CATHOLIC JUNIOR COLLEGE
General Certificate of Education Advanced Level
Higher 2
JC2 Preliminary Examination

MATHEMATICS 9740/01

Paper 1

22 AUGUST 2012

3 hours

Additional Materials: List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, arrange your answers in NUMERICAL ORDER.
Place this cover sheet in front and fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

Name: ____________________________ **Class:** ________________

Question	1	2	3	4	5	6	7	8	9	10	11	Total
Marks												
Total	6	9	10	11	10	7	12	8	9	10	8	100

This document consists of 6 printed pages.

Catholic Junior College

1 It is known that w satisfies the equation $|2w-1| \le |w+1|$.

(i) If $w \in \mathbb{R}$, find the range of values of w. [2]

Hence, state the range of values of w for which $\left|\dfrac{2w-1}{w+1}\right| \ge 1$. [1]

(ii) If $w \in \mathbb{C}$, find a cartesian equation for the locus of w and hence sketch it on an Argand diagram. [3]

2 (a) Mr Thrift makes use of a special offer from a bank to obtain an interest-free loan of \$2000. He decides to pay \$50 in the first month. On the first day of each subsequent month, he pays \$10 more than in the previous month. How many complete months would it take for him to fully repay the debt? [3]

(b) On 1 January 2012, Mr. Spendalot uses a credit card to borrow \$2000 from a bank, at an interest rate of 2% a month. He repays the bank \$50 on the 10$^{\text{th}}$ of each month. Interest is charged on the balance at the end of each month.

(i) Calculate the outstanding amount at 1 January 2013. [4]

(ii) How many months does he take to repay the entire loan? [2]

3 A sequence $u_1, u_2, u_3, \ldots$ is such that $u_1 = 0$ and

$$u_{n+1} = u_n + \frac{4+n-n^2}{[(n+1)(n+2)]^2}, \text{ for all } n \in Z^+.$$

(i) Prove by mathematical induction that $u_n = \dfrac{n-1}{(n+1)^2}$ for all positive integers n. [4]

(ii) Hence, find $\displaystyle\sum_{n=2}^{N} \frac{4+n-n^2}{[(n+1)(n+2)]^2}$ and state the sum to infinity. [3]

(iii) Hence, or otherwise, evaluate $\displaystyle\sum_{n=2}^{\infty} \frac{2+3n-n^2}{[n(n+1)]^2}$. [3]

4 Two lines, l_1 and l_2, have equations $\dfrac{x-1}{2} = -z-1,\ y=2$ and $x+3 = 4-y = z+2$ respectively.

(i) Determine whether l_1 and l_2 intersect, and state the coordinates of the intersection point, if any. [3]

(ii) Find the acute angle between l_1 and l_2. [2]

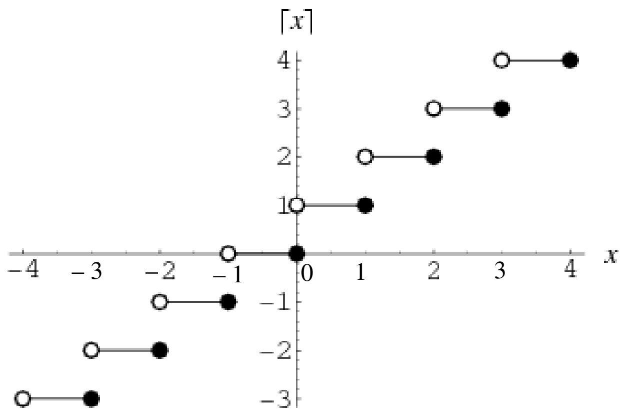
(iii) Find an equation for the line l_3, where l_3 is the reflection of l_1 about l_2. [6]

5 (a) The graph of a function f(x) has undergone the following transformations:

Step 1: Translation in the negative x-direction by 1 unit
Step 2: Translation in the positive y-direction by 5 units
Step 3: Scaling parallel to the x-axis by a factor of $\frac{1}{2}$

The function of the resulting graph is $\mathrm{h}(x) = 4x^2 + 4x + 9$.
Express h(x) in terms of f(x). Hence, obtain an expression for function f(x). [5]

(b) The **ceiling function,** $\lceil x \rceil$, is a function which gives the smallest integer greater than or equal to x, e.g. $\lceil 2.2 \rceil = 3$, $\lceil -2.7 \rceil = -2$. The graph of $y = \lceil x \rceil$ is shown below:



(i) Show that $2\lceil 2.5 \rceil + \lceil 3.09 \rceil = 10$. [1]

A function g(x) is defined by

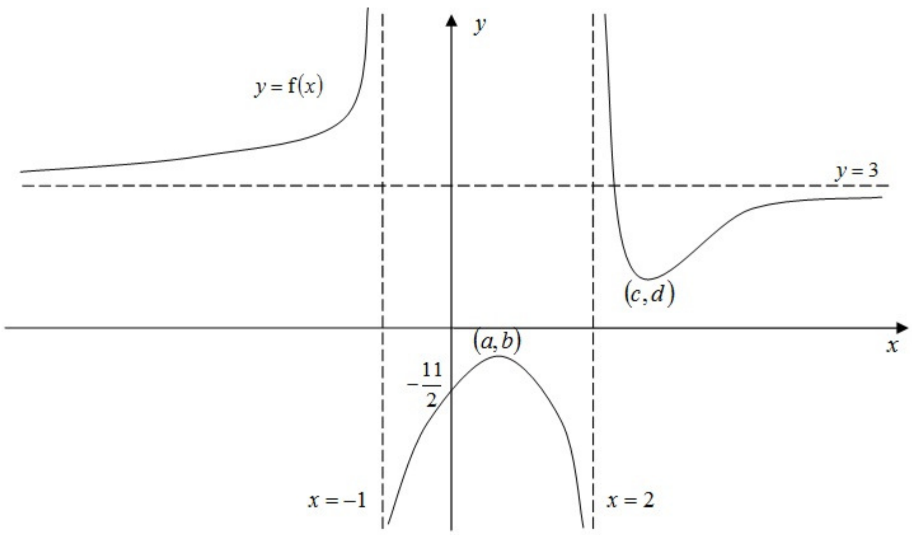
$$\mathrm{g} : x \mapsto \left\lceil \frac{x+1}{2} \right\rceil, \text{ where } x \in \mathbb{R}^+.$$

(ii) Evaluate $\mathrm{g}(0.12) + \mathrm{g}(3)$. [2]

(iii) Sketch the graph of $y = \mathrm{g}(x)$ for $0 < x \leq 4$. [1]

(iv) State the range of g for all positive real values of x. [1]

6 The sketch below shows the graph of $y = \mathrm{f}(x)$. The curve intersects the y-axis at the point $\left(0, -\dfrac{11}{2}\right)$, has a maximum point at (a,b) and a minimum point at (c,d). The equation of the asymptotes are $y = 3$, $x = -1$ and $x = 2$.



On separate diagrams, sketch the following graphs indicating the points corresponding to the stationary points, axial intercepts and asymptotes where necessary.

(i) $y = \mathrm{f}(|x|)$ [2]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$ [3]

(iii) $y = \mathrm{f}\,'(x)$ [2]

7 A curve C is defined by the parametric equations

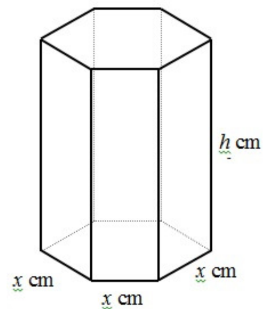
$$x = 2\cos\theta + 1, \quad y = \sin\theta, \quad \text{where } -\pi \le \theta \le \pi.$$

(i) Sketch the curve C, giving the coordinates of any points of intersection with the x- and y- axes. [2]

(ii) Find the area enclosed by the curve C for $-\pi \le \theta \le \pi$. [4]

(iii) The normal to the curve at the point $(2\cos\theta + 1, \sin\theta)$, where $0 < \theta < \dfrac{\pi}{2}$, meets the x- and y- axes at Q and R respectively. The origin is denoted by O. Find the area of triangle OQR, leaving your answer in terms of θ. [6]

8 Candy is being stored in a closed container in the form of a regular hexagonal prism, with sides x cm and height h cm (as shown in the figure below).



Given that the container has a volume of 972 cm^3,

(i) show that its base area given by $\dfrac{3\sqrt{3}x^2}{2}$ cm^2. [1]

(ii) Using differentiation, find the minimum area of the material, A cm^2, that is used to make the container, leaving your answer to 2 decimal places. [6]

(iii) Given that the cost of the material for the packaging is \$ 0.05 per 100 cm^2, find the minimum cost required for the container. [1]

9 Let x and y be variables such that $\ln(y+1) = 1 + \tan^{-1} x$.

(i) Show that $(1+x^2)\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = (1-2x)\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(ii) Find the Maclaurin series for y, up to and including the term in x^3, giving the coefficients in terms of e. [4]

(iii) Given that the first two non-zero terms in the Maclaurin series for y are equal to the first two non-zero terms in the series expansion of $\dfrac{1}{a+bx}$, where a and b are non-zero constants, find a and b in terms of e. [3]

10 (a) Find the roots of the equation $z^3 = 2 - 2\sqrt{3}i$ in exponential form. [3]

(b) Given that $z = 3i$ is a root of the equation

$$az^4 + z^3 + 11z^2 + bz + 18 = 0$$

(i) find the values of the real numbers a and b. [3]

(ii) Hence solve the equation exactly. [4]

11 On a single Argand diagram, sketch the following loci:

(i) $|z-3|=5$ [2]

(ii) $\arg(z-8)=\pi-\tan^{-1}\left(\frac{1}{2}\right)$ [2]

(iii) $|z+2|=|z-3+5i|$ [2]

State the complex number p that represents the point of intersection of the loci in (i) and (ii), in the form $a + ib$, where a and b are exact integers to be determined. [1]

The complex number w satisfies the inequalities

$$|w-3|\le 5$$

$$\arg(w-8)\le\pi-\tan^{-1}\left(\frac{1}{2}\right)$$

$$|w+2|\ge|w-3+5i|.$$

Illustrate the locus of w on the same Argand diagram. [1]

— **THE END** —