Paya Lebar Methodist Girls' School (Secondary)
Preliminary Examination 2017
Secondary 4 Express / 5 Normal Academic

Name: _____ ( ) Class: _____

Centre Number	S					Index Number				

MATHEMATICS 4048/02

Paper 2 21 August 2017

Additional Materials: Answer Paper (10 sheets) 2 hours 30 minutes
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number, name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in a loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 **(a)** **(i)** Factorise $16x^2y^2 - 40xy + 25$. [1]

(ii) If $16x^2y^2 - 40xy + 25 = k$, express y in terms of x and k, where $k > 0$. [2]

(b) Solve the equation $\frac{2x+15}{3} = 4 - \frac{2x-3}{6}$. [2]

(c) Simplify $\frac{8-2x^2}{12+4x-x^2}$. [2]

(d) Given that $\frac{1}{x} - \frac{1}{y} = 3$, find the value of $\frac{2x+3xy-2y}{x-2xy-y}$. [2]

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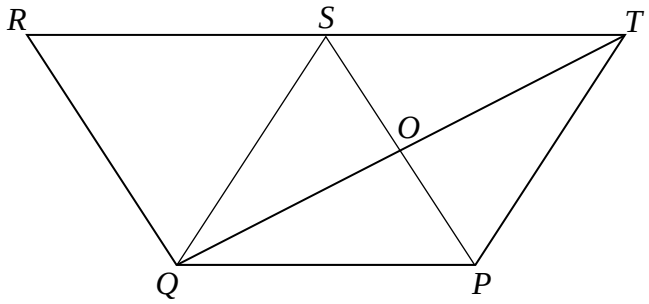
2 **(a)** Each exterior angle of an n-sided regular polygon is 16° greater than each exterior angle of another $3n$-sided regular polygon.

(i) Find n. [2]

(ii) Calculate an interior angle of the $3n$-sided regular polygon. [2]

(b) In the diagram, $PQRS$ is a rhombus and QS is a diagonal.
RS is produced to point T such that S is the midpoint of RT.
PS and QT intersect at O.

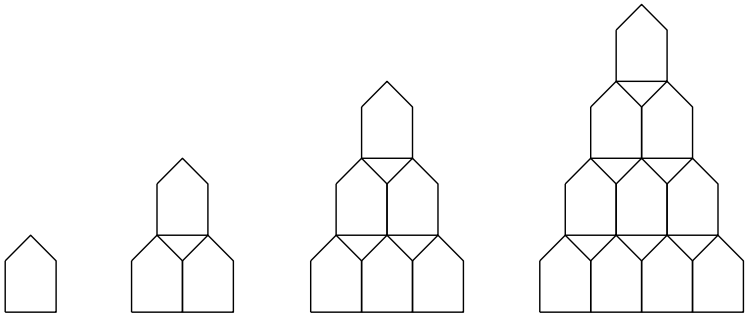


(i) Show that triangles QRS and PST are congruent. [3]

(ii) Explain why $PQST$ is a parallelogram. [2]

(iii) Show that triangles POQ and RQT are similar. [2]

3 Toothpicks are used to form a series of patterns as shown.



1st 2nd 3rd 4th

*n*th pattern	Total number of toothpicks used	Total number of regions formed within the pattern	Number of points at which 2 or more toothpicks meet
	T	*R*	*P*
1	5	1	5
2	14	4	11
3	27	9	19
4	44	16	29
⋮	⋮	⋮	⋮
6	*a*	*b*	*c*

(a) Write down the value of *a*, of *b* and of *c*. [3]

(b) Explain why the number 99 cannot appear in column *R*. [1]

(c) Express *P* in terms of *T* and *R*. [1]

(d) Show that the *n*th term of the pattern, T_n, is given by $T_n = 2n^2 + 3n$. [1]

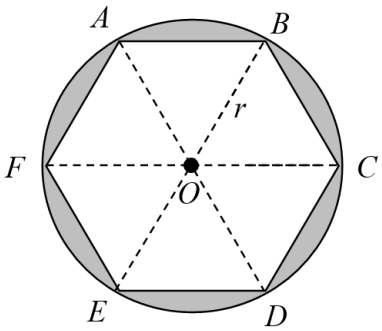
(e) T_{p+1} and T_p are consecutive terms in the pattern.

(i) Find and simplify an expression, in terms of *p*, for $T_{p+1} - T_p$. [2]

(ii) Explain why two consecutive terms of the pattern is not a multiple of 4. [1]

(f) Hence or otherwise, find the number of points at which 2 or more toothpicks meet in the 11th pattern. [1]

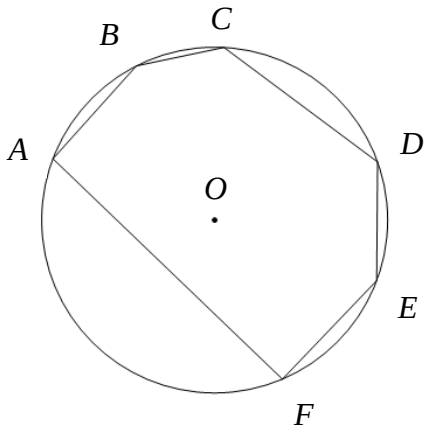
4 **(a)** The diagram shows a regular hexagon *ABCDEF* enclosed exactly by a circle of centre *O* and radius *r* cm.



(i) Show that angle *AOC* is $\frac{2}{3}\pi$ radians. [1]

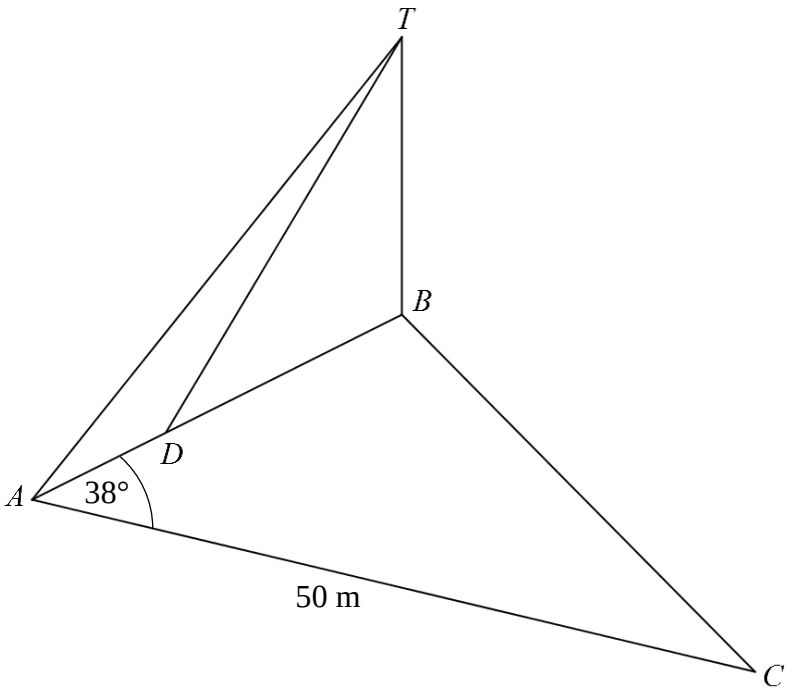
(ii) Find the shaded area in terms of *r*. [4]

(b) The diagram shows a circle with centre *O*.
A, *B*, *C*, *D*, *E* and *F* are points on the circle.



Show that angle *ABC* + angle *CDE* + angle *EFA* = 360°. [3]

5 *A*, *B* and *C* are three points on level ground.
Angle *BAC* = 38° and *AC* = 50 m.
T is the top of a vertical flagpole *TB*.



(a) The angle of elevation of the top of the flagpole from *A* is 24°.
The angle of elevation of the top of the flagpole from *D*, 6 m from *A*, is 32°.

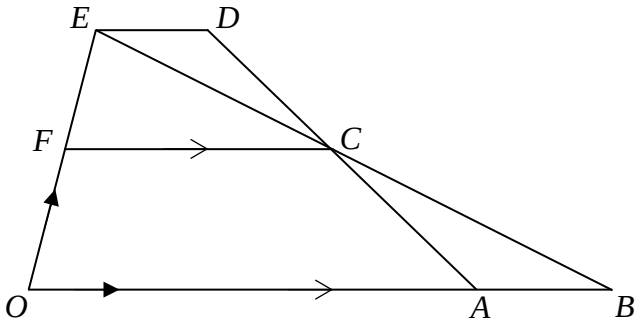
Calculate the height of the flagpole. [3]

(b) Find the length *BC*. [2]

(c) Another flagpole *EF* is to be positioned vertically on *AC* such that its base, *E*, is nearest to *B*.

Find the length *AE*. [2]

6 In the diagram, *OADE* is a trapezium and *FC* is parallel to *OB*.
It is given that $\overrightarrow{OF} = 4\mathbf{a}$ and $\overrightarrow{OB} = 7\mathbf{b}$.
$OF : OE = 3 : 7$ and $7OA = 5OB$.



(a) Express, as simply as possible, in terms of **a** and/or **b**,

(i) $\overrightarrow{FC}$, [1]

(ii) $\overrightarrow{CB}$, [1]

(iii) $\overrightarrow{EC}$, [1]

(iv) $\overrightarrow{ED}$. [1]

(b) What two facts can be deduced about *E*, *C* and *B*? [2]

(c) Find the ratio of

(i) area of triangle *EFC* : area of triangle *EOB*, [1]

(ii) area of triangle *EDC* : area of triangle *ABC*, [1]

(iii) area of quadrilateral *EDCF* : area of quadrilateral *OBCF*. [2]

7 Sticker printing machines print stickers at a constant rate.

(a) Machine *A* prints 35 stickers in one hour.
Machine *B* prints *y* stickers in one hour.

Machine *A* and machine *B* print a total of 38 stickers in 30 minutes.
Find *y*. [2]

(b) **(i)** Machine *C* prints *x* stickers in one hour.

Write an expression, in terms of *x*, for the number of hours taken by Machine *C* to print 37 stickers. [1]

(ii) Compared to machine *C*, Machine *D* prints 4 **more** stickers in one hour.

Write an expression, in terms of *x*, for the number of hours taken by Machine *D* to print 37 stickers. [1]

(iii) The difference in the timing taken by machine *C* and machine *D*, to each print 37 stickers is 15 minutes.

Write an equation in *x* to represent this information and show that it reduces to $x^2 + 4x - 592 = 0$. [3]

(iv) Solve the equation $x^2 + 4x - 592 = 0$. [3]

(v) Find the time taken by Machine *C* to print 37 stickers.
Give your answer in hours and minutes, correct to the nearest minute. [1]

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8 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation

$$y = 2\left(\frac{3}{x} + 2x - 5\right).$$

Some corresponding values of x and y, are given in the table below.

x	0.5	1	1.5	2	2.5	3	3.5	4
y	4	0	0	1	2.4	4	p	7.5

(a) Calculate the value of p. [1]

(b) Using a scale of 4 cm to represent 1 unit, draw a horizontal x-axis for $0 \le x \le 4$.
Using a scale of 2 cm to represent 1 unit, draw a vertical y-axis for $-1 \le y \le 8$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to

(i) find the minimum value of $2\left(\frac{3}{x} + 2x - 5\right)$ in the range $0 \le x \le 4$, [1]

(ii) find the solutions to the equation $\frac{3}{x} + 2x - 5 = 2$. [2]

(d) By drawing a tangent, find the gradient of the curve at (2, 1). [2]

(e) The line $y = -2x + a$ intersects the curve $y = 2\left(\frac{3}{x} + 2x - 5\right)$ at $x = 0.5$ and $x = b$.
On the same axes, draw the line $y = -2x + a$ for $0 \le x \le 4$.

Find the values of a and b. [3]

9 **(a)** A group of 50 students took the standing broad jump test.
The results are shown in the table.

Jumping distance (cm)	$80 < x \leq 100$	$100 < x \leq 120$	$120 < x \leq 140$	$140 < x \leq 160$
Number of students	20	16	$x + 10$	x

(i) Find

(a) the value of x, [1]

(b) the percentage of students who jumped more than a metre. [1]

(ii) Calculate

(a) the mean distance, [1]

(b) the standard deviation of the distances. [2]

(iii) The standard deviation of the distances achieved by a second group of students to complete the same standing broad jump test was 15 cm.
The mean distance achieved was 110.5 cm.

Use these information to comment on two differences between the two distributions. [2]

(b) Bag *A* contains 1 gold medal, 4 silver medals and 2 bronze medals.
Bag *B* contains 2 gold medals and 5 silver medals.

A medal is drawn at random from bag *A* and placed into bag *B*.
A medal is then drawn at random from bag *B*.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

(ii) Find, as a fraction in its simplest form, the probability that

(a) the medal drawn from bag *B* is a gold medal, [2]

(b) the medal drawn from bag *B* is a bronze medal. [1]

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10 With the higher demand for private hired cars, taxi companies introduced options and prices to cater to the different needs of the commuters.
The three most popular taxi companies are C-Taxi, G-Taxi and U-Taxi.

(a) A research was conducted to identify the cheapest fare provided by the three taxi companies.
The data shows the overall fare for a 5-km journey charged by each company.

Taxi Company	**C-Taxi**	**G-Taxi**	**U-Taxi**
Overall Fare ($)	9.80	7.84	6.02

A commuter chooses to use the taxi company that charges the cheapest fare.
Calculate the amount of fare saved as a percentage of the highest fare charged. [1]

The information on the fare charges of the 3 taxi companies is on the opposite page.

(b) The distance between Clarke Quay and Potong Pasir is 8 km.

(i) How much will it cost to take U-Taxi at 12.30 a.m., from Clarke Quay to Potong Pasir, if the whole journey takes 12 minutes? [2]

(ii) U-Taxi's fare emerged as the cheapest even though its fare includes Time fare per minute charges.

Suggest a reason for this. [1]

(c) Ms Seet will be starting work at her new office in Raffles Place next month.
She intends to commute to her office via taxi every day at 8.00 a.m.
She stays in Potong Pasir, which is 9.5 km from her new office.
The journey from Potong Pasir to Raffles Place takes 15 minutes.
Assume that there is a total waiting time of 5 minutes in the journey.

Suggest the taxi company that Ms Seet should use.
Justify the decision you make and show your calculations clearly. [6]

C-Taxi 	
Base fare (1 km or less)	$3.20
Every 400 m thereafter or less up to 10 km	$0.22
Every 350 m thereafter or less after 10 km	$0.22
Every 45 seconds of waiting or less	$0.22
Peak Period Surcharge	
Monday to Friday (Except Public Holidays): 6.00 a.m. – 9.29 a.m. Monday to Sunday & Public Holidays: 6.00 p.m. – 11.59 p.m.	25% of metered fare
Midnight – Before 5.59 a.m.	50% of metered fare
Monday to Sunday & Public Holidays: 5.00 p.m. – 11.59 p.m. ** Applicable at the time of boarding for taxis hired within the City Area and payable on top of the Peak Period Surcharge.*	$3 City Area Surcharge

G-Taxi 	
Base fare	$3.00
Per kilometre	$0.80
** Total charges are rounded off to the nearest dollar.*	

U-Taxi 	
Base fare	$3.00
Per kilometre	$0.45
Time fare per minute	$0.20

Note:

* Electronic Road Pricing charge is not included since it is levied on the customer by each company.

End of Paper