to show the variation with distance x from point P of the intensity I of the light on the screen. [3]

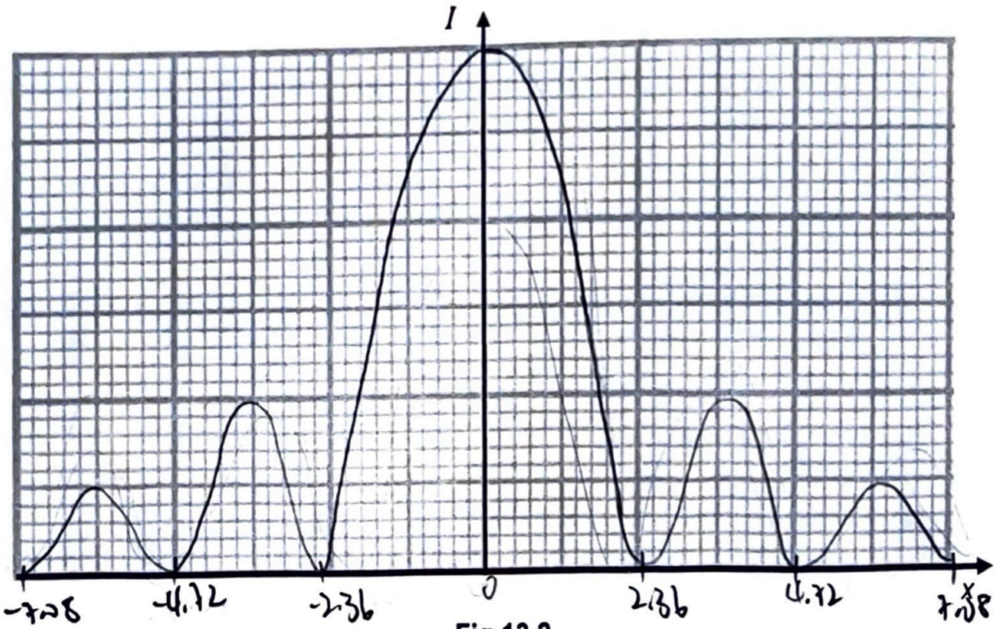


Fig 13.2

(b) Parallel light from a second source and of the same wavelength 590 nm is also incident on the slit. The angle between the two beams of light is θ, as shown in Fig. 13.3.

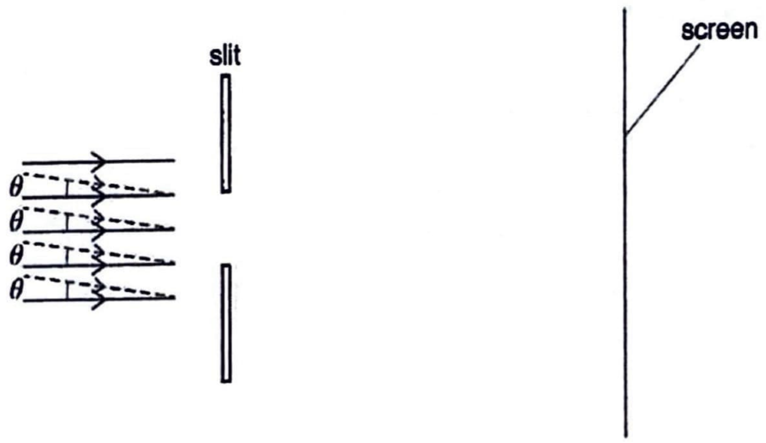


Fig. 13.3 (not to scale)

Each beam forms a separate interference pattern on the screen.

(i) Explain what is meant by the Rayleigh criterion for the resolution of the two patterns. [2]

(ii) Calculate the angle θ, in rad, such that the two interference patterns are just resolved. [2]

N2017/P3/Q5

D14 Two closely spaced monochromatic point sources of light are viewed through a single slit of width w.

(a) Sketch the appearance of the images observed for small and large values of w. [2]

(b) Sketch a graph showing the intensity variations along a line through the centres of the images for a critical value of w when the images are just resolvable. [2]

(c) Suggest a way to improve the resolving power of an imaging system which is used to observe small objects or small separation between objects. [1]

D15 A beam of monochromatic light of wavelength 600 nm is incident on a single slit of width 0.0200 mm.

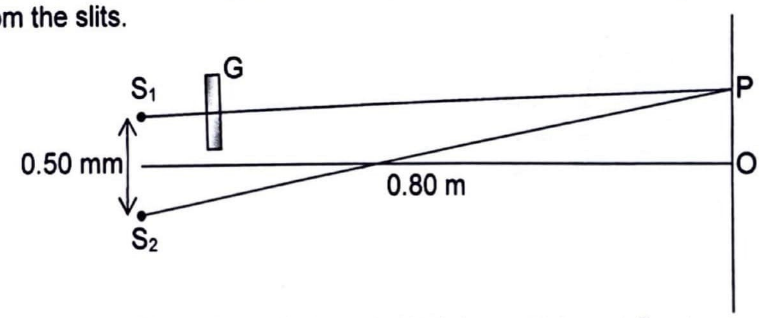
(a) Deduce the resolving power of the slit, [2]

(b) What is the minimum separation between two small objects placed 1.00 km from the slit, in order to be resolved by the slit? [2]

(c) Suppose the single slit is now brought into a medium of refractive index n = 1.33. What would be the answer for **(b)**? [3]

Challenging Questions

C1 In the figure below, S_1 and S_2 are two coherent monochromatic light sources separated by a distance of 0.50 mm. Position O is equidistant from S_1 and S_2, and the screen is 0.80 m from the slits.



When a thin parallel-sided piece of glass G of thickness 3.6×10^{-6} m is placed in front of S_1 as shown, the centre of the fringe system moves from O to a point P.

Calculate OP, if the wavelength of the two sources is 6.0×10^{-7} m and the refractive index of the glass is 1.5.

Ans: 2.88×10^{-3} m

C2 An oil film floating in water is illuminated by white light at normal incidence. The film is 280 nm thick and has a refractive index of 1.45. By considering interference of light rays which are reflected at the air-oil and oil-water boundary, find the dominant colour in the reflected light. Take the refractive indices of air and water to be 1.00 and 1.30, respectively.

Note: When light enters a denser medium from a less-dense medium, it will undergo a 180° phase shift. No such phase shift occurs when light enters an optically less dense medium from a denser one.

Ans: 541 nm (green)

Answers

SP1	(a) 2, (b) f_2L	**SP2**	125 Hz, 375 Hz	**SP3**	1.0×10^{10} Hz
SP4	A	**SP5**	A	**SP6**	0.184 m
SP7	62.5 m	**SP8**	7.2×10^{-4} m		
SP9	(a) 2.5×10^{-6} m, (b) 13.9°, (c) 9				
SP10	644 nm, 509 nm, 480 nm, 468 nm Max order = 3, 3, 4, 4			**SP11**	433 nm
SP12	D	**SP13**	C	**SP14**	5.77×10^5 m^{-1}
SP15	D	**SP16**	D	**SP17**	56 °

D2 (a) 1.8 cm, (b) 267 Hz
D3 (a) 341 m s^{-1}, (b) 272 mm, 556 mm
D4 (a)(ii) 335 m s^{-1}
D5 (c) 75 m , (d) 200
D6 (d) 664 nm
D7 (a) 589 nm , (b)(i) π rad, (ii) 1.7 rad
D8 (a) 1.13×10^{-3} m , (b) 0.179 I
D9 (c) 44
D10 (b)(i) 4.50×10^{-7} m, 5.99×10^{-7} m
D11 (a) 4
D12 (b) 17.7 lines per mm
D13 (a)(i) 4.72×10^{-3} m, (b)(ii) 9.83×10^{-4} rad
D15 (a) 3.00×10^{-2} rad , (b) 30.0 m , (c) 22.6 m

Self - Check Questions

S1 The principle of superposition states that when two or more waves of the same kind meet at a point in space, the resultant displacement at that point is equal to the vector sum of the displacements of the individual waves at that point.

S2 A stationary wave is the result of the superposition of two progressive waves of the same type, frequency, amplitude and speed, travelling along the same line but in the opposite directions.

S3

Property	Progressive Wave	Stationary Wave
Waveform	Propagates with the velocity of the wave.	Does not propagate.
Energy	Transports energy.	Does not transport energy.
Amplitude	Every point oscillates with the same amplitude.	Amplitude varies from 0 at the nodes to the maximum at the antinodes.
Phase	All particles within one wavelength have different phases.	All particles between two adjacent nodes have the same phase. Particles in adjacent segments have a phase difference of π rad.
Frequency	All points vibrate in s.h.m. with the frequency of the wave.	Except for the nodes which are at rest, all points vibrate in s.h.m. with the same frequency as the progressive waves that give rise to it.
Wavelength	Equals to the distance between adjacent points which have the same phase.	Equals to twice the distance between a pair of adjacent nodes or antinodes.

S4 Twice the length of the string, four times the length of the pipe, twice the length of the pipe.

S5 Diffraction is the bending of waves after passing through an aperture or round an obstacle.

S6 Diffraction effects are only significant when the aperture size is comparable to the wavelength of the waves. The wavelengths of visible light are very short (~10^{-7} m) compared the size of 'ordinary' openings such as doors and windows. Hence, diffraction of light is not easily observed

S7 *Interference* is the superposition of two or more waves to give a resultant wave whose resultant amplitude is given by the principle of superposition.

Constructive (Destructive) interference occurs when two coherent waves arrive at the same point with a phase difference of *zero* (*π rad*) to produce a *maximum (minimum)*.

Phase difference between two waves at a point is the fraction of a cycle which one wave is ahead of the other.

Path difference between two waves is a measure of the fraction of a cycle which one is ahead of the other.

Coherence refers to the condition in which two sources or waves have a constant phase difference.

S8
- The waves must be **coherent**.
- The waves must have **similar amplitude**.
- The waves must **overlap** and be of the **same type**.
- For electromagnetic waves, they must be **unpolarised** or **polarised** in the same plane.

S9 Separate light sources of the same frequency do not produce coherent waves because light pulses from the sources have short durations and abrupt change in phase. Since the human eye cannot cope with the rapid changes in phase, the pattern is not observable.

On the other hand, two loudspeakers connected to the same signal generator produce long continuous wave-trains which have a constant phase difference. Hence, the interference pattern does not fluctuate.

S10
$$x = \frac{\lambda D}{a}$$

This relationship is only valid when the rays are almost parallel (ie, a << D) and the angle of the fringes from the normal is small (ie, a >> λ)

S11 $d \sin\theta = n\lambda$, where d is the slit-separation, n is the order being observed, θ is the angle of the n^{th}-order maximum from the straight-through, λ the wavelength of incident waves. This equation assumes that the rays through the slits are parallel, ie d << D, where D is the distance between the grating and the screen.

S12 From $\sin\theta = \dfrac{n\lambda}{d} < 1 \Rightarrow n < \dfrac{d}{\lambda}$.

S13 $\sin\theta = \dfrac{\lambda}{b}$, where b is the width of the slit

The equation is only valid when the distance of the screen from the single-slit is much larger than the width of the single-slit as the derivation assumed parallel rays.

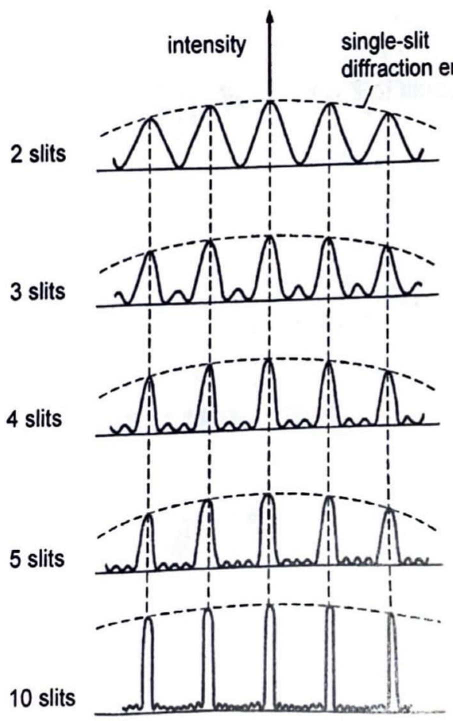
The width of the central bright fringe is directly related to θ which is inversely related to the width of the aperture b.

For small angles, the width y of the central bright fringe on a screen a distance D away is $y = \dfrac{2D\lambda}{b}$.

S14 Rayleigh criterion states that two images are just resolved by an aperture when the central maximum of one image falls on the first minimum of the other image.

This condition is satisfied when the limiting angle of resolution θ (or the minimum angular separation of two objects) follows $\theta \approx \dfrac{\lambda}{b}$, where λ is the wavelength of the light radiating or reflected from the two objects and b is the width of the rectangular slit.

S15



Self - Practice Questions

SP1 (a) At the fundamental frequency, $\lambda_1 = 2L$

At the first overtone, $\lambda_2 = L$

Since the speed of the wave propagating along the string is a constant,

$$v_1 = v_2 \Rightarrow f_1\lambda_1 = f_2\lambda_2 \Rightarrow \frac{f_2}{f_1} = \frac{\lambda_1}{\lambda_2} = 2$$

(b) $v = f_2 \times \lambda_2 = f_2 L$.

SP2 Since this is a closed pipe, the two lowest frequencies will be the first $\left(L = \frac{1}{4}\lambda_1\right)$ and the third $\left(L = \frac{3}{4}\lambda_3\right)$ harmonics.

The wavelengths are $\lambda_1 = 4L = 2.4$ m and $\lambda_2 = 4L/3 = 0.8$ m respectively.

Hence, the frequencies are:

First Harmonic: $f_1 = v/\lambda_1 = 300 \div 2.4 = 125$ Hz

Third Harmonic: $f_3 = v/\lambda_3 = 300 \div 0.8 = 375$ Hz

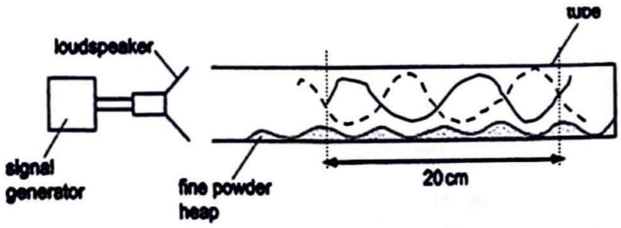
SP3 Successive nodes are 1.5 cm apart. Hence, $\lambda = 2 \times 1.5 = 3.0$ cm.

From $v = f \times \lambda$,

$\Rightarrow$ $3.0 \times 10^8 = f \times 0.030$

$\therefore$ $f = 1.0 \times 10^{10}$ Hz

SP4



As shown in the figure, 20 cm corresponds to 2 wavelengths. Hence, $\lambda = 20/2 = 10$ cm

$$f = \frac{v}{\lambda} = \frac{330}{0.10} = 3300 \text{ Hz} = 3.3 \text{ kHz}$$

Sound waves are longitudinal waves.

Answer: A

SP5 The mode of vibration corresponds to the 1st overtone or 3rd harmonic. Hence, $L = \frac{3}{4}\lambda$

Pressure change is maximum at the nodes.

Note: Pressure antinodes occur at displacement nodes, and vice versa.

Answer: A

SP6 Minimum at detector corresponds to destructive interference

$\therefore$ Path difference = $\pi r - 2r = \frac{1}{2}\lambda$

$\therefore$ r = 0.184 m

SP7

$$x = \frac{\lambda D}{a}$$

$$0.80 = \frac{\left(50 \times 10^{-3}\right)(1000)}{a}$$

$a = 62.5$ m

SP8 Using $x = \frac{\lambda D}{a}$,

fringe separation for 480 nm $= \frac{4.8 \times 10^{-7} \times 1}{5.0 \times 10^{-4}}$
$= 9.6 \times 10^{-4}$ m

fringe separation for 600 nm $= \frac{6.0 \times 10^{-7} \times 1}{5.0 \times 10^{-4}}$
$= 1.2 \times 10^{-3}$ m

Hence, separation between third-order interference fringes $= 3\left(1.2 \times 10^{-3} - 9.6 \times 10^{-4}\right)$
$= 7.2 \times 10^{-4}$ m

SP9 (a) N = 400 lines per mm = 400 000 lines per metre.
Grating spacing $d = 1/N = 1 \div 400\,000 = 2.5 \times 10^{-6}$ m

(b) $d \sin\theta = n\lambda$

$2.5 \times 10^{-6} \sin\theta = 1 \times \left(600 \times 10^{-9}\right)$

$\theta = 13.9^\circ$

(c) $\sin\theta = \frac{n\lambda}{d} \quad \Rightarrow \frac{n\lambda}{d} < 1$

$\Rightarrow n < \frac{d}{\lambda} = \frac{2.5 \times 10^{-6}}{6 \times 10^{-7}} = 4.17$

Hence, $n = 4$. Adding the maxima of the other side and that at the straight-through position, the total number of maxima is 4 + 4 + 1 = 9

SP10 $d = 1/500 = 0.00200$ mm $= 2.00 \times 10^{-6}$ m

colour	**angle / °**	**(a) $\lambda = d\sin\theta$ / nm**	**(b) $n_{max} < d/\lambda$**
red	18.78	644	3.1 ~ 3
green	14.74	509	3.9 ~ 3
light blue	13.89	480	4.2 ~ 4
dark blue	13.53	468	4.3 ~ 4

SP11 If two wavelengths λ_x and λ_y are diffracted to the same angle, then

$d\sin\theta = n_x\lambda_x = n_y\lambda_y$

$\Rightarrow \quad 3 \times \lambda_x = 2 \times 650$

$\Rightarrow \quad \lambda_x = 433$ nm

SP12 As point source P is not equidistant from slits S_1 and S_2, path difference Δx of the 2 rays must be determined from source P to point of observation Q:

$$\Delta x = (l_1 + l_3) - (l_2 + l_4)$$

Since destructive interference takes place at point Q, the path difference Δx must be an odd multiple of half-wavelength:

$$\Delta x = (2m + 1)\,\lambda/2 \text{ or } (m + ½)\,\lambda$$

Answer: D

SP13 $n = 2$, $\theta = \alpha/2$

Hence, $d = \dfrac{n\lambda}{\sin\theta} = \dfrac{2\lambda}{\sin(\alpha/2)}$

Answer: C

SP14 From $d \sin\theta = n\lambda$, it can be seen that the longest wavelength deviates the most.

$\Rightarrow d = \dfrac{n\lambda}{\sin\theta} = \dfrac{2(7.5\times10^{-7})}{\sin 60°} = 1.73\times10^{-6}$ m

$\therefore N = 1/d = 5.77\times10^{5}$ m^{-1}

SP15 For the first minimum to occur at P, the path difference between wavelets from R and the mid-point to P is $\lambda/2$. Similarly, the path difference between wavelets from the mid-point and S to P is also $\lambda/2$. Hence, the path difference between SP and RP is λ. Angle θ for the first minimum is such that $\sin\theta = \lambda/a$.

Answer: D

SP16 When the width of the slit is reduced, less energy passes though the slit and hence the intensity of all peaks decreases.

Since $\sin\theta = \dfrac{\lambda}{b}$, when b decreases, θ increases and hence the width of the pattern also increases.

Answer: D

SP17 $\theta = \sin^{-1}\left(\dfrac{1.0}{1.2}\right) = 56°$