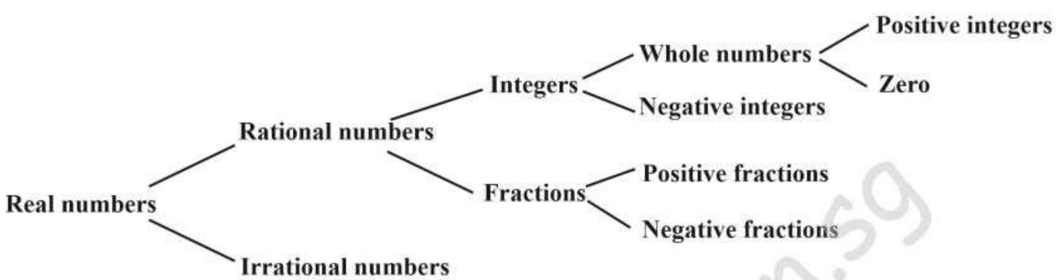
TOPIC-1

Numbers, Time and Percentage

Revision Notes:

1. The Number System



(a) Positive integers : 1, 2, 3, ….

(b) Whole numbers : 0, 1, 2, ….

(c) Negative integers : –1, –2, -3, ….

(d) Integers : …., 2, –1, 0, 1, 2, ….

(e) Rational numbers: $\frac{a}{b}$ where a, b are integers and $b \neq 0$.

(f) An irrational number is a number which cannot be expressed in the form $\frac{a}{b}$ where a, b are integers and $b \neq 0$

(g) Real numbers are the set of rational and irrational numbers.

2. Prime Numbers and Prime Factorisation

(a) A prime number is a number which has exactly 2 different factors, 1 and the number itself.
E.g., Prime numbers are 2, 3, 5, 7, 11, 13, ___

(b) A composite number is a number with more than two different factors.
E.g., Composite numbers are 4, 6, 8, 9, 10, 12, 14, ___

(c) The number 1 is neither prime nor composite.

(d) In prime factorisation, a composite number is expressed as a product of prime numbers.
E.g., Prime factorisation of 120 is $120 = 2^3 \times 3 \times 5$

3. HCF and LCM

(a) The HCF of two or more given numbers is the greatest number which is a common factor of these numbers.
E-g. HCF of 15 and 25 is 5.

(b) The LCM of two or more given numbers is the smallest number which is a common multiple of these numbers.
E-g. LCM of 16 and 24 is 48.

4. Squares, Square Roots. Cubes and Cube Roots

(a) If a number y can be expressed as $y = x^2$, then y is the square of x and x is the square root of y.
E.g. The square of 6 is $6^2 = 36$
The square of –6 is $(-6)^2 = 36$
The square roots of $36 = \pm\sqrt{36} = \pm 6$

5. Percentage

(a) A percentage is a fraction with the denominator 100.
E.g., $\frac{1}{100} = 1\%$, $\frac{45}{100} = 45\%$, $\frac{75}{100} = 75\%$.

(b) Fraction to Percentage
E.g., $\frac{5}{8} = \frac{5}{8} \times 100\% = 62.5\%$

(c) Decimal to Percentage
E.g., $0.65 = 0.65 \times 100\% = 65\%$

(d) Percentage to Fraction or Decimal
E.g., $12\% = \frac{12}{100} = \frac{3}{25}$
$22\frac{1}{2}\% = 22.5\% = \frac{22.5}{100} = 0.225$

(e) $\text{Percentage increase} = \frac{\text{increase}}{\text{original amount}} \times 100\%$
$\text{Percentage decrease} = \frac{\text{decrease}}{\text{original amount}} \times 100\%$

Worked Examples:

Example 1

The numbers 60 and 126, written as products of their prime factors, are $60 = 2^2 \times 3 \times 5$, $126 = 2 \times 3^2 \times 7$. Use these results to find
(a) the HCF of 60 and 126
(b) the LCM of 60 and 126,
(c) the smallest positive integer k, such that $1226k$ is a perfect square.

Solution:

(a) HCF = $2 \times 3 = 6$

(b) LCM = $2^2 \times 3^2 \times 5 \times 7 = 1260$

(c) $126k = 2 \times 3^2 \times 5 \times 7 \times k$

For $126k$ to be a perfect square, the smallest $k = 2 \times 7 = 14$

Example 2

Estimate $\frac{11.9 \times 0.598}{23.6}$, giving your answer correct to one significant figure.

Solution:

$$\frac{11.9 \times 0.598}{23.6} = \frac{12 \times 0.6}{24} = 0.3$$

TOPIC-2

Ratio, Rate, Proportion, Speed and Map Problems

Revision Notes:

1. Ratio

(a) The ratio of two similar quantities is what fraction one quantity is of the other. The ratio of a to b is written as a: b or $\frac{a}{b}$.

(b) To find the ratio of two quantities, we must first express them in the same units.

(c) A ratio has no unit.

2. Rate

Rate is used to describe how a quantity is changing with another quantity.

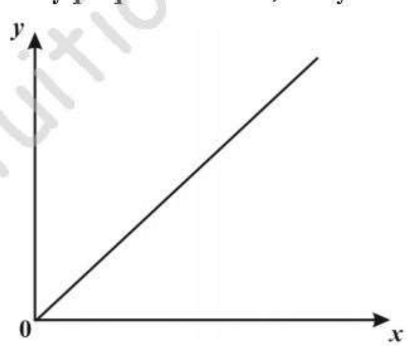
E.g., Speed is the rate of distance travelled per unit of time.

3. Proportion

(a) Direct Proportion

(i) Two quantities x and y are in **direct proportion** if corresponding pairs of values are in the same ratio i.e. $\frac{x_1}{x_2}=\frac{y_1}{y_2}$, where x_1 and x_2 are any two values of x and y_1 and y_2 are the corresponding values of y.

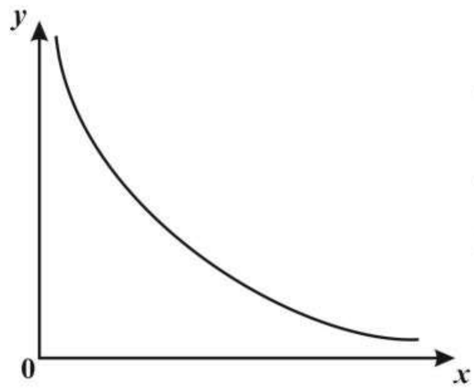
(ii) If y is **directly proportional** to x, then $y = kx$ where k is a constant.



(b) Inverse Proportion

(i) Two quantities x and y are in **inverse proportion** if one is increases in the same ratio as the other decreases. If x_1 and x_2 are any two values of x and y_1 and y_2 are the corresponding values of y, then $\frac{x_2}{x_1}=\frac{y_1}{y_2}$

(ii) If y is **inversely proportional** to x, then $y = \frac{k}{x}$ where k is a constant.



$$y_1 = \frac{k}{x_1}$$
$$y_2 = \frac{k}{x_2}$$
$$y_2 x_2 = y_1 x_1$$

4. Speed

(a) Distance = Speed × Time

(b) Average speed = $\dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

(c) Conversion of units

E.g., $9\text{m/s} = \dfrac{9m}{1s} = \dfrac{\frac{9}{1000}km}{\frac{1}{3600}h} = \dfrac{9}{1000} \times \dfrac{3600}{1} = 32.4$ km/h

E.g., $72\text{km/h} = \dfrac{72\text{km}}{1\text{h}} = \dfrac{72 \times 1000\text{m}}{60\,\text{min}} = 1200$ m/min

5. Scales and Maps

(a) Scale drawings are representation of the dimensions of a plan to the actual dimensions of the original figure. The scale is generally expressed as a ratio 1: n. The scale represented in the form 1: 1 000 000 means that 1 unit length (such as 1 cm) on the map represents 1 000 000 units on the actual diagram.

The scale of 1: n can also be expressed as a representative fraction (R.F.) of $\dfrac{1}{n}$.

(b) The area scale of a map is the square of its linear scale. If the linear scale of map is 1: n, then the area scale is 1: n^2.

E.g., If the linear scale is 1 cm represents 2 km, then 1 cm^2 on the map will represent (2 km)^2, i.e., 4km^2.

Thus, the area scale is 1cm^2: 4 km^2 while the linear scale is 1 cm: 2 km.

Worked Examples:

Example 1

A car and a van were travelling towards each other at uniform speeds. There were 300. km apart at 1100 and passed each other at 1300. If the speed of the car was 90 km/h, find the speed of the van.

Solution:

Distance covered by car in 2 hours = $90 \times 2 = 180$ km

Distance covered by van in 2 hours = $300 - 180 = 120$ km

Speed of van = $\dfrac{120}{2} = 60$ km/h

Example 2

A container is unloaded by 9 men in 24 minutes. Given that all the men work at the same rate, find how long it would take 12men to unload the same container.

Solution:

Inverse Proportion

9 men $\rightarrow$ 24 min

12 men$\rightarrow$ x min

$$\frac{9}{12} = \frac{x}{24}$$

$$x = \frac{9 \times 24}{12} = 18$$

12 men would take 18 minutes to unload the container

TOPIC-3

Simple Interest, Compound Interest and Financial Transactions

Revision Notes

1. Profit = Selling price – Cost price

   Loss = Cost price − Selling price

2. For a principal amount of $\$P$ at an interest rate of R% per annum for T years, the simple interest $\$I$, is $I = \frac{PRT}{100}$

3. For a principal amount of $\$P$ at an interest rate of r% per compound period for n compound periods, total amount = $P\left(1+\frac{r}{100}\right)^n$

Worked Examples

Example 1

A hi-fi set with a marked price of $1950 may be purchased using either one of the schemes below.

(i) For cash payment, an 8% deduction on the marked price is given.

(ii) For hire purchase, a down payment of 10% of the marked price and a monthly instalment of $ 165 for 12 months are required. Find the difference between the cash price and hire-purchase price.

Solution:

Cash payment: Deduction = $\frac{8}{100} \times \$1950 = \$156$

Amount paid = $1950 – $156 = $1794

Hire Purchase: Down payment = $\frac{10}{100} \times \$1950 = \$195$

Payment on 12 instalments = $165 × 12 = $1980

Amount paid = $1980 4 + $195 = $2175

Difference = $2175 – $1794 =$381

Example 2

John borrowed $20 000 from a bank over a period of 4 years. If the interest rate is 3% p.a, find

(a) the simple interest he must pay,

(b) the total amount he must pay at the end of the loan period.

Solution:

(a) Simple interest, $I = \dfrac{PRT}{100}$

$= \dfrac{\$20\,000 \times 3 \times 4}{100}$

$= \$2400$

(b) Total amount = \$20 000 + \$2400 = \$22 400

TOPIC-4

Algebra

Revision Notes

1. Evaluation of Algebraic Expressions

E.g., To evaluate $a + 2b - 3c$ when $a = 2$, $b = 3$ and $c = 4$,

$a + 2b - 3c = -2 + 2(3) - 3(4) = -8$

2. Simplifying Linear Algebraic Expressions

E.g., To simplify $-2(3x - 5) + 4x$,

$-2(3x - 5) + 4x = -6x + 10 + 4x$

$= -2x + 10$

E.g., To simplify $\frac{2x}{3} - \frac{3(x-5)}{2}$,

$$\frac{2x}{3} - \frac{3(x-5)}{2} = \frac{2x}{3} - \frac{3x-15}{2}$$

$$= \frac{4x}{6} - \frac{9x-45}{6}$$

$$= \frac{4x-9x+45}{6}$$

$$= \frac{-5x+45}{6}$$

3. Three Useful Formula for Expansion

(i) $(a + b)^2 = a^2 + 2ab + b^2$ E-g. $(x + 2)^2 = x^2 + 4x + 4$

(ii) $(a - b)^2 = a^2 - 2ab + b^2$ E.g. $(x - 3)^2 = x^2 - 6x + 9$

(iii) $(a + b)(a - b) = a^2 - b^2$ E-g. $(3x + 4)(3x - 4) = 9x^2 - 16$

4. Factorisation

(i) Find common factors

E.g., $ax + ay = a(x + y)$

(ii) Regroup terms

E.g., $ax - bx - kay + kby = x(a - b) - ky(a - b)$

$= (a - b)(x - ky)$

(iii) Use the formula

E.g., $49x^2 - 25y^2 = (7x + 5y)(7x - 5y)$

E.g., $4x^2 + 12x + 9 = (2x + 3)^2$

E.g., $25x^2 - 10x + 1 = (5x - 1)^2$

(iv) Use Trial and error method

E.g., $6x^2 - 17x + 12 = (2x - 3)\,(3X - 4)$

$2x$	-3	$-9x$
$3x$	-4	$-8x$
$6x^2$	$+12$	$-17x$

5. Simplifying Algebraic Fractions

E.g., $\dfrac{3a}{4} \div \dfrac{9a^2}{10} = \dfrac{3a}{4} \times \dfrac{10}{9a^2} = \dfrac{5}{6a}$

E.g., $\dfrac{1}{x-2} + \dfrac{2}{x-3} = \dfrac{x-3}{(x-2)(x-3)} + \dfrac{2(x-2)}{(x-2)(x-3)}$

$= \dfrac{x-3+2x-4}{(x-2)(x-3)}$

$= \dfrac{3x-7}{(x-2)(x-3)}$

E.g., $\dfrac{1}{x^2-9} + \dfrac{2}{x-3} = \dfrac{1}{(x-3)(x+3)} + \dfrac{2}{x-3}$

$= \dfrac{1}{(x-3)(x+3)} + \dfrac{2(x+3)}{(x-3)(x+3}$

$= \dfrac{1+2x+6}{(x-3)(x+3)}$

$= \dfrac{2x+7}{(x-3)(x+3)}$

E.g., $= \dfrac{1}{x-3} + \dfrac{2}{(x-3)^2} = \dfrac{x-3}{(x-3)^2} + \dfrac{2}{x-3)^2}$

$= \dfrac{x-1}{(x-3)^2}$

6. Change of Subject of a Formula.

The subject of a formula is the variable written explicitly in terms of other given variables.

A variable is made the subject of a formula by isolating it on one side of the formula.

E, g. To make u the subject of the formula $v^2 = u^2 + 2as$,

$v^2 = u^2 + 2as$

$v^2 - 2as = u^2$

$u = \pm\sqrt{v^2 - 2as}$

7. Solving Linear Equations

E.g. $2(3x-4)-4x=12$

$6x-8-4x=12$

$2x-8=12$

$2x=20$

$x=10$

E.g. $\frac{x}{3}+\frac{x-2}{4}=3$

$12\times\frac{x}{3}+12\times\frac{x-2}{4}=12\times3$

$4x+3(x-2)=36$

$4x+3x-6=36$

$7x-6=36$

$7x=42$

$x=6$

E.g. $\frac{3}{x-2}=6$

$6(x-2)=3$

$6x-12=3$

$6x=15$

8. Simultaneous Linear Equations

Simultaneous linear equations can be solved by (i) elimination method,
(ii) substitution method

E.g., Solve the simultaneous equations.

$5x+3y=3$ ………….... (1)

$3x+2y=-1$ ………….... (2)

(i) ***Elimination method***

(1) × 2 $10x+6y=6$ ………..... (3)

(2) × 3 $9x+6y=-3$ ………..... (4)

(3) – (4) $10x-9x=6-(-3)$

$x=9$

Sub, x = 9 into (1),

$5(9)+3y=3$

$3y=-42$

$y = -14$

$\therefore x = 9, y = -14$

(ii) *Substitution method*

From (1), $y = \dfrac{3-5x}{3}$ ……………..(3)

Sub, $y = \dfrac{3-5x}{3}$ into (2),

$$3x + 2\left(\frac{3-5x}{3}\right) = -1$$

$$9x + 6 - 10x = -3$$

$$-x = -9$$

$$x = 9$$

Sub, $x = 9$ into (3),

$$y = \frac{3-5(9)}{3}$$

$$y = -14,$$

$\therefore x = 9, y = -14$

9. Quadratic Equations

An equation of the form $ax^2 + bx + c = 0$ is a quadratic equation, where a, b and c are constants, and $a \neq 0$. Quadratic equations can be solved by.

(i) factorisation, (ii) using the formula, (iii) completing the square,

(a) Factorisation

E.g. $2x^2 + x - 15 = 0$

$(2x-5)(x+3) = 0$

$2x - 5 = 0$ or $x + 3 = 0$

$\therefore x = 2\frac{1}{2}$ or $x = -3$

(b) Using the Formula $x = \dfrac{-b \pm \sqrt{b^2 4ac}}{2a}$

E.g. $5x^2 + x - 1 = 0$

Substitute $a - 5$, $b - 1$, $c = -1$,

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 5 \times (-1)}}{2 \times 5}$$

$x = 0.358$ or -0.558

(c) By completing the Square

E.g. $x^2 + 7x + 5 - 0$

Step 1: Bring the constant term to the RHS,

$x^2 + 7x = -5$

Step 2: Divide the coefficient of x by 2 and add the square of this value to both sides of the equation such that LHS becomes a perfect square,

$$x^2 + 7x + \left(\frac{7}{2}\right)^2 = -5 + \left(\frac{7}{2}\right)^2$$

$$\left(x + \frac{7}{2}\right)^2 = \frac{29}{4}$$

Step 3: Take the square root of both sides of the equation, we have

$$x + \frac{7}{2} = \pm\sqrt{\frac{29}{4}}$$

$$x = -\frac{7}{2} + \frac{\sqrt{29}}{2} \quad \text{or } x = -\frac{7}{2} - \frac{\sqrt{29}}{2}$$

$\therefore x = -0.807$ or $x = -6.19$

10. Solving Fractional Equations

E.g., Solve $= \dfrac{6}{x+4} = x + 3$

$(x + 4)(x+3) = 6$

$x^2 + 3x + 4x + 12 = 6$

$x^2 + 7x + 6 = 0$

$(x + 1)(x + 6) = 0$

$x = -1$ or $x = -6$

E.g., Solve $\dfrac{1}{x-2} + \dfrac{2}{x-3} = 5$

Multiply both sides by $(x - 2)(x - 3)$,

$$\frac{1}{x-2} \times (x-2)(x-3) + \frac{2}{x-3} \times (x-2)(x-3) = 5(x-2)(x-3)$$

$x - 3 + 2(x - 2) = 5(x^2 - 5x + 6)$

$3x - 7 = 5x^2 - 25x + 30.$

$5x^2 - 28x + 37 = 0$

$$x = \frac{28 \pm \sqrt{(-28)^2 - 4 \times 5 \times 37}}{2 \times 5}$$

$x = 3.46$ or $x = 2{:}14$

11. Solving Linear Inequalities

(a) Solving linear inequalities is similar to solving equations. We need to reverse the inequality sign if the *y* inequality is multiplied or divided throughout by a negative number.

E.g. $3x + 2 > 8 + 5x$

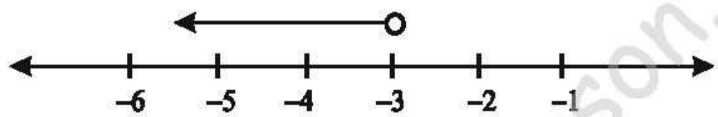
$3x - 5x > 8 - 2$

$-2x > 6$

$x < -3$

(b) When the solution set of an inequality is represented on a number line, a dot * is used if the value is included, otherwise an empty circle o is used.

E.g. The solution set for $x < -3$ is as follow.



12. Laws of Indices

(a) $a^m \times m^n = a^{m+n}$ E.g., $23 \times 24 = 2^7$

(b) $\frac{a^m}{a^n} = a^{m-n}$ E.g., $\frac{2^8}{2^3} = 3^5$

(c) $a^0 = 1$ E.g., $2^0 = 1$, $\left(3/4\right)^0 = 1$

(d) $a^{-m} = \frac{1}{a^m}$ E.g., $2^{-5} = \frac{1}{2^5}$

(e) $(a^m)^n = a^{mn}$ E.g., $(2^4)^3 = 2^{12}$

(f) $a^m b^m = (ab)^m$ E.g., $2^3 \times 5^3 = 10^3$

(g) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$ E.g., $\frac{18^5}{6^5} = 3^5$

(h) $a^{\frac{1}{n}} = \sqrt[n]{a}$ E.g., $5^{\frac{1}{2}} = \sqrt{5}, 5^{\frac{1}{3}} = \sqrt[3]{5}$

(i) $a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$ E.g., $64^{\frac{2}{3}} = \left(\sqrt[3]{64}\right)^2 = 4^2 = 16$

TOPIC-5

Number Sequence and Problem Solving

Revision Notes:

A number sequence is a set of numbers arranged in such a way that each successive number follows the preceding of according to a certain rule. The numbers in a sequence is called the terms of the sequence.

One way to define a sequence is to give a formula for the n^{th} term.

E.g. The n^{th} term of the sequence 3, 5,7,9, ___ is $2n + 1$.

Worked Example

Example 1

Find the nth term of the following sequence:

(a) 5, 8,11,14, (b) 4,9,16,25,.....

Solution:

(a) $3n + 2$ (b) $(n \div 1)^2$

Example 2

(a) The nth term of a sequence is given by $2n^2 + 1$. Write down the first 4 terms.

(b) The first 4 terms of another sequence are 1, 7, 17, 31, ...

(i) Write down the next term.

(ii) By comparing this sequence with your answer to (a), write down the nth term.

Solution:

(a) $2(1)^2 + 1,\ 2(2)^2 + 1,\ 2(3)^2 + 1,\ 2(4)^2 + 1 = 3, 9, 19, 33$

(b) (i) 49

(ii) nth term $= 2n^2 + 1 - 2 = 2n^2 - 1$

Example 3

Consider the pattern

$1^2 - 0^2 = 1,$

$2^2 — 1^2 = 3,$

$3^2 - 2^2 = 5$

$4^2 - 3^2 = 7$

$x^2 - y^2 = 101$

(a) Write down the 7th fine in the pattern.

(b) Find the value of $143^2 - 142^2$.

(c) Find integer values of x and y which satisfy the equation $x^2 - y^2 = 101$

Solution:

$43^2 - 6^2 = 13$

$43^2 - 142^2 = 143 + 142 = 285$

$x^2 - y^2 = 101$

$(x + y)(x - y) = 101$

We need to find two consecutive integers whose sum is 101 and the difference is 1.

Since $101 \div 2 = 50\frac{1}{2}$ we choose $x = 51$ and $y = 50$.

Example 4

Which sticks are used to form the patterns below.

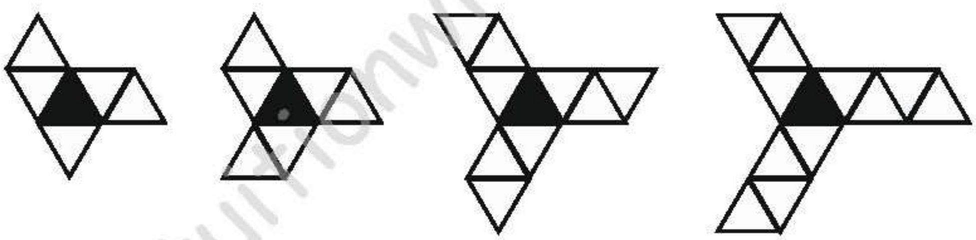


Fig. 1 Fig. 2 Fig 3 Fig 4

Fill in the next two rows in the table below.

Figure number, n	No. of small triangles, t
1	4
2	7
3	10
4	13

Find the relationship between n and t.

One of the patterns contains 61 triangles. Which figure is it?

Given that $1 + 2 + 3 + ... + 98 + 99 + 100 = 5050$, find the total number of triangles from figure

1 to figure 100.

Solution:

Figure number, n	No. of small triangles, t
1	4
2	7
3	10
4	13
5	16
6	19

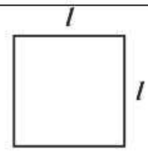
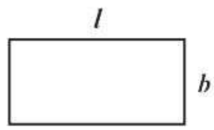
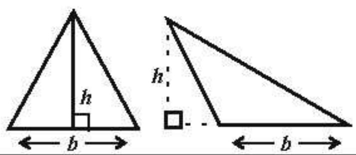
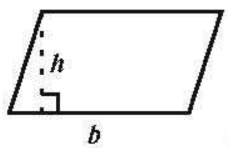
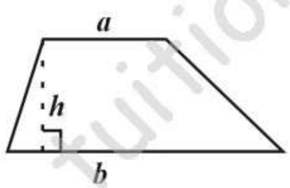
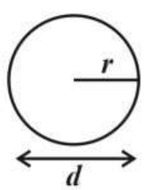
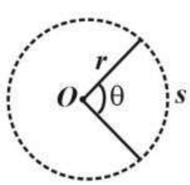
$t = 3n + 1$

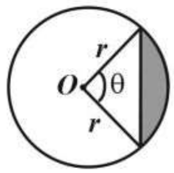
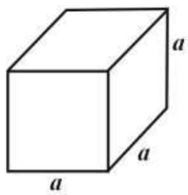
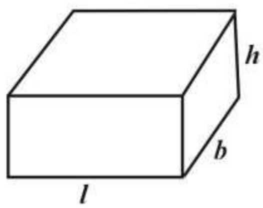
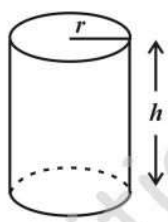
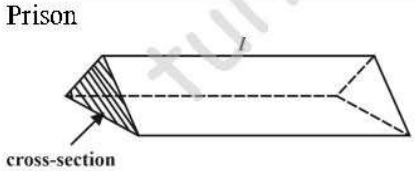
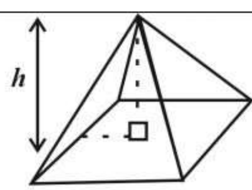
TOPIC-6

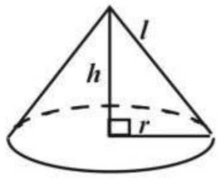
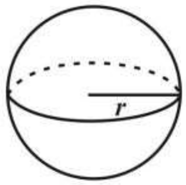
Mensuration

Revision Notes

Formula

Square 	Area = l^2 Perimeter = $4l$
Rectangle 	Area = $l \times b$ Perimeter = $2\ (l \times b)$
Triangle 	Area = $\frac{1}{2} \times b \times h$
Parallelogram 	Area = $b \times h$
Trapezium 	Area = $\frac{1}{2} \times h \times (a + b)$
Circle 	Area = πr^2 Circumference = $2\pi r = \pi d$
Sector 	Arc length, s = $\frac{\theta}{360°} \times 2\pi r$ (θ is in degrees) $= r\theta$ (θ is an radians) Area of sector = $\frac{\theta}{360°} \times \pi r^2$ (θ is in degrees)

	$= \frac{1}{2} r^2\theta$ (θ is in radians)
Segment 	Area of segment = Area of sector – Area of triangle $= \frac{\theta}{360°} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$ (θ is in degrees) $= \frac{1}{2} r^2\theta - \frac{1}{2} r^2 \sin\theta$ (θ is in radians)
Cube 	Volume = a^3 Total surface area = $6a^2$
Cuboid 	Volume = $l \times b \times h$ Total surface area = $2(lb + lh + bh)$
Cylinder 	Volume = $\pi r^2 h$ Curved surface area = $2\pi rh$ Total surface area = $2\pi rh + 2\pi r^2$
Prison 	Volume = area of cross-section × length
 Pyramid	Volume = $\frac{1}{3}$ × base area × height

Cone 	Volume = $\frac{1}{3}\pi r^2 h$ Curved surface area = πrl (opened) Curved surface area = $rl + \pi r^2$ (enclosed)
Sphere 	Volume = $\frac{4}{3}\pi r^3$ Surface area = $4\pi r^2$

2. Relation between Degree and Radian

π radians = 180°

$\therefore$ 1 radian = $\dfrac{180°}{\pi}$

$1° = \dfrac{\pi}{180°}$ radian

To convert radians to degrees, multiply the angle by $\dfrac{180°}{\pi}$.

To convert degrees to radians, multiply the angle by $\dfrac{\pi}{180°}$.

TOPIC-7

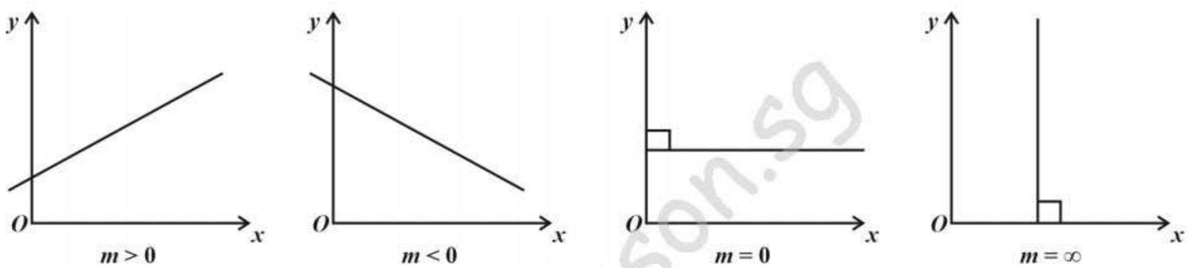
Coordinate Geometry

Revision Notes:

1. Distance between A (x_1, y_1) and B (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ or $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

2. Gradient of the line joining A (x_1, y_1) and B (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{y_1 - y_2}{x_1 - x_2}$

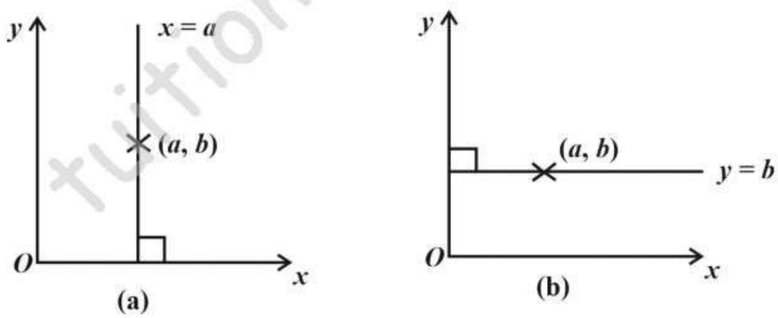
3. Gradient of different types of straight lines



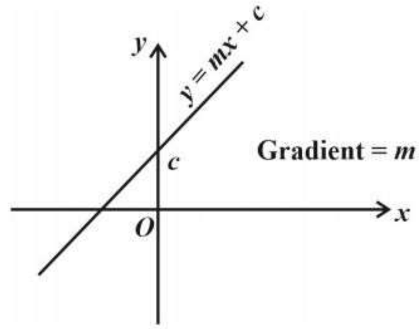
(undefined)

4. (a) The equation of a straight line which is parallel to the y axis and which passes through the point (a, b) is $x = a$

   (b) The equation of a straight line which is parallel to the x–axis and which passes through the point (a, b) is $y = b$.



5. The equation of a straight line having a gradient m and passing through $(0, c)$ is $y = mx + c$.



TOPIC- 8

Properties of Angles, Angle Properties of Polygons, Geometrical Construction

Revision Notes:

1. Types of Angles

(a) An acute angle is less than 90°.

(b) An obtuse angle is greater than 90° but less than 180°.

(c) A reflex angle is greater than 180° but less than 360°.

(d) $\angle a$ and $\angle b$ are complement if $\angle a + \angle b = 90°$.

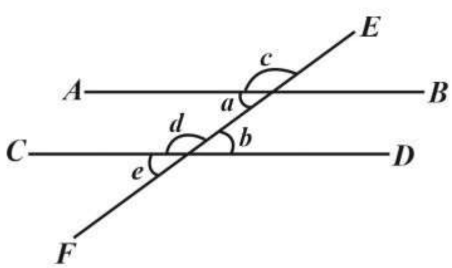
(e) $\angle a$ and $\angle b$ are supplementary if $\angle a + \angle b = 180°$.

2. Properties of Angles

(a) The sum of all the angles at a point is 360°.

(b) The sum of adjacent angles on a straight line is 180°.

(c) If AB and CD arc parallel, then



$\angle a = \angle b$ (alternate angles)

$\angle c = \angle d$ (corresponding angles)

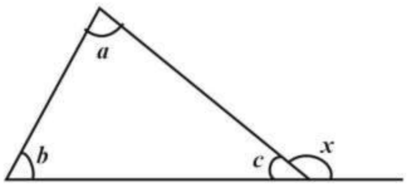
$\angle b = \angle e$ (vertically opposite angles)

$\angle a + \angle d = 180°$ (interior angles)

3.(a) Classification of Triangles

By sides	Equilateral triangle	Three equal sides
	Isosceles triangle	Two equal sides
	Scalene triangle	No equal sides
By angles	Acute–angled triangle	Three acute angles
	Obtuse–angled triangle	One obtuse angle
	Right–angled triangle	One right angle

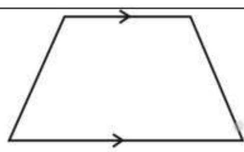
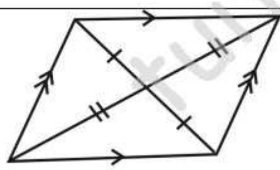
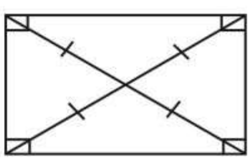
(b) Angle Properties

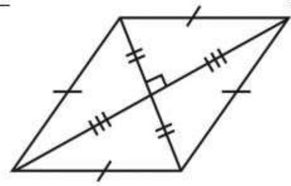
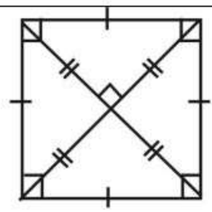
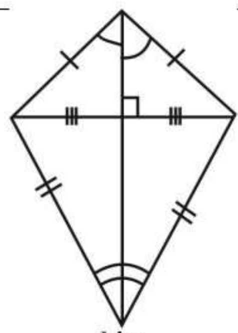


$\angle a + \angle b + \angle c = 180°$ ($\angle$ sum of Δ)

$\angle x = \angle a + \angle b$ (ext. $\angle s$ of Δ)

Quadrilaterals

 **trapezium**	(a) Only one pair of opposite Sade are parallel.
 **parallelogram**	(a) Opposite sides are parallel. (b) Opposite sides are equal in length. (c) Opposite angles are equal. (d) Diagonals bisect each other.
 **rectangle**	(a) Opposite sides are parallel. (b) Opposite sides are equal in length. (c) All angles are right angles. (d) Diagonals bisect each other. (e) Diagonals are equal in length.

 rhombus	(a) Opposite sides are parallel. (b) All sides are equal in length. (c) Opposite angles are equal. (d) Diagonals bisect each other at right angle. (e) Diagonals bisect the interior angles.
 square	(a) Opposite sides are parallel. (b) All sides are equal in length. (c) All angles are right angles. (d) Diagonals are equals in length. (e) Diagonals bisect each other at right angles. (f) Diagonals bisect the interior angles.
 kite	(a) No parallel sides. (b) Two pairs of equal adjacent sides. (c) Diagonals bisect each other at right angle. (d) One diagonal bisects the interior angles.

5. Polygons

A plane figure with three or more straight edges as its sides is called a polygon. A regular polygon is one in which all its sides and all its angles are equal.

Number of Sides	Name of Polygons
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
10	decagon

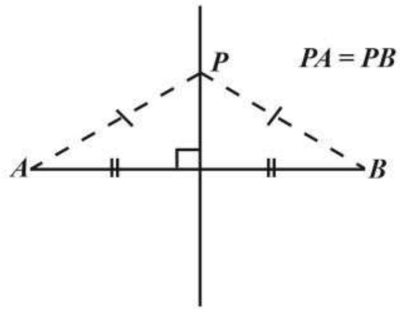
6. Angle Properties of Polygons

(a) The sum of the interior angles of an n–sided polygon is $(n - 2) \times 180°$.

(b) The sum of the exterior angles of n-sided polygon is 360°.

7. Perpendicular Bisectors

(a) The perpendicular bisector of the line *AB* is a line bisects and perpendicular to *AB*. Any point on the perpendicular bisector is equidistant from *A* and *B*.

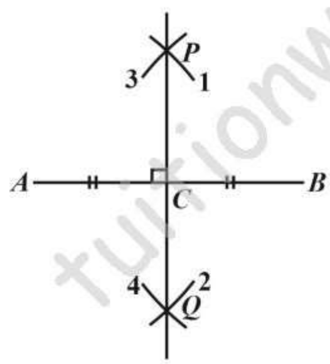


(b) To construct a perpendicular bisector of *AB*,

(i) place the compass point at *A* and make arc 1 above the line *AB* and make arc 2 below the line *AB* as shown,

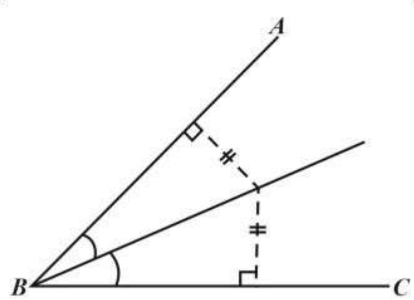
(ii) Using the same radius as (i) and with the compass point at *B*, make arc 3 to cut arc 1 at *P* and arc 4 cut arc 2 at *Q*.

(iii) join *PQ* to cut *AB* at *C*, *PQ* is the perpendicular bisector of *AB*.



Angle Bisectors

(a) The angle bisector is a line that divides an angle into two equal parts. Any point on the angle bisector is equidistant from the two sides of the angle?



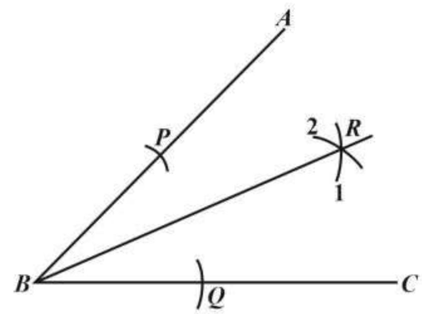
(b) To construct the angle bisector of $\angle ABC$.

(i) place the compass point at **B**, draw ah arc to cut *AB* at *P* and *BC* at *Q*.

(it) with *P* as centre, draw an arc 1 as shown,

(iii) with *Q* as centre, draw an arc 2 to cut arc 1 at **R**,

(iv) join **BR**, **BR** is the angle bisector of $\angle$ABC.



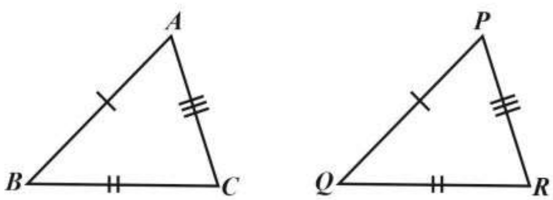
TOPIC-9

Congruence and Similarity

Revision Notes:

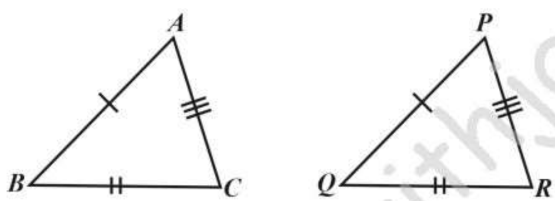
1. Congruent Triangles

If ΔABC and ΔPQR are congruent, then $AB = PQ$, $BC = QR$, $CA = RP$, $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle B$ and area of $\angle ABC$ = area of $\angle PQR$.

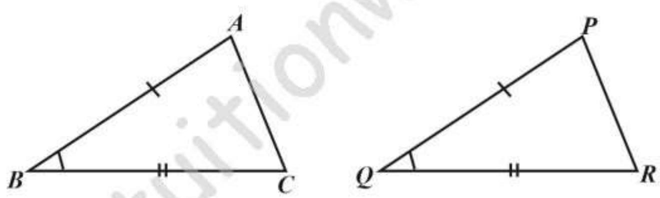


2. Test of Congruent Triangles

(a) All three corresponding sides are equal (SSS)

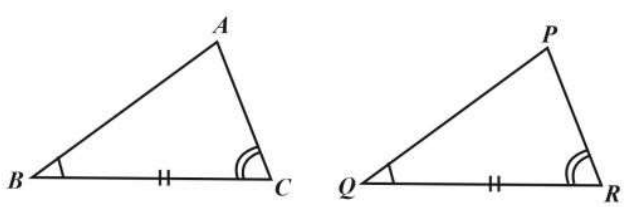


(b) Two sides and included angle are equal. (SAS)

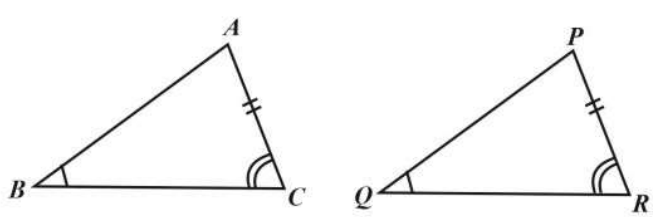


(c) Two angles and a corresponding side are equal. (ASA, AAS or SAA)

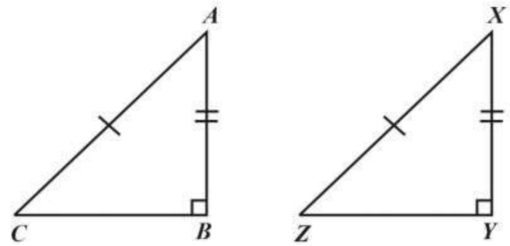
(i) ASA



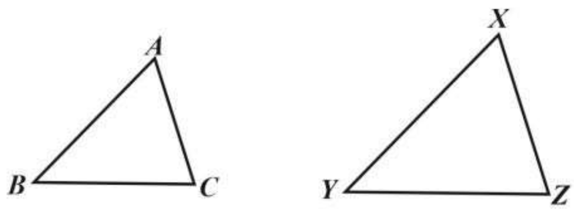
(ii) AAS or SAA



(d) The hypotenuse and a side of right–angled triangles are equal. (RHS)



3. Similar Triangles

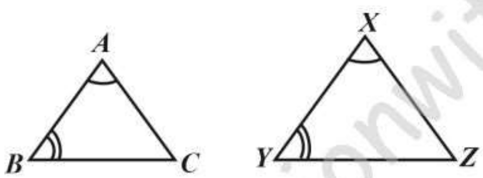


If ΔABC and ΔXYZ are similar, then

$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$ and

$\angle A = \angle X$, $\angle B = \angle Y$, $\angle C = \angle Z$.

4. Tests for Similar Triangles



ΔABC and ΔXYZ are similar if

(a) Two pairs of corresponding angles are equal, i.e., $\angle A = \angle X$, $\angle B = \angle Y$.

(b) Three pairs of corresponding sides are in the same ratio, i.e., $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$.

(c) Two pairs of corresponding sides are in the same ratio and a pair of included angles is equal, i.e., $\frac{AB}{XY} = \frac{AC}{XZ}$ and $\angle A = \angle X$.

5. Area of Similar Figures

For two similar plane figures, the ratio of their areas is the square of the ratio of their corresponding length, i.e., $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

6. Volume of Similar Solids

For two geometrically similar solids, the ratio of their volumes is the cube of the ratio of their corresponding length, i.e., $\frac{V_1}{V_2}=\left(\frac{l_1}{l_2}\right)^3$.

TOPIC-10

Properties of Circle

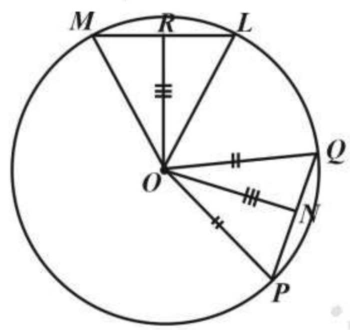
Revision Notes:

1. Symmetry Properties of Circles

(a) Equal chords are equidistant from the centre of the circle. Conversely, chords which are equidistant from the centre of the circle are equal in length.

(i) If PQ = LM, then $ON = OR$.

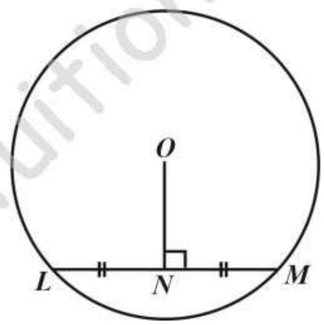
(ii) If $ON = OR$. then $PQ = LM$.



(b) The perpendicular bisector of a chord passes through centre of the circle. Conversely, the line joining the midpoint of chord to the centre of the circle is perpendicular to the chord.

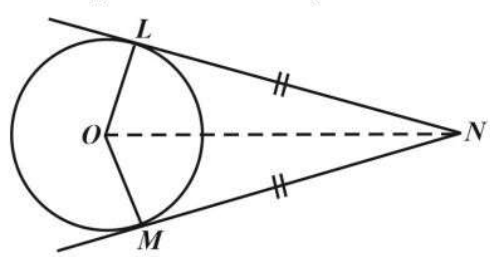
(i) If $LN = NM$, then ON is perpendicular to LM.

(ii) If ON is perpendicular to LM, then $LN = NM$.



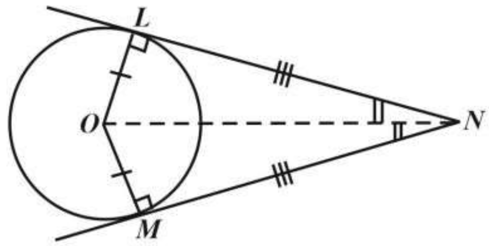
(c) Tangents from external points are equal in length.

If MN and LN are tangents from external point N, then $LN = NM$.



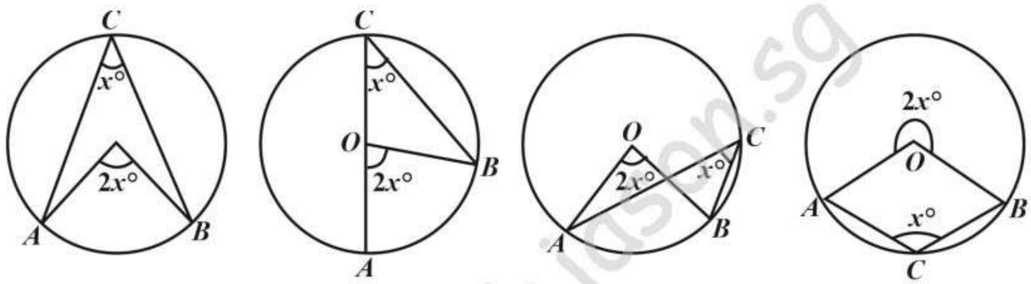
(d) The line joining the external point to the centre of the circle bisects the angle between the tangents.

If LN and MN are tangents and OL and OM are radii, then $\angle LNO = \angle MNO$.

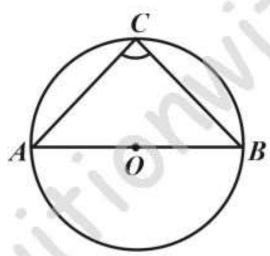


2. Angle Properties of Circles

(a) The angle at the centre of a circle is twice the angle at the circumference subtended by the same arc, i.e., $\angle AOB = 2\angle ACB$.

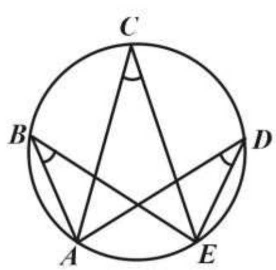


(b) The angle in a semi–circle is a right angle If AB is the diameter, then $\angle ACB = 90°$.



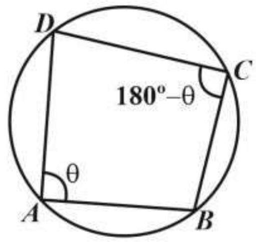
(c) Angles in the same segment are equal.

The minor arc AE, subtends equal angles at B, C and D. i.e., $\angle ABE = \angle ACE = \angle ADE$.

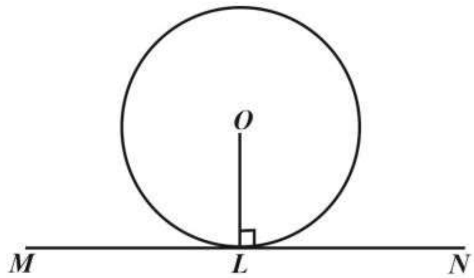


(d) Angles in Opposite Segments are supplementary,

i.e., $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$.



(e) A tangent to a circle is perpendicular to the radius of the circle at the point of contact.



If OL, is the radius, MN is the tangent,

L is the point of contact,

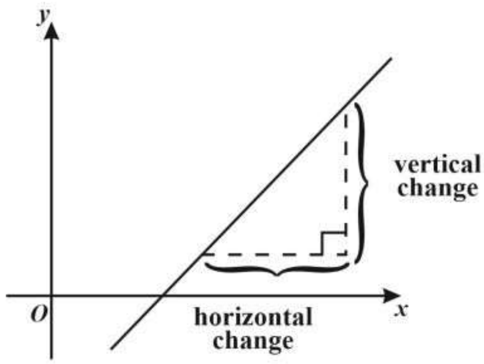
then OL is perpendicular to NM.

TOPIC- 11

Graphs and Graphical Solutions

Revision Notes:

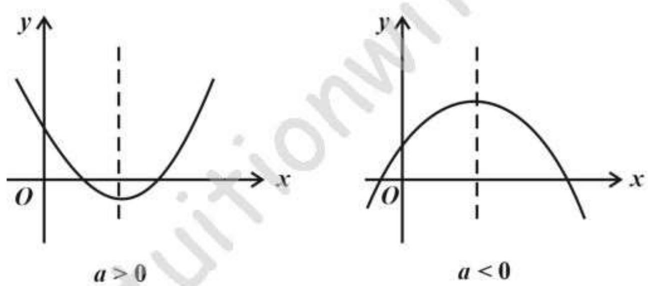
1. Gradient of a Linear graph



$$\text{Gradient} = \frac{\text{vertical change}}{\text{horizontal change}}$$

2. Graphs of Quadratic Functions

(a) $y = ax^2 + bx + c$

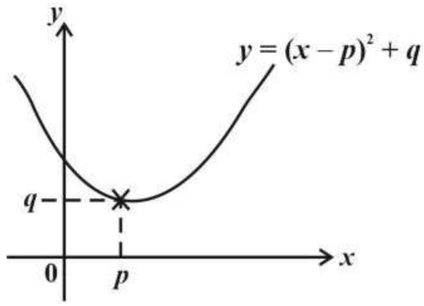


(i) If $a > 0$, the graph has a minimum point

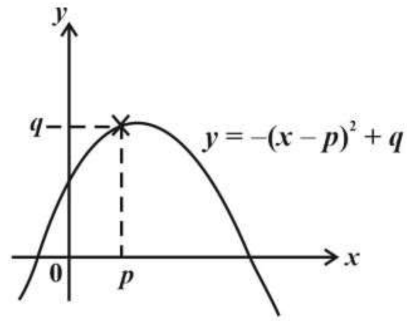
(ii) If $a < 0$, the graph has a maximum point

(iii) The graph of $y - ax^2 + bx + c$ has an axis of symmetry which passes through the minimum or maximum point

(b) $y = (x - p)^2 + q$ *or* $y = (x - p)^2 + q$

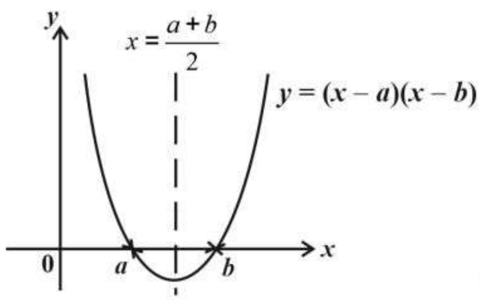


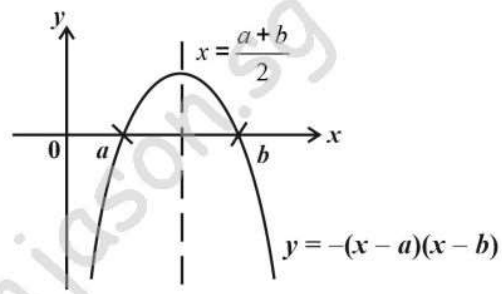
Minimum point = (*p*, *q*)



Maximum point = (*p*, *q*)

(c) $y = (x - a)(x - b)$ or $y = -(x - a)(x - b)$



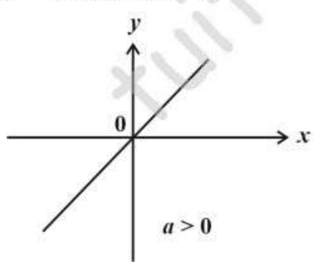


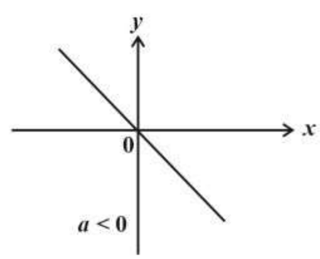
(i) x–intercepts are $x = a$ and $x = b$

(ii) Equation of axis of symmetry is $x = \dfrac{a+b}{2}$

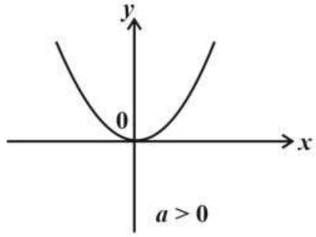
3. Graphs of $y = ax^n$ where n = ±1, ±2, 3

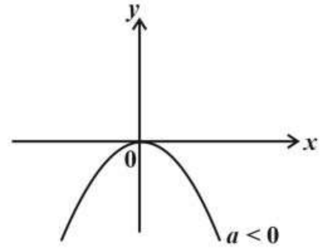
(i) $y = ax$, when $n = 1$



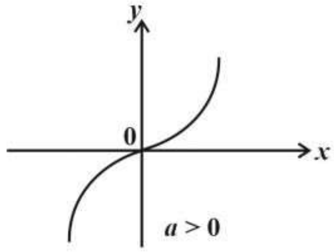


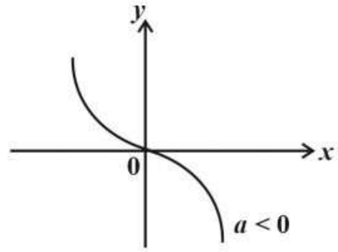
(ii) $y = ax^2$, when $n = 2$



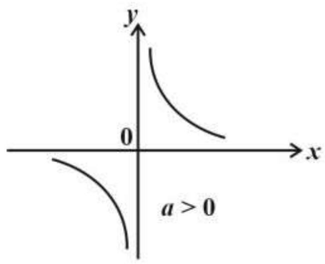


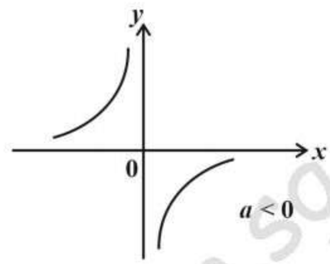
(iii) $y = ax^3$, when $n = 3$



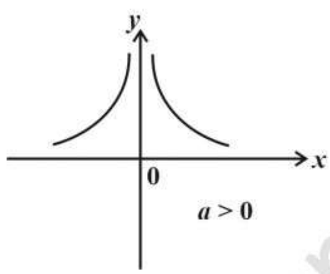


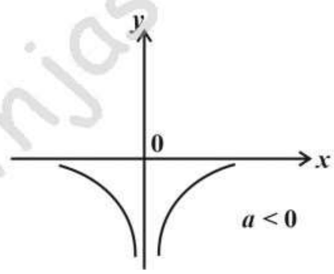
(iv) $y = \frac{a}{x}$, when $n = -1$



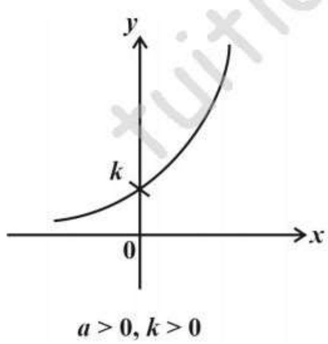


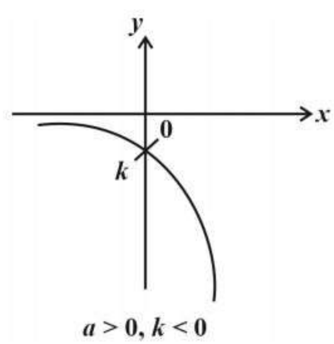
(v) $y = \frac{a}{x^2}$, when $n = 2$



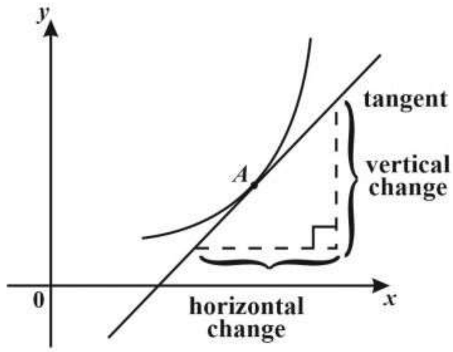


4. Graphs of exponential functions $y = ka^x$ where a is a positive integer





5. Gradient of a curve



Gradient of a curve at a point A = gradient of the tangent to the curve at point A

$$= \frac{\text{vertical change}}{\text{horizontal change}}$$

6. Graphical Solution of Equations

(a) Simultaneous linear Equations

The solution of a pair of simultaneous linear equations is given by the coordinates of the point of intersection of their graphs.

(b) Quadratic Equations

(i) For quadratic equations of the form $ax^2 + bx + c = 0$, the solution in x can be obtained from the x – coordinates of the point(s) of intersection of the graph of $y = ax^2 + bx + c$ and the x–axis.

(ii) For quadratic equations of the form $ax^2 + bx + c = dx + e$, the solution in x can be obtained from the x – coordinates of the point(s) of intersection of the graph of $y = ax^2 + bx + c$ and the line $y - dx + e$.

TOPIC-12

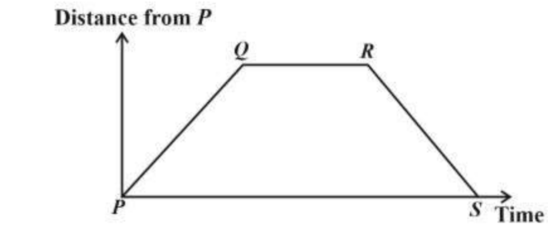
Graphs in Practical Situations

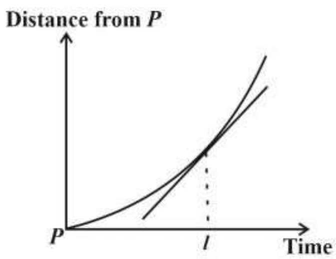
Revision Notes

1. Distance Time Graphs

(a) The absolute value of the gradient represents speed.

(b) The sign of the gradient indicates the direction of travel.





Positive gradient (PQ) : Object travelling away from A The tangent to the curve at times

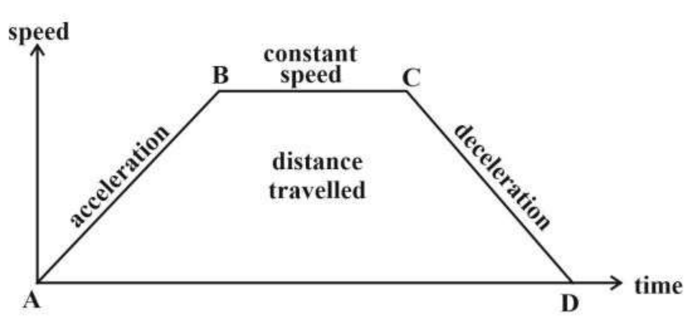
Zero gradient (QR) : Object is stationary represents the speed at time t.

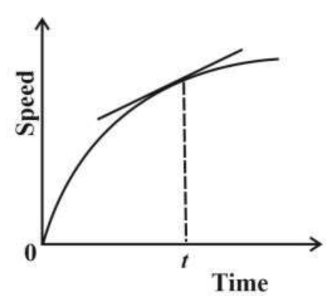
Negative gradient (R) : Object travelling towards A

2. Speed Time Graphs

(a) The gradient of a speed–time graph represents acceleration.

(b) The area under a speed–time graph gives the distance travelled.





Positive gradient (AB) : Speed is increasing The tangent to the curve at times

Zero gradient (*BC*) : Speed is constant represents the acceleration.

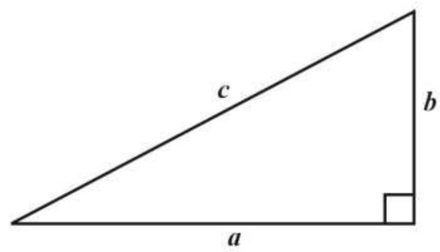
Negative gradient (*CD*) : Speed is decreasing

TOPIC-13

Trigonometry

Revision Notes

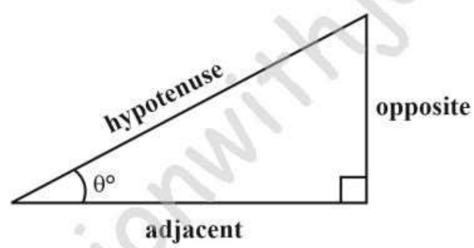
1. Pythagoras' theorem



In a right–angled triangle, the square of the hypotenuse (side opposite right angle) is equal to sum of squares of the other two sides, i.e., $c^2 = a^2 + b^2$.

Conversely, if a triangle has sides a, b and c such that $c^2 = a^2 + b^2$, then the triangle is a right angled triangle.

2. Trigonometrical Ratios of Acute Angles



For a right–angle triangle, PQR, with reference to the angle θ, the side opposite the right angle is called the hypotenuse (the longest of the 3 sides of the triangle). Opposite refers to the side that is directly opposite the angle θ and adjacent the third side.

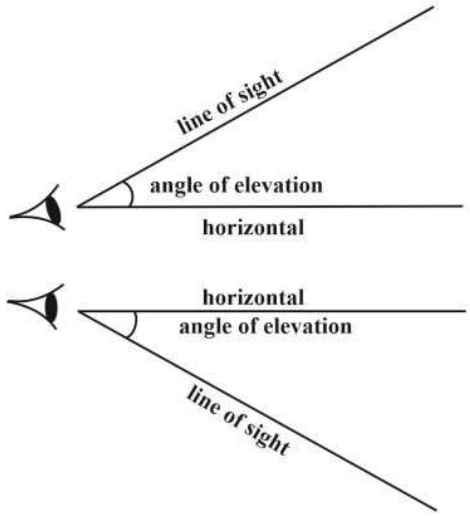
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

3. Angle of Elevation and Depression

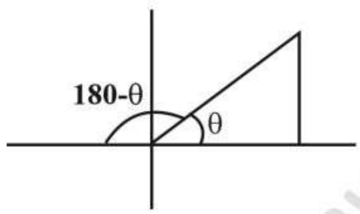
When we look up to view an object, our line of sight makes an angle with the horizontal. We call this angle, the angle of elevation.



Similarly, the angle through which an observer's eye must be lowered from the horizontal in order to view an object is called the angle of depression.

Note: The angles of elevation and depression are always measured from the horizontal.

4. Sine and Cosine of an Obtuse Angle

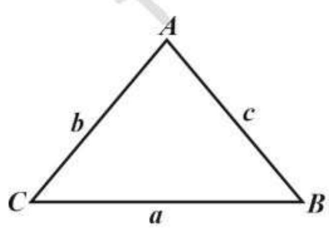


For any acute angle θ,

$\sin(180° - \theta) = \sin\theta$,

$\cos(180° - \theta) = -\cos\theta$,

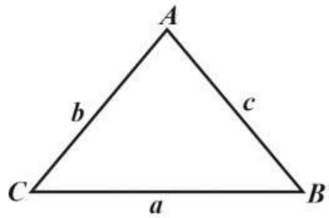
5. The Sine Rule



For a triangle *ABC*, the Sine Rule states that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \text{ or } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

6. The Cosine Rule



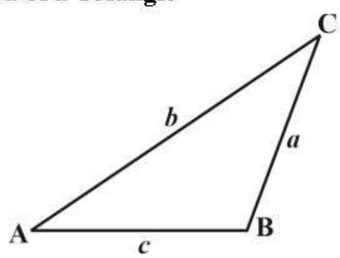
For a triangle ABC, the Cosine Rule states that.

$a^2 = b^2 + c^3 - 2bc \cos A$

$a^2 = a^2 + c^3 - 2ac \cos B$

$c^2 = a^2 + b^3 - 2ab \cos C$

7. Area of a Triangle



For any triangle ABC,

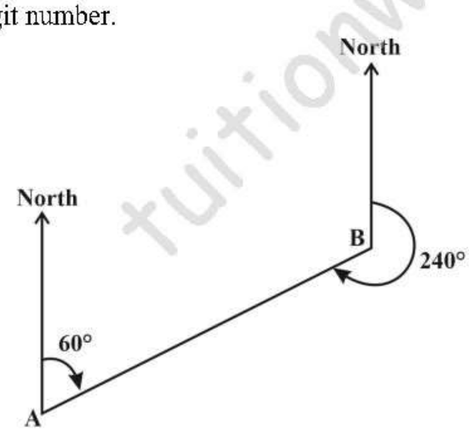
area = $\frac{1}{2} bc \sin A$

= $\frac{1}{2} ac \sin B$

= $\frac{1}{2} ab \sin C$

8. Bearings

Bearings are always measured from, the North and in a clockwise direction and stated as a three–digit number.



E.g. The bearing of B from A is 060°.

E.g. The bearing of A from B is 240°.

TOPIC-14

Sets and Venn Diagrams

Revision Notes

Sets

A set is a collection or group of things. The things that make up the set are called the elements or members of the set.

E.g. A = {1, 3, 5,7} is a set 1, 3, 5 and 7 are elements Of *A*.

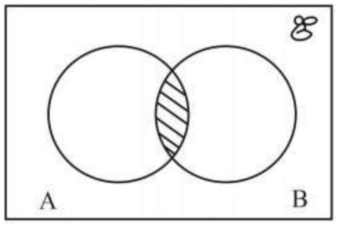
Set Language and Notations

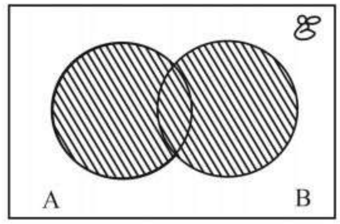
Set Language	Notation
"...is an element of.."	$\in$
"...is not an element of..."	$\notin$
Number of element in set *A*	$n(A)$
Universal set	$\mathscr{E}$
The empty set	$\varnothing$
A is a subset of *B*	$A \subseteq B$
A is not a subset of *B*	$A \not\subset B$
A is a proper subset of *B*	$A \subset \text{B}$
A is not a proper subset of *B*	$\text{A} \not\subset \text{B}$
Union of sets *A* and *B*	$\text{A} \cup \text{B}$
Intersection of sets *A* and *B*	$\text{A} \cap \text{B}$
Complement of set *A*	A'

Venn Diagrams

A simple way of showing the relationship between sets is by the use of Venn diagrams.

E.g. $A \cap B$ $A \cup B$





Worked Examples:

Example 1

It is given that $b = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$, $A = \{x : x$ is a prime number), $B = \{x : x$ is a multiple of 4$\}$ $C = \{x : x$ is a factor of 12$\}$–

(i) Find (a) $n(A)$, (b) $n(A \cap B)$, (c) $n(C)$

(ii) List the element(s) of x such that $x \in$ A and $x \notin B \cup C$.

Solution:

(i) (a) $A = \{2, 3, 5, 7, 11\}$

$n(A) = 5$

(b) $B = \{4, 8\}$

$A \cap B = \varnothing$

$n(A \cap B) = 0$

(c) $C = \{2, 3, 4, 6\}$

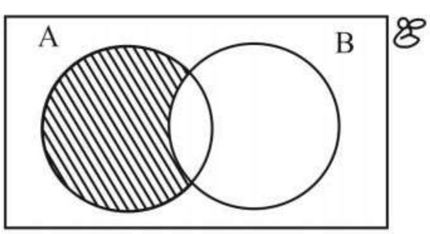
$n(C) = 4$

(ii) $\{5, 7, 11\}$

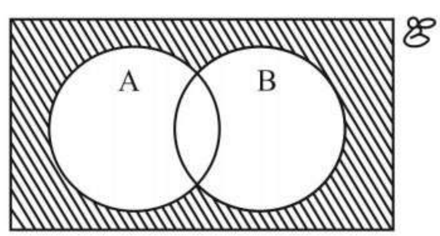
Example 2

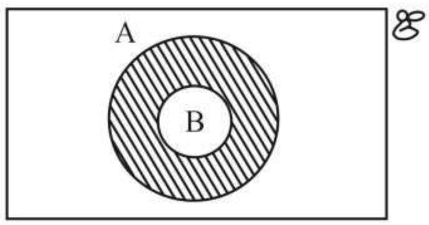
Express in set notation, the set shaded in the following Venn diagrams.

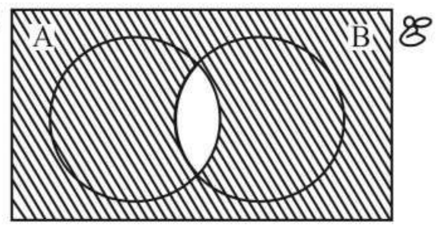
(i)



(ii)



(iii)


(iv)


Solution:

(i) $A \cap B'$ (ii) $(A \cup B)'$ (iii) $A \cap B'$ (iv) $(A \cap B)'$

Example 3

The sets b, C and P are given by b = {students in a class}, C = {students who study Chemistry} and P = {study who study Physics}. Express in words, the students that belong to set.

(a) $C \cap P$, (b) $(C \cup P)'$, (c) $C \cap P'$

Solution:

(a) Students who study both Chemistry and Physics.

(b) Students who did not study either Chemistry or Physics.

(c) Students who study Chemistry but not Physics.

Example 4

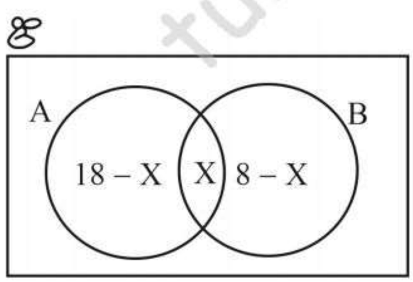
The sets A and B are such that $n(A) = 18$ and $n(B) = 8$. Given that $n(\xi) = 20$, find the smallest possible value of $n(A \cap B)$.

The smallest possible value of $n(A \cap B)$

The largest possible value of $n(A \cup B)$,

Solution:

Let $n(A \cap B) = x$



The smallest possible value of $n(A \cap B)$ occurs when $A \cup B = \xi$

$n(\xi) = 20$

$18 - x + x + 8 - x = 20$

$26 - x = 20$

$x = 26 - 20 = 6$

The largest possible value of $n(A \cap B)$ is 6. ξ

The largest possible value of $n(A \cup B)$ occurs when $A \cup B = \xi$.

The largest possible value of $n(A \cup B) = n(\xi) = 20$.

TOPIC-15

Matrices

Revision Notes:

1. Matrices

A matrices is a rectangle array of numbers arranged in rows and columns. The order of a matrix is defined as the number of rows by the number of columns. An $m \times n$ matrix has m rows and n columns. The numbers in the matrix are called elements.

E.g. $\begin{pmatrix} 1 & 3 & 4 \\ 3 & 6 & 8 \end{pmatrix}$ is a 2×3 matrix since there are 2 rows and 3 columns.

2. Equality of Matrices

Two matrices are equal if they have the same order and their corresponding elements are equal.

E.g. If $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, then $a = 1$, $b = 2$, $c = 3$ and $d = 4$.

3. Addition and Subtraction of Matrices

Two matrices can be added and subtracted only if they have the same order.

E.g. $\begin{pmatrix} 1 & 3 \\ 4 & 7 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1+1 & 3+3 \\ 4+2 & 7+1 \end{pmatrix} = \begin{pmatrix} 2 & 6 \\ 6 & 8 \end{pmatrix}$

E.g. $\begin{pmatrix} 1 & 3 \\ 4 & 7 \end{pmatrix} - \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1-1 & 3-3 \\ 4-2 & 7-1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 2 & 6 \end{pmatrix}$

4. Multiplication of a Matrix by a Scalar

When a matrix is multiplied by a number k, each element in the matrix is multiplied by k.

E.g.2. $\begin{pmatrix} 2 & 4 & 6 \\ 2 & 3 & 7 \\ 4 & 5 & 1 \end{pmatrix} = \begin{pmatrix} 2\times2 & 2\times4 & 2\times6 \\ 2\times2 & 2\times3 & 2\times7 \\ 2\times4 & 2\times5 & 2\times1 \end{pmatrix} = \begin{pmatrix} 4 & 8 & 12 \\ 4 & 6 & 14 \\ 8 & 10 & 2 \end{pmatrix}$

5. Multiplication of Matrices

2 matrices can be multiplied if and only if number of columns of the first matrix is the same as number of rows, of the second matrix. The order of the resulting matrix is given by the number of rows of the first matrix by the number of columns of the second matrix. In general, a $m \times n$ matrix multiplied by $n \times q$ matrix will give a $m \times q$ matrix.

E.g. (3×3 matrix) $\times$ (3×2 matrix) $= 3 \times 2$ matrix

(1×2 matrix) $\times$ (2×3 matrix) $= 1 \times 3$ matrix

(1 × 2 matrix) × (1 × 2 matrix) = Not possible

The resulting matrix has elements where the corresponding elements from the multiplied with those from the respective columns of the second matrix.

E.g. $\begin{pmatrix}1 & 3\\ 2 & 1\end{pmatrix}\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{pmatrix}=\begin{pmatrix}1\times1+3+4 & 1\times2+3\times5 & 1\times3+3\times6\\ 2\times1+1\times4 & 2\times2+1\times5 & 2\times3+1\times6\end{pmatrix}=\begin{pmatrix}13 & 17 & 21\\ 6 & 9 & 12\end{pmatrix}$

Note : In general, AB ≠ BA

6. Zero Matrices

A zero matrix is a matrix with all its elements equal to zero.

E.g. $\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$ is 2 × 2 zero matrix and (0 0) is 1 × 2 zero matrix.

7. Identity Matrices

An identity matrix is *a n* × *n* matrix whose elements on the main diagonal are 1 and all other elements are zero.

E.g. $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ is a 2 × 2 identity matrix and $\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$ is a 3 × 3 identity matrix.

Note : If A is a matrix and I is an identity matrix of the same order, then $AI = IA = A$.

Worked Examples:

Example 1

If $A=\begin{pmatrix}-2 & 3\\ 0 & 1\end{pmatrix}, B=\begin{pmatrix}3 & 2\\ 6 & 5\end{pmatrix}$ and $C=\begin{pmatrix}4 & 2 & 1\\ -1 & 0 & 5\end{pmatrix}$, evaluate the following matrix products where possible.

(i) AB (ii) BA (iii) BC (iv) CB

Solution:

(i) $AB=\begin{pmatrix}-2 & 3\\ 0 & 1\end{pmatrix}=\begin{pmatrix}3 & 2\\ 6 & 5\end{pmatrix}=\begin{pmatrix}-2\times3+3\times6 & -2\times2+3\times5\\ 0\times3+1\times6 & 0\times2+1\times5\end{pmatrix}=\begin{pmatrix}12 & 11\\ 6 & 5\end{pmatrix}$

(ii) $BA=\begin{pmatrix}3 & 2\\ 6 & 5\end{pmatrix}\begin{pmatrix}-2 & 3\\ 0 & 1\end{pmatrix}=\begin{pmatrix}3\times(-2)+2\times0 & 3\times3+2\times1\\ 6\times(-2)+5\times0 & 6\times3+5\times1\end{pmatrix}=\begin{pmatrix}-6 & 11\\ -12 & 23\end{pmatrix}$

(iii) $BC=\begin{pmatrix}3 & 2\\ 6 & 5\end{pmatrix}\begin{pmatrix}4 & 2 & 1\\ -1 & 0 & 5\end{pmatrix}=\begin{pmatrix}3\times4+2\times(-1) & 3\times2+2\times1 & 3\times1+2\times5\\ 6\times3+5\times(-1) & 6\times2+5\times0 & 6\times1+5\times5\end{pmatrix}=\begin{pmatrix}10 & 6 & 13\\ 19 & 12 & 31\end{pmatrix}$

(iv) CB is not defined since the number of columns of C is not equal to number of now of B.

Example 2

Given that $\begin{pmatrix} 2 & x & 2x-1 \\ 8 & 3y & -2 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ 4 \end{pmatrix}$, find the values of x and y.

Solution:

$$\begin{pmatrix} 2 & x & 2x-1 \\ 8 & 3y & -2 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 6+2x+2x-1 \\ 24+6y-2 \end{pmatrix} = \begin{pmatrix} 9 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 4x+5 \\ 22+6y \end{pmatrix} = \begin{pmatrix} 9 \\ 4 \end{pmatrix}$$

Comparing corresponding elements, we have $4x + 5 - 9$ and $22 + 6y = 4$

$4x - 4$ $\qquad$ $6y = 18$

$x = 1,$ $\qquad$ $y = -3$

TOPIC- 16

Vectors

Revision Notes:

1. Vectors

A vector is a quantity which has both magnitude and direction. .

E.g. velocity is a vector but temperature is not

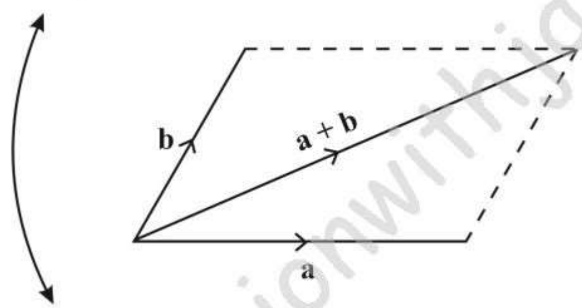
2. Magnitude of Vectors

If $x = \begin{pmatrix} x \\ y \end{pmatrix}$, then its magnitude | r | = $\sqrt{x^2 + y^2}$ units.

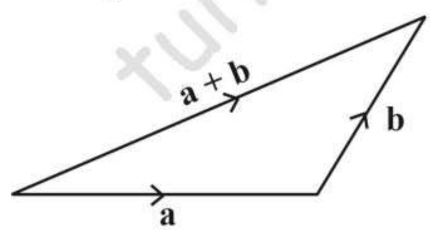
E.g. If $x = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, then | r | = $\sqrt{4^2 + 3^2}$ = 5 units

3. Vector Addition

(a) Triangle law of vector addition

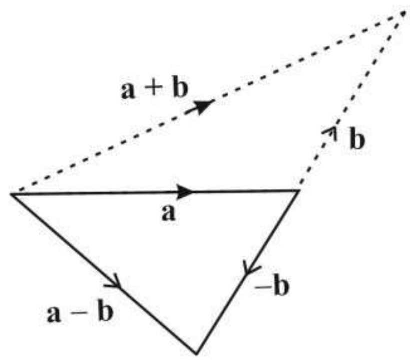


(b) Parallelogram law of vector addition

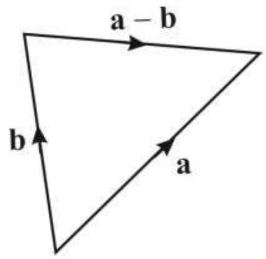


4. Vector Subtraction

The subtraction of vectors a – b; can be expressed as the addition of vectors, a + (–b).



The subtraction of vectors a – b, can also be obtained as follow



5. **Solar Multiplication of Vectors**

if $a = kb$, where a is a non–zero vector and k is any non–zero real number,

(i) if $k > 0$, then a has the same direction as **b** and has magnitude k times that of **b**,

(ii) if $k < 0$, then a is in opposite direction as **b** and has magnitude k times that of **b**.

6. **Vector as Sum or Difference of Two Coplanar Vectors**

Any given vector AB can be expressed as.

$AB = AP + PB$ or $AB = PB – PA$

where P is any point on the same plane.

7. **Position Vectors**

The position vector of a point P(x, y) is $\overrightarrow{OP} = \begin{pmatrix} x \\ y \end{pmatrix}$, where O is the origin.

Worked Examples:

Example 1

Given that $\overrightarrow{OA} = \begin{pmatrix} P \\ 4 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$, find

(i) the value of $|\overrightarrow{OB}|$,

(ii) the possible values for if $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are two sides of a rhombus.

Solution:

(i) $\overrightarrow{OB}=\begin{pmatrix}5\\0\end{pmatrix}$

$|\overrightarrow{OB}|=\sqrt{5^2+0^2}=5$ units

(ii) $|\overrightarrow{CA}|=|\overrightarrow{OB}|$

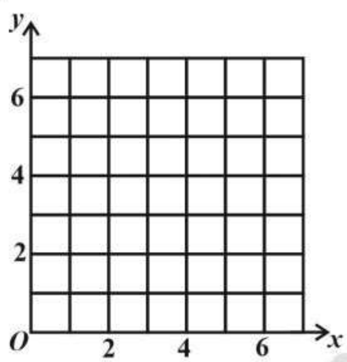
$\sqrt{p^2+4^2}=5$

$p^2+4^2=5^2$

$p^2=9$

$p=\pm 3$

Example 2



TOPIC-17

Probability

Revision Notes:

1. Definition

The probability that an event, A, will happen is written as $P(A)$ and is given by

$$P(A) = \frac{\text{number of times event } A \text{ can occur}}{\text{total number of occurrences}}$$

2. Properties of Probability

(i) If A is any event, $0 \leq P(A) \leq 1$.

If A is an impossible event, then $P(A) = 0$

If A is a sure event, then $P(A) = 1$.

(ii) The probability that the event A, does not happen is called the complement of A and is written as $P(A)'$ and is given by $P(A') = 1 - P(A)$.

(iii) Two events are mutually exclusive if the event of one happening excludes the other of happening. In other words, they both cannot happen at the same time.

If A and B are two mutually exclusive events, then

$P(A \text{ or } B) = P(A) + P(B)$

(iv) Two events are independent if the occurrence of one happening does not affect the occurrence of the other. If A and B are independent events, then

$P(A \text{ and } B) = P(A) \times P(B)$.

3. Possibility Diagrams

A possibility diagram uses dot or a table to list all the possible outcomes of an experiment.

E.g. The four faces of a red die are marked 1, 2, 2 and 3. The four faces of a blue, die are marked 1, 2, 5 and When such a die is thrown, the score is the number on the face on which it lands. The two dice are thrown together and their scores added. The possibility diagram below shows all the possible outcomes.

Red

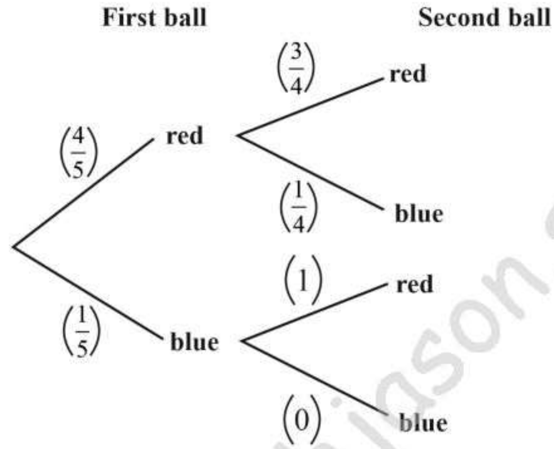
	+	1	2	2	3
	1	2	3	3	4
Blue	2	3	4	4	5
	5	6	7	7	8
	6	7	8	8	9

4. Tree Diagrams

In a tree diagram, the outcomes are written at the ends of the branches, and the corresponding probabilities by the side of each branch.

The probability of any outcome in a tree diagram is calculated by multiplying the probabilities along the relevant branches leading to that outcome.

E.g. When two balls are drawn at random from a bag contains 1 blue ball and 4 red balls, the probability tree diagram can be shown as follow.



P(two red balls are drawn) $\frac{4}{5}\times\frac{3}{4}=\frac{3}{5}$.

P(one ball of each colour is drawn)

= P(first ball is red and second ball is blue) + P(first ball is blue and second ball is red)

$=\frac{4}{5}\times\frac{1}{4}+\frac{1}{5}\times 1=\frac{2}{5}$

Worked Examples:

Example 1

Sets $\mathcal{E}$, P and Q are given by $\mathcal{E}$ = {x : x is an integer, $1 \le x \le 20$}, P = {x : x is an integer, $\frac{1}{2}x \le 4$} and {x : x is an integer, $3x > 16$}. A number n is chosen at random from $\mathcal{E}$.

Find the probability that

(a) $n \in \mathbf{P}$, (b) $n \in P'$, (c) $n \in P \cap Q$ (d) $n = 22$.

Solution :

$P = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$P(n \in \mathbf{P}) = \frac{8}{20}=\frac{2}{5}$

$P(n \in P') = 1 - P(n \in P)$

$= 1 - \frac{2}{5} = \frac{3}{5}$

$Q = \{6, 7, 8,9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19,20\}$

$P \cap Q = (6, 7, 8)$

$P(n \cap \mathbf{P} \cap \text{Q}) = \frac{3}{20}$

$P(n = 22) = \frac{2}{20} = 0$

TOPIC-18

Statistics

Revision Notes

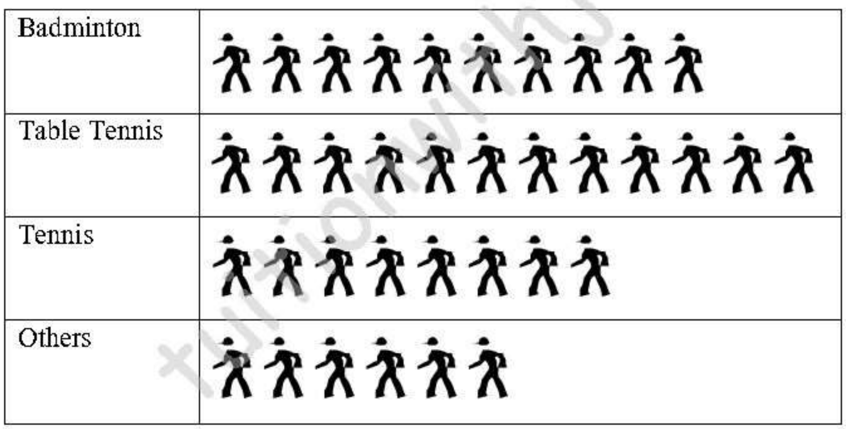
1. Presentation of Data

(a) Tabulation

Game	Badminton	Table Tennis	Tennis	Others
Number of pupils	100	120	80	60

(b) Pictogram

A pictogram is a diagram using pictures or symbols to represent data.

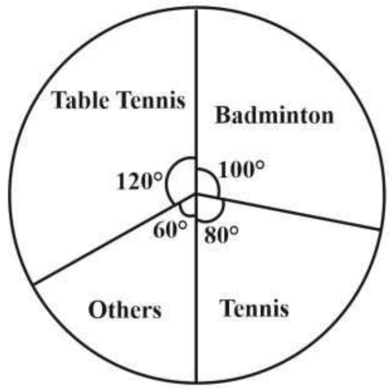


Each  represents 10 pupils

(c) Pie Chart

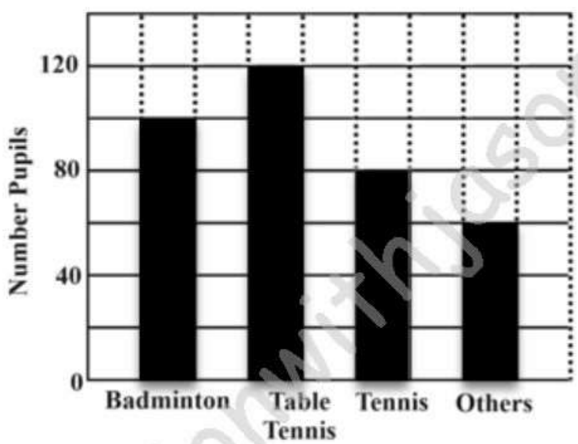
In a pie chart, the whole set of data is represented by a circle. The angle of the sector is proportional to quantity of the category, represented by the sector.

E.g. Angle of sector for Tennis = $\frac{80}{360} \times 360° = 80°$



(d) Bar Chart

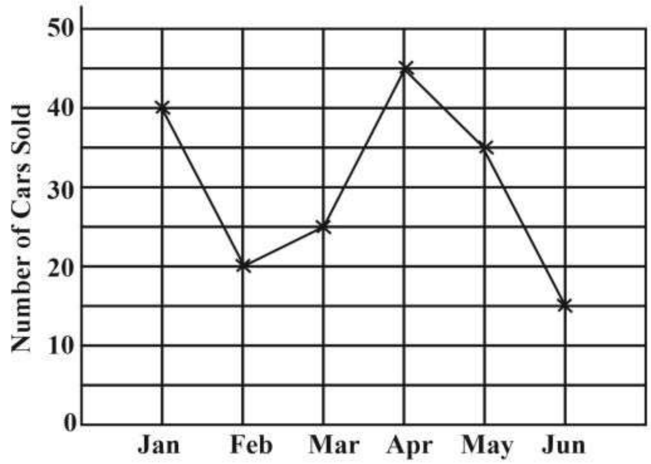
A bar chart is a diagram using bars to show data. It can be vertical or horizontal. All the bars are of equal width and they are spaces of equal width between them.



(e) Line Graph

A line graph is usually used to show trends or changes in a certain variable, over a period of time. On a line graph, the values between two successive readings may not have any meaning.

E.g., The line graph shows the sales of a motor company in the first six months of a year.

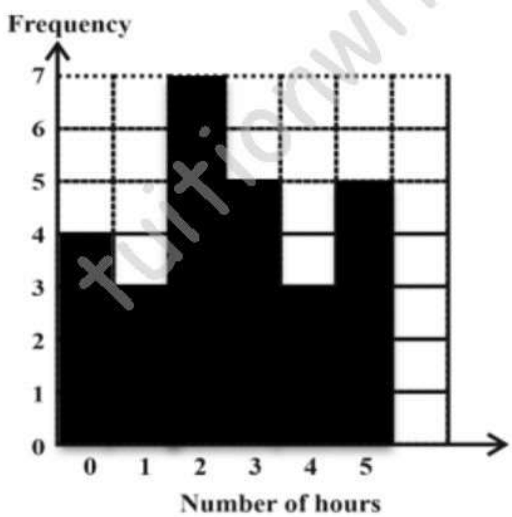


(f) Histogram

A histogram is used to represent a frequency distribution. It is similar to the bar chart except that there is no gap between the rectangular columns.

The area of each rectangle, represents the frequency. If all the classes have die same width then all the rectangles will be the same width and the frequencies are then represented by the height of the rectangles.

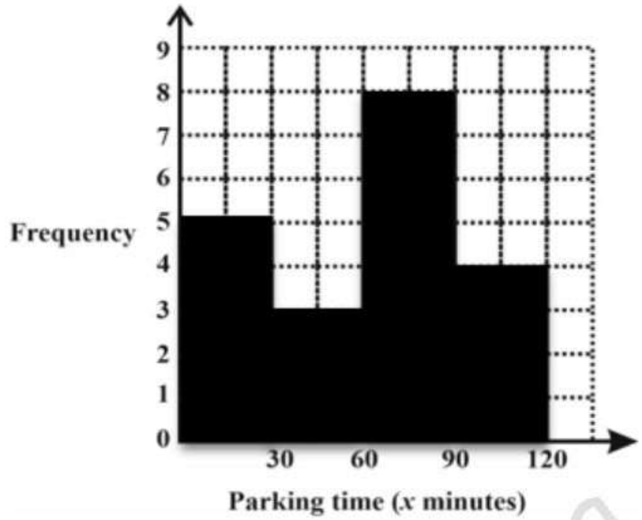
E.g. The histogram shows the number of hours spend doing homework bn a particular day by 25 pupils.



E.g., The frequency table below shows the parking times, in minutes, of 20 cars.

Parking time (x minutes)	$0 < x \leq 30$	$30 < x \leq 60$	$60 < x \leq 90$	$90 < x \leq 120$
Frequency	5	3	8	4

The table can be represented using a histogram.



(g) Choice of Statistical Graphs

(i) A pictogram is more suitable when the data needs to be presented in a lively and interesting manners. Although it is not a very, accurate way of representing data, it can still be more, informative effective than other methods for presenting data to the general public.

(ii) A bar chart and a histogram are useful for comparing quantities in the different categories and they represent data more accurately.

(iii) Compared with a bar chart or a histogram, a pie chart is a better way of show how a total is broken into different parts, it clearly compares the sizes of these parts to the whole.

(iv) A line graph is suitable to show data that changes with time.

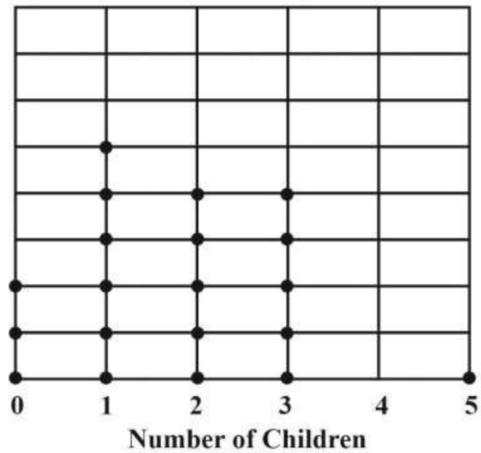
2. Dot Diagram

A dot diagram is drawn by placing. dots that the values of a set of data above the horizontal line. The number, dots above each valve indicates how many times each value occurred.

E.g., The number of children in each of 20 families is given below.

3	0	2	1	1
5	3	2	1	0
1	1	3	2	3
2	2	1	3	0

The dot diagram below displays this information.



Stem–and–Leaf Diagram

A stem and leaf diagram will split each data or number into two pail the leaf. A stem and leaf diagram will have one column for stems and one column for leaves. Fore can be many leaves, one leaf or no leaf.

E.g. The weight of 21 students in a class were recorded as follow:

58	81	60	68	67	74
73	59	79	65	74	64
87	62	77	77	83	63
63	76	75			

The Stem and Leaf diagram is shown below.

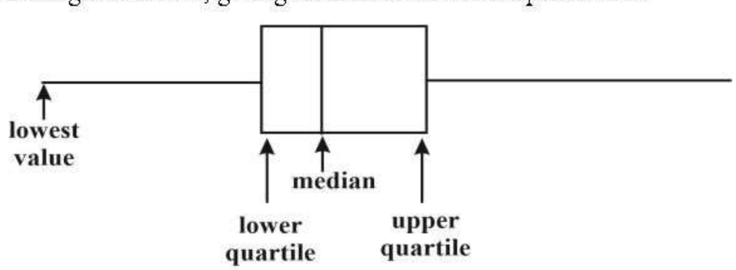
Stem	Leaf
5	8 9
6	0 2 3 3 4 4 5 7 8
7	3 4 5 6 7 7 9
8	1 3 7

5 | 8 represents.

Box and Whisker Plot (or Box Plot)

A box and whisker plot summarises a data set in terms of its median age.

The 'box' contains the median, lower and upper quartiles and the (hiskers') extend to the lowest and the highest values, giving an indication of the spread of th



E.g., Construct a box and whisker plot of the following data.

5, 24, 27, 35, 42, 48, 54, 57, 64, 63, 69, 72, 79, 86, 92

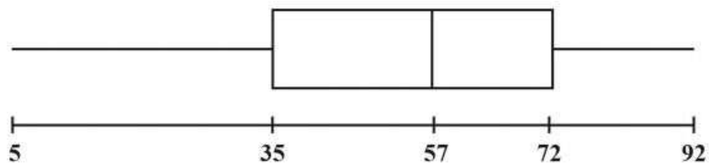
Minimum value = 5

Maximum value = 92

Lower quartile = 35

Median = 57

Upper quartile = 72



5. Averages

Three commonly used averages are the mean, mode and median. These averages are also called measures location or measures of central tendency.

(a) Mean, $\overline{x}$

(i) For a set of values, $\overline{x} = \frac{\Sigma x_i}{n}$

(ii) For an ungrouped frequency distribution, $\overline{x} = \frac{\Sigma f_i x_i}{f_i}$

(iii) For a grouped frequency distribution, $\overline{x} = \frac{\Sigma f_i x_i}{f_i}$

where x_1 is the mid– value of each class is taken to represent each class.

(b) Median

For a set of values arranged in ascending or descending order:

(i) the median is the middle value if there is an odd number of values,

(ii) the median is the mean of the two middle values if there is an even number of values.

(c) Mode

(i) The mode of a set of values is the value which occurs most frequently.

(ii) The modal class is the class interval with the highest frequency.

(d) Purposes and Use of Mean, Mode and Median

(i) The mean takes into account all items in the data set. When there are no extreme values, it is the common kind of average that is used. However, the disadvantage of the mean is that it is distorted extreme values in tire data set.

(ii) The median can be easily obtained when the values arranged in ascending or descending order. The useful way of averaging because it avoids the problem of distortion associated with the mean.

(iii) The mode is; the appropriate average to use when the more typical value is required. The mode can used not only as an average for numerical data but also as an average for non–numerical data sub favourite colour. Furthermore, the mode always represents an actual recorded value and is not distance by extreme values.

6. Measures of Spread

The measure of spread shows how data are dispersed around an average.

(a) Range

The range of a set of data values is given by

Range = highest value – lowest value

(b) Quartiles

The lower quartile (Q_1), median (Q_2) and the upper quartile (Q_3) divide a set of ordered data into four equal parts.

The lower quartile also known as first quartile has 25% of the data–values below it and 75%. above.

The upper quartile also known as third quartile has 75% of the data values below it and 25% above.

The second quartile is actually the median since it is greater than 50% of the data values.

(c) Interquartile range

The interquartile range measures spread by finding the difference between the upper quartile and the lower quartile.

Interquartile range = upper quartile – lower quartile = $Q_3 - Q_1$.

(d) Percentiles

The lower quartile is 25^{th} percentile, the median is 50^{th} percentile and the upper quartile is 75^{th} percentile. The 90^{th} percentile in a list of numbers is the smallest number for which at least 90% of the numbers are less than or equal to it. In general, the n^{th} percentile in a list of numbers is the smallest number for which at least n% of the numbers are less than or equal to it.

(e) Standard Deviation

The standard measures how widely the data spread around the mean.

(i) ***Ungrouped data***

For a set of n numbers $x_1, x_2, \ldots x_n$ whose mean is $\overline{x}$, the standard deviation, s is given by

$$s = \sqrt{\frac{\Sigma(x - \overline{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2}$$

(ii) ***Frequency distribution***

For data given in a frequency table in which $x_1, x_2, \ldots x_n$ occur with frequencies $f_1, f_2, \ldots f_n$, the standard deviation, s is given by

$$s=\sqrt{\frac{\Sigma f(x-\overline{x})^2}{\Sigma f}}=\sqrt{\frac{\Sigma fx^2}{\Sigma f}-\left(\frac{\Sigma fx}{\Sigma f}\right)^2}=\sqrt{\frac{\Sigma fx^2}{\Sigma f}-\overline{x}^2}$$

(iii) ***Group data***

Take the midpoint of a class interval to represent that class, then use one of the formula as given in (ii).

Cumulative Frequency Curve

n = total number of items,

Q_1 = lower quartile or 25th percentage,

Q_2 = median or 50th percentage,

Q_3 = Upper quartile or 75th percentage.