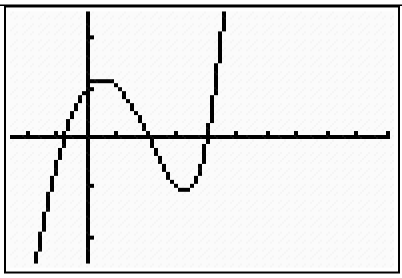
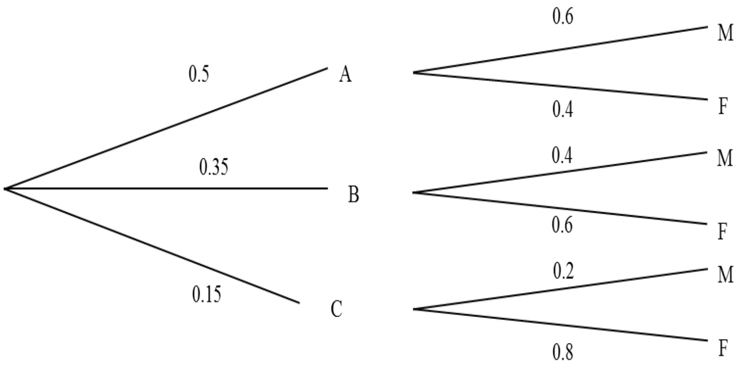
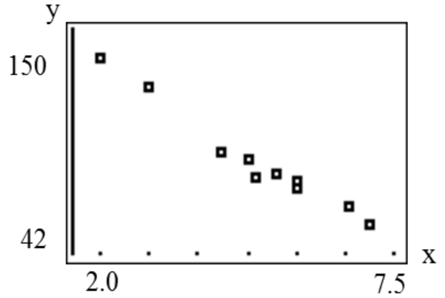
2012 H1 A Level Solution

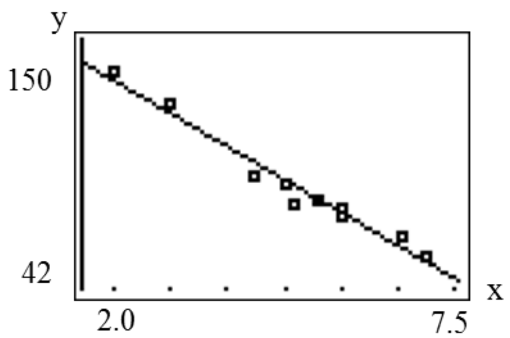
1	Let $e^{2x}=u$ $3u=4(u^{-1}-1)$ $3u^2=4(1-u)$ $3u^2+4u-4=0$ Solving, $u=-2$ or $u=\frac{2}{3}$ $e^{2x}=-2$ or $e^{2x}=\frac{2}{3}$ (rejected) $2x=\ln\frac{2}{3}$ $x=\frac{1}{2}\ln\frac{2}{3}$
2	$3x+y+20+x+y=100$ $4x+2y=80$ $2x+y=40-(1)$ $(x+20)(y)=3(20x)$ $y=\frac{60x}{x+20}-(2)$ Solving (1) and (2): $2x+\frac{60x}{x+20}=40$ $2x^2+40x+60x=40x+800$ $2x^2+60x-800=0$ $x^2+30x-400=0$ Solving, $x=-40$ or $x=10$ (rejected) When $x=10$, $y=\frac{60(10)}{10+20}=20$ Length of HF = 10 + 20 = 30
3(i)	$\frac{3}{4}k^2=k^2-x^2$ $x^2=\frac{1}{4}k^2$ $x=\pm\frac{1}{2}k$

3(ii)	$\int_{-\frac{1}{2}k}^{\frac{1}{2}k}(k^2-x^2)-\frac{3}{4}k^2dx$ $=2\int_{0}^{\frac{1}{2}k}\left(\frac{1}{4}k^2-x^2\right)dx$ $=2\left[\frac{1}{4}k^2x-\frac{x^3}{3}\right]_0^{\frac{1}{2}k}$ $=2\left[\frac{1}{8}k^3-\frac{1}{24}k^3\right]$ $=\frac{1}{6}k^3$
4(i)a)	$\frac{d}{dx}2\ln\left(3x+2\right)$ $=2\times\frac{d}{dx}\ln(3x+2)$ $=2\times\frac{3}{3x+2}$ $=\frac{6}{3x+2}$
4(i)b)	$\frac{d}{dx}\frac{4}{2x+1}$ $=\frac{d}{dx}4(2x+1)^{-1}$ $=4(-1)(2x+1)^{-2}\times 2$ $=\frac{-8}{(2x+1)^2}$
4(ii)	$\int_2^4\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2dx$ $=\int_2^4(x-2+\frac{1}{x})dx$ $=\left[\frac{x^2}{2}-2x+\ln x\right]_2^4$ $=\left(\frac{16}{2}-8+\ln 4\right)-\left(\frac{4}{2}-4+\ln 2\right)$ $=\ln 4-(-2+\ln 2)$ $=2+\ln 4-\ln 2$ $=2+\ln\frac{4}{2}$ $=2+\ln 2$

5(i)	 Coordinates of axis intercepts = (-0.767, 0), (2, 0), (4, 0), and (0, 1).
5(ii)	When $x = 1.5$, gradient of C = -1.0395
5(iii)	When $x = 1.5$, $y = 0.57843$ $0.57843 = -1.0395(1.5) + c$ $c = 2.1377$ $\therefore y = -1.0395x + 2.1377$
5(iv)	At A: $x = 0, y = 2.1377$ At B: $x = y = 1.0481$ Length of AB = $\sqrt{(1.0481-0)^2 + (2.1377-1.0481)^2} = 1.51 (3s.f)$
6(i)	It is a method to obtain a sample of size n from a population of size N via systematic sampling, where we line up the population and *randomly* select a first member out of the first k members, where $k \approx \frac{N}{n}$. Then we *automatically* select every kth member following this first chosen member.
6(ii)	Advantage: This method is cost effective as it takes less time and effort to carry out. Disadvantage: The sample may not be representative of the town people as it may not include adults from different strata in a correct proportion. E.g: Too many men.
6(iii)	Instead of standing outside the main supermarket, he can obtain a list of adults who use computers and choose 100 people from the list to do his survey.
7(i)	A and B independent => $P(A \cap B) = P(A)P(B)$ $P(A) + P(B) - P(A \cup B) = P(A)P(B)$ $p + p - \frac{5}{9} = p^2$ $p^2 - 2p + \frac{5}{9} = 0$
(ii)	$p = \frac{5}{3}$ (rejected since $0 < p < 1$) or $p = \frac{1}{3}$ $P(A \cap B) = p^2 = \frac{1}{9}$

<table>
<tr><td>8(i)</td><td>

$P(\text{vote for A and male}) = 0.5 \times 0.6 = 0.3$</td></tr>
<tr><td>(ii)</td><td>$P(\text{female}) = 0.5(0.4) + 0.35(0.6) + 0.15(0.8)$
$= 0.53$</td></tr>
<tr><td>(iii)</td><td>$P(\text{vote for C}|\text{male}) = \dfrac{P(\text{vote for C and male})}{P(\text{male})}$
$= \dfrac{0.15 \times 0.2}{1 - 0.53}$
$= 0.0638$</td></tr>
<tr><td>9(i)</td><td>
</td></tr>
<tr><td>(ii)</td><td>$r = -0.984$
This value indicates that there is a strong negative linear correlation between the age of ‘Pluto’ cars and advertised price. The advertised price decreases at a constant rate as age of the car increases.</td></tr>
<tr><td>(iii)</td><td>$y = -19.21x + 183.12$


</td></tr>
<tr><td>(iv)</td><td>(a) when $x = 4$, $y = -19.213(4) + 183.12$
$= 106.27$
Advertised price = $10 600 (corrected to 3 s.f)</td></tr>
</table>

	(b) when $x=9, y=-19.213(9)+183.12$ $=10.203$ Advertised price = \$1 020 (corrected to 3 s.f)
(v)	For (a), since $\|r\|$ is close to 1 and $x=4$ is within the data range, the estimate is reliable. For (b), $x=9$ is outside the data range, thus the estimate is not reliable.
10(i)	Let S be the r.v. "no of Sunbrite plants that will flower in one tray" $S \sim B(12, 0.8)$ $P(S=10)=0.283$
(ii)	$P(S<8)=P(S\le 7)=0.0726$
(iii)	Let X be the r.v. "no of Sunbrite plants that will flower in 8 trays" $X \sim B(96, 0.8)$ Since $n=96>50$ is large , $np=76.8>5, nq=19.2>5,$ $X \sim N(76.8, 15.36)$ approximately $E(X)=76.8$ $V(X)=15.36$ $P(X>75) \xrightarrow{c.c} P(X>75.5)=0.62994 \approx 0.630$
(iv)	Let Y be the r.v. "no of gardeners out of 3 with more than 75 of their plants will flower" $Y \sim B(3, 0.62994)$ $P(Y\ge 2)=1-P(Y\le 1)$ $=0.691$
11(i)	$\bar{x}=\frac{\sum(x-300)}{100}+300$ $=\frac{-60}{100}+300$ $=299.4$ $s^2=\frac{1}{100-1}(1240-\frac{(-60)^2}{100})$ $=12.162\approx 12.2$
(ii)	Let X be the r.v. "length and string" and μ be the population mean. $H_0: \mu=300$ $H_1: \mu<300$ Under H_0, since $n=100>50$ is large, $\bar{X} \sim N(300, \frac{12.162}{100})$ approximately by CLT p-value = 0.0427 < 0.05 Therefore, reject H_0 and conclude that at 5% significance level, there is sufficient evidence that the average length of string in a ball is at least 300m.
(iii)	Under H_0,

	$\bar{X} \sim N(300, \frac{12.1}{100})$ approximately by CLT Do not reject H_0, p-value > sign level $P(\bar{X} < k) > 0.10$ Using GC, $k > 299.554$ Hence, the least possible value of k = 299.56
12(i)	Let A be the r.v. "mass, in kg, of grapefruit of type A" Let B be the r.v. "mass, in kg, of grapefruit of type B" $A_1 + \ldots + A_{10} \sim N(2.5, 0.004)$ $P(A_1 + \ldots + A_{10} < 2.4) = 0.0569$
(ii)	$D = (A_1 + \ldots + A_6) - (B_1 + \ldots + B_5) \sim N(-0.25, 0.0069)$ $P(-0.2 < D < 0.2) = 0.274$
(iii)	Let W be the r.v. "amount that Mrs Woo pay" $W = 1.5(A_1 + A_2 + A_3) - 2.4(B_1 + B_2 + B_3) \sim N(3.645, 0.018252)$ Let T be the r.v. "amount that Mr Tan pay" $T = 1.5(A_1 + \ldots + A_{10}) \sim N(3.75, 0.009)$ $W - T \sim N(-0.105, 0.027252)$ $P(W - T > 0) = 0.262$