Anglo-Chinese School

(Independent)

THE BEST IS YET TO BE

PRELIMINARY EXAMINATION 2019

YEAR 6 IB DIPLOMA PROGRAMME

MATHEMATICS

HIGHER LEVEL

PAPER 2

24 September 2019

2 hours

Candidate Session Number

0	0	2	3	2	9				

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the writing paper provided. Fill in your session number on each answer sheet, and attach them to this examination paper using the string provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics HL and Further Mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**

Section A (50 Marks)		Section B (50 Marks)	
Question	**Marks**	**Question**	**Marks**
1		**9**	
2			
3		**10**	
4			
5			
6		**11**	
7			
8			
Subtotal		**Subtotal**	
TOTAL	**/ 100**		

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This question paper consists of 13 printed pages including this cover page.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

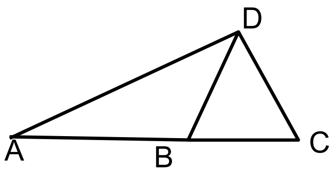
Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 5]

ACD is a triangle and B is a point on AC such that $BC = x$ cm and $AB = (x+2)$cm .

$\angle BCD = 48^o$ and $\angle BDC = 50^o$. If the area of $\Delta ABD = 23.06$ cm^2, find the value of x .



2. [Maximum mark: 7]

Find the value of n if the coefficient of x^3 in the expansion of $(2 + 3x + x^2)^n$ is 1560 .

3. [Maximum mark: 5]

Solve the following system of equations:

$$\begin{cases} \ln x = y - 2 \\ \ln y = x - 2 \end{cases}$$

4. [Maximum mark: 8]

(a) Prove that $\cos t+\sqrt{3}\sin t=2\cos\left(t-\frac{\pi}{3}\right)$. [3]

(b) Particle P travels with velocity $v=\cos t+\sqrt{3}\sin t$ m/s for $0\le t\le\frac{5\pi}{6}$.

(i) Find the maximum velocity.

(ii) Find the exact time when it is first travelling with velocity $\sqrt{2}$ m/s. [3]

(c) Another particle, Q travels with velocity v such that $v=\frac{1}{\cos t+\sqrt{3}\sin t}$ for $0\le t\le\frac{\pi}{2}$.

Find the total distance travelled by particle Q. [2]

5. [Maximum mark: 6]

Let w be one of the non-real solutions to $z^3 = 1$.

(a) Determine the value of $1 + w + w^2$. [2]

(b) Hence evaluate $\left(w - 2w^2\right)\left(w^2 - 2w\right)$. [4]

6. [Maximum mark: 7]

(a) Show that $\dfrac{5x^2 - 2x + 3}{(1 - x)(2x^2 + 1)} = \dfrac{2}{1 - x} + \dfrac{1 - x}{2x^2 + 1}$. [2]

(b) Hence find $\displaystyle\int \dfrac{5x^2 - 2x + 3}{(1 - x)(2x^2 + 1)}\, dx$. [5]

7. [Maximum mark: 6]

(a) Find the value(s) of k for which $\dfrac{4x-4}{2x+3}=k$ has no solutions. [3]

(b) Find the values of x such that $1<\dfrac{4x-4}{2x+3}<3$. [3]

8. [Maximum mark: 6]

6 men and 4 women participated in a sports competition which consisted of earlier elimination rounds.

(a) In the first round the 10 contestants are grouped into two groups of 5. In how many ways can the two groups be formed if each group must have at least one woman? [3]

(b) A particular male contestant was eliminated in the first round and the remaining 9 people were asked to do a photo shoot. 9 chairs were arranged with 4 chairs in the first row and 5 chairs in the second row. In how many ways can the 9 remaining contestants be arranged such that the men and women occupy alternate seats? [3]

Do **not** write solutions on this paper.

Section B

Answer **all** questions in the answer sheets provided. Please start each question on a new page.

9. [Maximum mark: 18]

(a) The number of cars sold in a car showroom in one week has a Poisson distribution with mean 3.2.

(i) Show that the probability that at least 7 cars are sold in a week is 0.0446, to 3 significant figures.

(ii) A random sample of 150 weeks is taken. Find the probability that at least 7 cars are sold per week in no more than 4 of the weeks.

(iii) For two randomly chosen weeks, find the probability that 1 car is sold in one of the weeks given that a total of 3 cars are sold in the two weeks.

[12]

(b) The price of cars follows a normal distribution with mean μ and variance $30\ 000^2$. The top 10% of cars that cost above \$200 000 are classified as 'expensive', the next 75% are classified as 'moderate' and the rest are classified as 'value for money'.

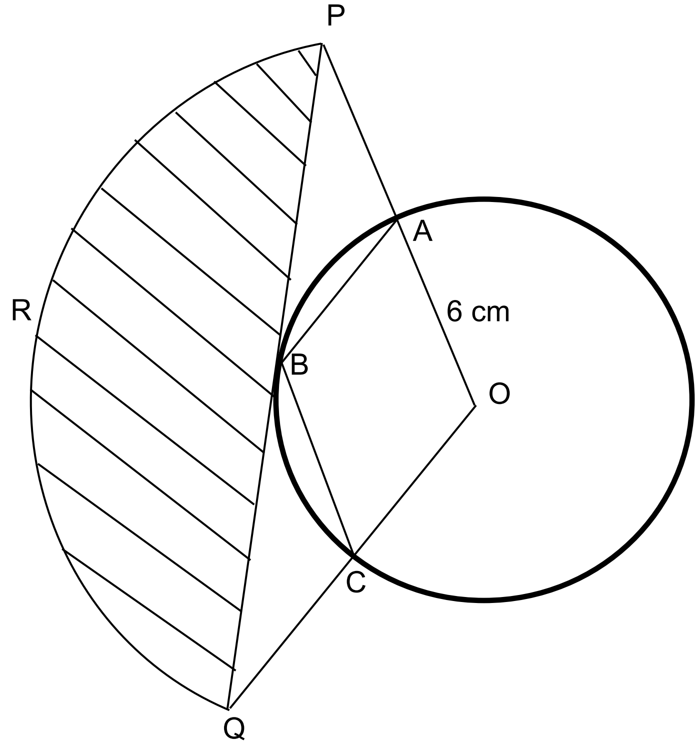
(i) Find the value of μ to the nearest dollar.

(ii) Find, to the nearest dollar, the minimum price of a car for it to be classified as 'moderate'.

[6]

10. [Maximum mark: 11]

(a) The diagram shows a circle with centre O and radius 6 cm. PBQ is a tangent to the circle at B and OP and OQ intersect the circle at A and C respectively. OABC is a rhombus and PRQ is an arc of another circle with centre O. Calculate the area of the shaded segment PRQ. [4]



(b)

(i) Find the area bounded by the curve $y = \arccos x$ and the axes.

(ii) The region enclosed by $y = \arccos x$, $y = \dfrac{\pi}{3}$, $y = \dfrac{\pi}{6}$ and the y-axis is rotated 2π radians about the y-axis. Find the exact volume of the solid generated. [7]

11. [Maximum mark: 21]

(a) $$f(x) = \begin{cases} -x+1, & x \le 2 \\ -x^2 + hx - k, & 2 < x \le 5 \end{cases}$$

where $h, k \in R$

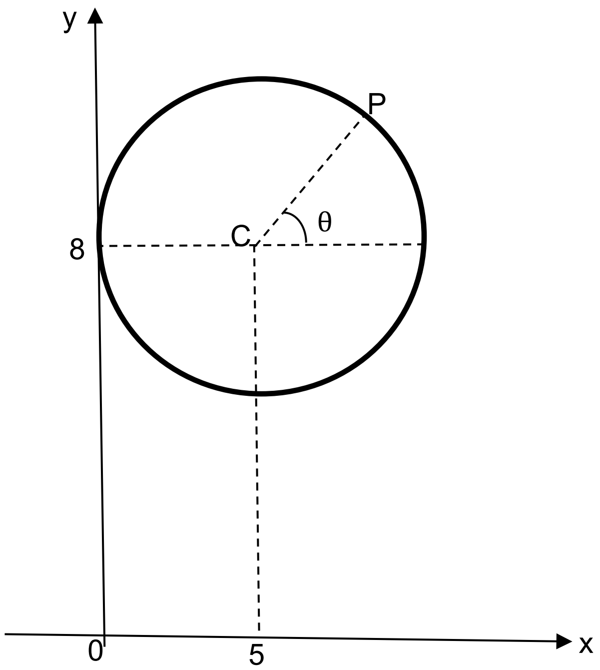
(i) Given that f and its derivative, $f\,'$, are continuous for all values in the domain of f, find the values of h and k.

(ii) Show that f is a one-to-one function.

(iii) Obtain expressions for the inverse function f^{-1} and state their domains.

[9]

(b)



The diagram shows a circle with centre C (5, 8) in the x-y plane and vector equation given by **r** = $(5 + 5cos\theta)$**i**+$(8 + 5sin\theta)$**j**. P is a general point on the circle and the angle between the unit vector **i** and CP, measured in an anti-clockwise direction from the horizontal to CP is θ. Given a line *l* with vector equation **r** = $2t$**i**+$(\mathbf{8} - \mathrm{t})$**j**

where $t \in$ R,

(i) find the perpendicular distance of C (5, 8) to the line *l*.

(ii) Find the coordinates of the 2 points of intersection of the circle and the line *l* .

(iii) Another line l_1 has vector equation $\mathbf{r} = (6 - 3s)\mathbf{i} + (\mathbf{9} + s)\mathbf{j} + (\mathbf{8} - s)\mathbf{k}$ where $s \in$ R,

find the cartesian equation of the plane that contains both *l* and l_1 .

[12]

END OF PAPER