H

TEMASEK JUNIOR COLLEGE

2024 JC1 END OF YEAR EXAMINATION

Higher 2

MATHEMATICS **9758**

27 Sep 2024

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Answer all questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

1 The equation of a curve C is $\frac{x^2}{25} - (y-7)^2 = 4$.

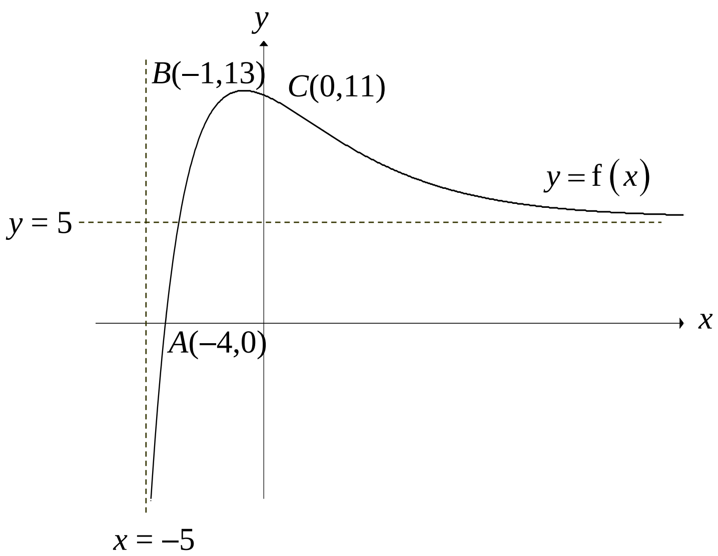
State precisely a sequence of transformations by which the curve C may be obtained from the graph of $x^2 - y^2 = 1$. [3]

2 Mr Chan started selling three types of roasted meat rice in a hawker centre. For each plate of chicken rice, duck rice and pork rice, he makes a profit of exactly 50 cents, 80 cents and $1.10 respectively.

On a particular day, Mr Chan made a profit of $167.40, and he earned $28.80 more in profit from the sale of pork rice than the chicken rice. The total number of plates of rice sold that day is found to be 3 times the plates of duck rice sold on that day.

Find the number of plates of chicken rice, duck rice and pork rice that Mr Chan sold that day. [4]

3 The diagram below shows the graph of $y = \mathrm{f}(x)$. The curve intersects the x-axis and y-axis at $A(-4, 0)$ and $C(0, 11)$ respectively and has a maximum point at $B(-1, 13)$. The lines $y = 5$ and $x = -5$ are asymptotes of the curve.



Sketch, on separate diagrams, the graphs of

(a) $y = \mathrm{f}'(x)$, [2]

(b) $y = \dfrac{1}{\mathrm{f}(-x)}$. [3]

4 **(a)** Find $\int \frac{x^2+4}{x^2-2x+5}\,\mathrm{d}x$. . [4]

(b) Find $\frac{\mathrm{d}}{\mathrm{d}x}\sec^2 3x$. [1]

Hence find $\int x\tan 3x\sec^2 3x\,\mathrm{d}x$. [3]

5 **(a)** Differentiate with respect to *x*,

(i) $\ln\left(\frac{1+\sin x}{1-\sin x}\right)$, leaving your result in a form involving a single trigonometric function. [3]

(ii) $\tan^{-1}\left(\frac{1}{\sqrt{1-x}}\right)$ where $x<1$, simplifying your answer. [3]

(b) It is given that $y=x^{3x}$ for $x > 0$. By taking logarithm first, find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of *x*. [3]

6 **(a)** Sketch the graph of $y=\frac{x^2+3x+3}{x+1}$, indicating clearly the equations of any asymptotes, coordinates of any stationary points and any points where the curve crosses the axes. [3]

` **(b)** By adding a suitable graph on the same diagram, solve the inequality, $\frac{x^2+3x+3}{x+1}\geq\left|5\ln(x+3)-1\right|$. [4]

(c) Hence solve the inequality

$$\frac{\sin^2 x-3\sin x+3}{1-\sin x}\geq\left|5\ln(3-\sin x)-1\right|, \text{ where } -\frac{\pi}{2}\leq x\leq\frac{\pi}{2}. \quad [2]$$

7 **(a)** The vectors **a**, **b** and **c** are such that $\mathbf{a}+\mathbf{b}=\frac{1}{2}\mathbf{c}$.

(i) Show that $\mathbf{a}\times\mathbf{c}=\mathbf{c}\times\mathbf{b}$. [2]

(ii) It is given that **c** is a unit vector and that **a** and **c** are perpendicular. If the angle between **b** and **c** is $\frac{\pi}{3}$, show that **b** is also a unit vector. [3]

(b) The points *D*, *E* and *F* have position vectors $-3\mathbf{i}+4\mathbf{j}-3\mathbf{k}$, $5\mathbf{i}+\mathbf{k}$ and $\mathbf{2i}+4\mathbf{j}+\mathbf{k}$ respectively. The point *P* lies in the line segment *DE* and is 3 unit away from *F*. Find all possible position vectors of *P*. [5]

[Turn over

8 **(a)** The function f is defined by

$$\mathrm{f}(x) = 1 - \cos^{-1} x,\ 0 \le x < 1.$$

(i) By using differentiation, show that the function f^{-1} exist. Hence find the value of $\mathrm{f}^{-1}\left(1 - \frac{\pi}{2}\right)$. [4]

(ii) Sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ on the same diagram. [2]

(b) The functions g and h are defined by

$$\mathrm{g}(x) = 1 + \frac{1}{x-2} \quad \text{for } x \in \mathbb{R},\ x \ne 2$$

$$\mathrm{h}(x) = \lambda - (x-1)^2 \quad \text{for } x \in \mathbb{R}, \text{ where } \lambda \text{ is a positive integer.}$$

(i) Find the range of h in terms of λ. [1]

(ii) Hence find the largest value of λ such that the composite function gh exists and find the corresponding range of gh. [3]

9 The curve C has parametric equations

$$x = \frac{3t-1}{t-1}, \quad y = \frac{t^2-1}{k}, \text{ where } t \in \mathbb{R},\ t \ne 1,$$

and k is a positive real constant.

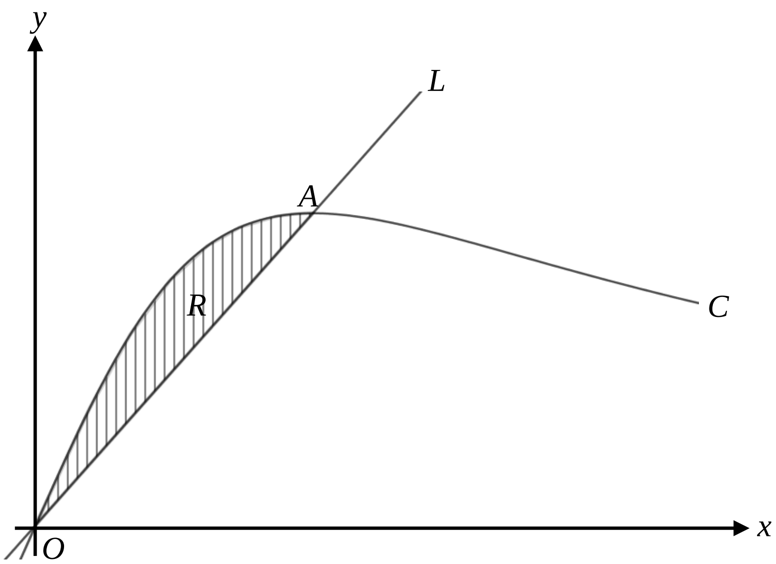
(a) C cuts the the x-axis at the point A. Show that A has coordinates (2, 0). [2]

(b) Find the equation of the tangent T to C at the point A. [4]

(c) T cuts C again at the point B. Find the coordinates of B in terms of k. [3]

(d) Find the area of the triangle OAB. [1]

10 The diagram below shows curve C with equation $y=\dfrac{2x}{4+x^2}$, where $x \geq 0$ and the line L with equation $y=\dfrac{1}{4}x$. The region R is bounded by C and L.



(a) Without the use of a graphing calculator, find the exact area of R. [4]

(b) Use the substitution $x = 2\tan\theta$ to find $\displaystyle\int \frac{x^2}{\left(4+x^2\right)^2}\,\mathrm{d}x$ in terms of θ.

Hence find the volume, in term of π, of the solid generated when the region R is rotated completely about the x-axis. [6]

11 The line l_1 has a vector equation $\mathbf{r}=\begin{pmatrix}-2\\0\\-1\end{pmatrix}+\lambda\begin{pmatrix}3\\0\\4\end{pmatrix}$, $\lambda \in \mathbb{R}$. Another line l_2 cuts l_1 at the point A with coordinates $(-2, 0, -1)$ and l_2 makes an angle of $\dfrac{\pi}{3}$ with l_1.

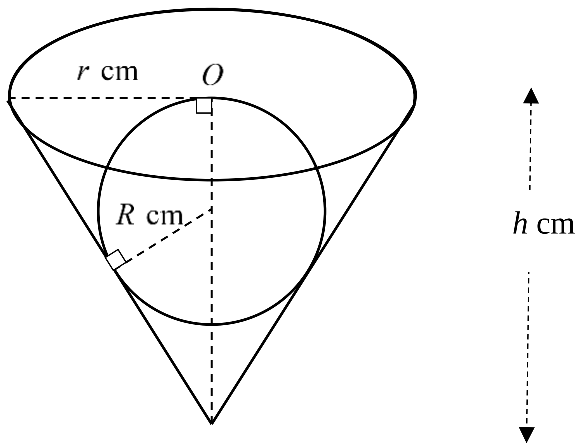
(a) Given that l_2 also passes through the point B with coordinates $(0, k, 0)$ where k is a positive real number, show that the exact value of k is $\sqrt{11}$. [3]

(b) The point N is the foot of perpendicular of B onto the line l_1. Find the position vector of N. [4]

(c) Hence find the exact area of triangle ABN. [3]

12 [It is given that the volume of cone with radius r and height h is $\frac{1}{3}\pi r^2 h$]

A water filter system takes the shape of an inverted right circular cone with a circular top of radius r cm and height h cm. It contains a spherical filter which the water passes through during filtration. The sphere has a constant radius R cm and is inscribed in the cone such that the sphere is in contact with the center of the circular top, O, and the slant surface of the cone as shown in the diagram below.



(a) Show that $r^2 = \dfrac{h^2R^2}{h^2 - 2hR}$. [3]

(b) As h varies, find the minimum volume of the cone in terms of R. [5]

[You do not need to show that the volume is minimum]

Filtered water from the filtration system is treated with two chemicals X and Y. The amount of chemical X and Y that are added to the water at time t seconds are x cm^3 and y cm^3 respectively.

The amount of the chemicals is controlled by the following equation

$$\frac{1}{x^2} - \frac{1}{y^2} = \frac{1}{a^2} \text{ where } a \text{ is a positive constant.}$$

(c) Given that X is added at a rate of 2 cm^3 per second, find the exact rate at which Y is added when $y = 2a$. [4]

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