Candidate Index Number

Anglo - Chinese School

(Independent)

FINAL EXAMINATION 2022

YEAR 3 INTEGRATED PROGRAMME

CORE MATHEMATICS

PAPER 1

Friday **30th September 2022** **1 hour 30 minutes**

Candidates answer on the Question Paper.
No additional materials are required.

INSTRUCTIONS TO CANDIDATES

- Write your index number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Answer all questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- The maximum mark for this paper is 80.

For Examiner's Use

This question paper consists of **18** printed pages.

[Turn over

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Answer **all** the questions in the spaces provided.

1. [Maximum mark: 3]

Express $\frac{1}{x^2+2x-3}-\frac{x^2+1}{2x^4-2}$ as a single fraction in its simplest form.

2. [Maximum mark: 4]

Given that $\begin{pmatrix} a & 3a \\ 3 & 4 \end{pmatrix} - 2\begin{pmatrix} b & 2b \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 13 \\ 1 & 0 \end{pmatrix}$, find the value of a and of b.

3. [Maximum mark: 5]

A right cone has base diameter 6 cm and height of 4 cm.

(a) Find, in terms of π, the total surface area of the cone. [3]

(b) A solid hemisphere has the same surface area as the cone. Find the radius of the hemisphere in the form $a\sqrt{2}$ cm, where a is a constant. [2]

4. [Maximum mark: 5]

(a) Solve $27\left(\frac{1}{3}\right)^x = 81^{\frac{5}{4}}$. [2]

(b) Simplify $\log_4 a \times \log_a 64 - \log_{\sqrt{2}} 4$. [3]

5. [Maximum mark: 6]

(a) Solve the simultaneous equations $xy = 16$ and $y = x^3$. [3]

(b) On the same axes, sketch the graphs of $xy = 16$ and $y = x^3$, labelling the axes-intercept(s) and point(s) of intersection clearly. [3]

6. [Maximum mark: 7]

(a) Simplify $\frac{2}{\sqrt{3}}-\frac{\sqrt{108}}{4}+\frac{5}{1+\sqrt{3}}$, leaving your answer in the form $a+b\sqrt{3}$, where a and b are constants. [4]

(b) Given that $t=\frac{1}{\sqrt{3}}$, express $\frac{t-1}{2t-1}$ the form $p+q\sqrt{3}$, where p and q are constants. [3]

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7. [Maximum mark: 8]

(a) Given that a and b are integers such that $-2 \le a < 5$ and $-3 \le b \le 7$, find the

(i) greatest possible value of $a^2 - \frac{b^2}{2}$, [2]

(ii) smallest possible value of $\frac{ab}{(a-1)^2}$. [2]

(b) Solve the inequalities $4x - 17 \le 5x - 12 < 3x + 50$. [4]

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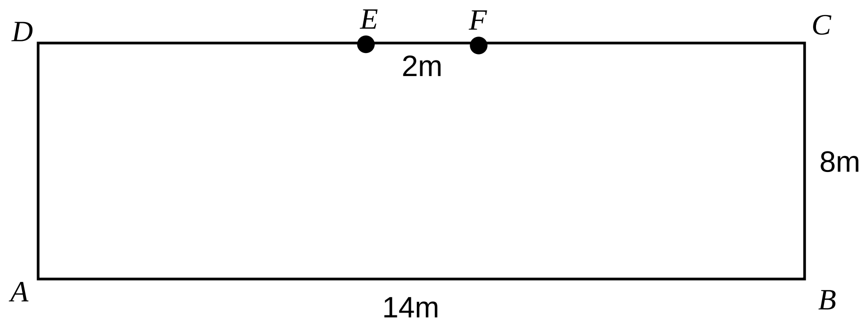
8. [Maximum mark: 10]

(a) If A is an obtuse angle and $\cos A = -\frac{12}{13}$, find the value of each of following:

(i) $\tan A$, [2]

(ii) $\sin A + \cos A$. [2]

(b) The diagram below shows an 8 metres by 14 metres rectangular assembly area where A, B, C and D are points on level ground. Two flagpoles stand at E and F, such that $DE = FC$ and the flagpoles stand 2 metres apart from each other. The height of each flagpole is 7 metres and the top of the flagpole at F is denoted as G.



Find

(i) the length BG, giving your answer in surd form, [3]

(ii) $\tan \measuredangle GBF$, [1]

(iii) $\cos \measuredangle DFB$. [2]

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9. [Maximum mark: 15]

A company uses the function $P = -n^2 + 10n - 21$ to model its profits, P, in **thousands** of dollars, where n **hundred** units of Product A are produced and sold.

(a) Explain the meaning of "-21" in the context. [1]

(b) The company makes a profit by producing 400 units of Product A. A director suggests doubling the number of units produced. Explain if this is advisable, justifying your answer clearly. [2]

(c) State the maximum profit and the corresponding number of units to produce. [2]

(d) Sketch the graph of $P = -n^2 + 10n - 21$, clearly labelling the coordinates of the axes-intercepts and turning point. [3]

To meet demands, the company has to increase its production capacity to produce Product B. The costs, in **thousands** of dollars, C, of producing n **hundred** units of Product B is given by $C = kn^2 - 10n + 30$.

(e) If $k = 1.5$, explain why it is not possible to have a production cost of $10,000. [3]

(f) Find the range of values of k, such that the production cost is always more than $20,000. [4]

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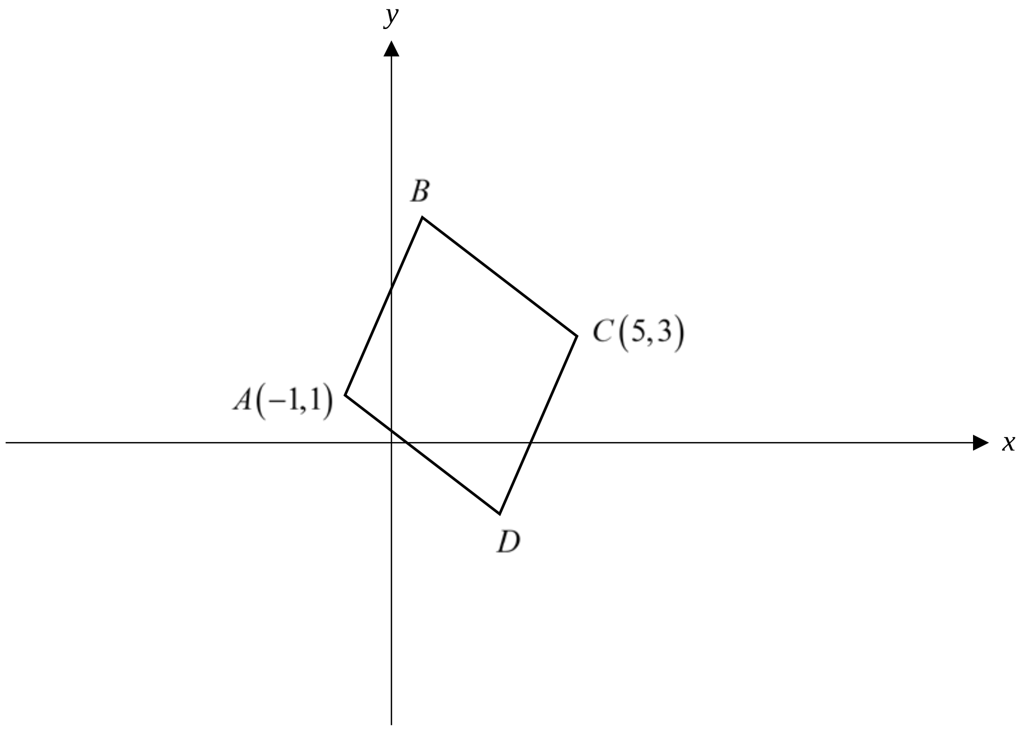
10. [Maximum mark: 5]

The roots of the quadratic equation $9x^2 = kx - 1$ are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ where $\alpha, \beta > 0$. Show that $\alpha + \beta = \sqrt{k+6}$.

11. [Maximum mark: 12]

Solutions to this question by accurate drawing will not be accepted.
In the diagram below, $A(-1,1)$ and $C(5,3)$ are two vertices of a parallelogram $ABCD$. AB has a gradient of 2 and the perpendicular bisector of AB passes through the center of $ABCD$.



Find

(a) the equation of AB, [2]

(b) the equation of the perpendicular bisector of AB, [2]

(c) the coordinates of B, [3]

(d) the coordinates of D, [2]

(e) the area of $ABCD$. [3]

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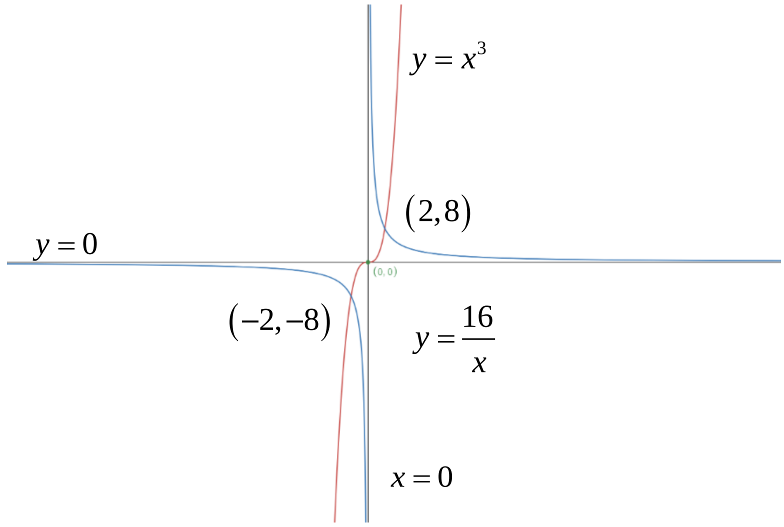
---**END OF PAPER 1**---

<u>**Answers**</u>

1. $\frac{1}{2(x+1)(x+3)}$
2. $a=3, b=-1$
3. (a) 24π (b) $a=2$
4. (a) -2 (b) -1
5. (a) $x=2, y=8$; $x=-2, y=-8$

   (b)



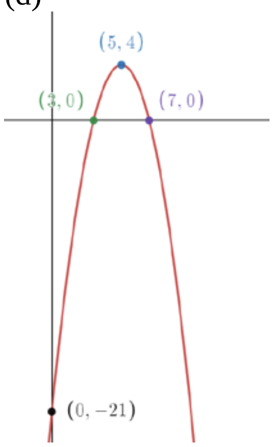
6. (a) $a=-\frac{5}{2}, b=\frac{5}{3}$ (b) $p=-1, q=-1$
7. (a)(i) 16 (ii) -6 (b) $-5 \le x < 31$
8. (a)(i) $-\frac{5}{12}$ (ii) $-\frac{7}{13}$

   (b)(i) $\sqrt{149}$ (ii) $\frac{7}{10}$ (iii) $-\frac{3}{5}$
9. (a) Starting cost of $21000 / Loss of $21000 when no goods were produced yet

   (b) The company makes a profit of $3000 when producing 400 units, but a loss of $5000 when producing 800 units. Hence, this not advisable.

   (c) $4000, 500 units

   (d)



   (e) $b^2-4ac=-20.$ Since discriminant is negative, there are no real roots so it is not possible.

   (f) $k>\frac{5}{2}$

10. –

11. (a) $y = 2x + 3$

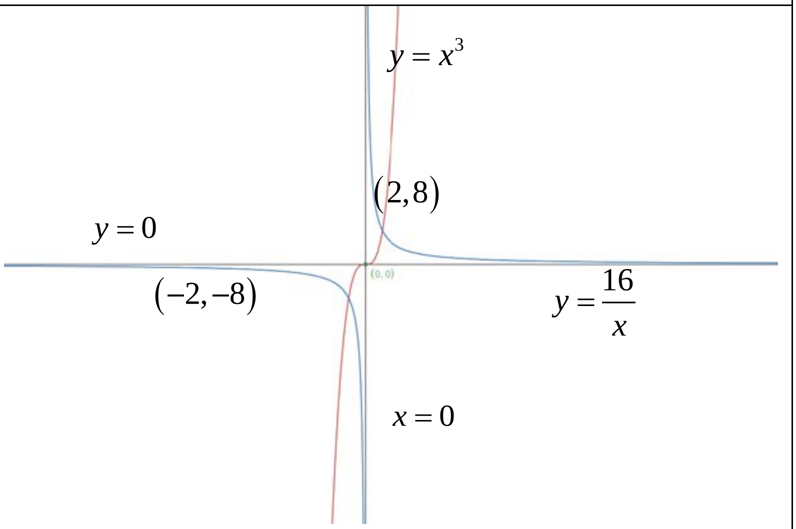
(b) $y = -\frac{1}{2}x + 3$

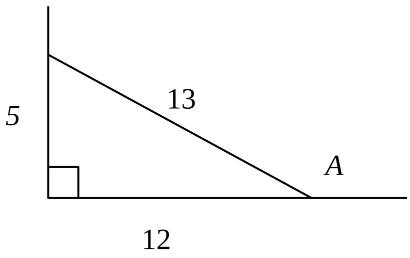
(c) $B(1, 5)$

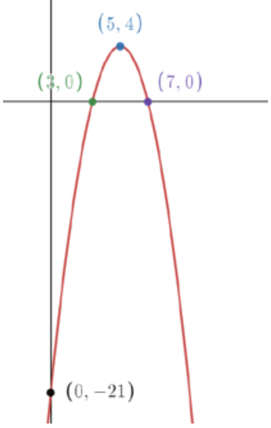
(d) $D(3, -1)$

(e) 20 units^2

Qn	Worked Solutions
1a	$\frac{1}{x^2+2x-3}-\frac{x^2+1}{2x^4-2}$ $=\frac{1}{x^2+2x-3}-\frac{x^2+1}{2\left(x^2+1\right)\left(x^2-1\right)}$ $=\frac{1}{(x+3)(x-1)}-\frac{1}{2(x+1)(x-1)}$ $=\frac{2x+2-(x+3)}{2(x+1)(x+3)(x-1)}$ $=\frac{x-1}{2(x+1)(x+3)(x-1)}$ $\frac{1}{2(x+1)(x+3)}$
2	$a-2b=5$ ___(1) $3a-4b=13$ ___(2) $(2)-(1)\times 2: a=3, b=-1$
3a	Slant height $=\sqrt{3^2+4^2}=5$ cm Total surface area of cone $=\pi(3^2)+\pi(3)(5)=24\pi$
3b	$3\pi(r^2)=24\pi$ $r=\sqrt{8}=2\sqrt{2}$ cm $a=2$
4a	$27\left(\frac{1}{3}\right)^x=81^{\frac{5}{4}}$ $3^3(3^{-x})=3^5$ $3-x=5$ $x=-2$
4b	$\log_4 a\times\log_a 64-\log_{\sqrt{2}} 4$ $=\frac{\lg a}{\lg 4}\times\frac{\lg 64}{\lg a}-4$ $=\frac{3\lg 4}{\lg 4}-4$ $=-1$
5a	$y=\frac{16}{x}$ Substitute into $y=x^3$. $\frac{16}{x}=x^3$ $x^4=16$ $x=4$

	$x=2, y=8$ $x=-2, y=-8$
5b	
6a	$\frac{2}{\sqrt{3}}-\frac{\sqrt{108}}{4}+\frac{5}{1+\sqrt{3}}$ $=\frac{2\sqrt{3}}{3}-\frac{6\sqrt{3}}{4}+\frac{5\left(1-\sqrt{3}\right)}{1+\sqrt{3}\left(1-\sqrt{3}\right)}$ $=\frac{2\sqrt{3}}{3}-\frac{6\sqrt{3}}{4}+\frac{5-5\sqrt{3}}{-2}$ $=\frac{8\sqrt{3}-18\sqrt{3}-30+30\sqrt{3}}{12}$ $=\frac{-30+20\sqrt{3}}{12}$ $=-\frac{5}{2}+\frac{5}{3}\sqrt{3}$ $a=-\frac{5}{2}, b=\frac{5}{3}$
6b	$\frac{\frac{1}{\sqrt{3}}-1}{\frac{2}{\sqrt{3}}-1}$ $=\frac{\frac{1-\sqrt{3}}{\sqrt{3}}}{\frac{2-\sqrt{3}}{\sqrt{3}}}$ $=\frac{1-\sqrt{3}}{2-\sqrt{3}}$ $=\frac{1-\sqrt{3}}{2-\sqrt{3}}\left(\frac{2+\sqrt{3}}{2+\sqrt{3}}\right)$

	$= \frac{2-\sqrt{3}-3}{4-3}$ $= -1-\sqrt{3}$ $p=-1, q=-1$
7ai	$4^2-0=16$
7aii	$\frac{(2)(-3)}{(2-1)^2}=-6$
7b	$4x-17 \le 5x-12$ and $5x-12 < 3x+50$ $x \ge -5$ and $2x < 62$ $-5 \le x < 31$
8ai	 $\tan A = -\tan(180° - A)$ $= -\frac{5}{12}$
8aii	$\sin A = \frac{5}{13}$ $\sin A + \cos A = \frac{5}{13} - \frac{12}{13}$ $= -\frac{7}{13}$
8bi	$CF = \frac{14-2}{2} = 6$ $BF = \sqrt{6^2+8^2}$ $=10$ $BG = \sqrt{10^2+7^2}$ $=\sqrt{149}$
8bii	$\tan \measuredangle GBF = \frac{7}{10}$
8biii	$\cos \measuredangle DFB = -\cos(180° - \measuredangle CFB)$ $= -\frac{6}{10}$ $= -\frac{3}{5}$
9a	Starting cost of \$21000 / Loss of \$21000 when no goods were produced yet

9b	The company makes a profit of \$3000 when producing 400 units, but a loss of \$5000 when producing 800 units. Hence, this not advisable.
9c	Maximum profit of \$4000 when 500 units produced
9d	
9e	$10 = 1.5n^2 - 10n + 30$ $1.5n^2 - 10n + 20 = 0$ $b^2 - 4ac = 100 - 120 = -20$ Since discriminant is negative, there are no real roots so it is not possible.
9f	$kn^2 - 10n + 30 > 20$ $kn^2 - 10n + 10 > 0$ $k > 0$ and $b^2 - 4ac = 100 - 40k < 0$ $k > \frac{5}{2}$
10	$9x^2 - kx + 1 = 0$ $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$ $\frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = \frac{k}{9}$ $\frac{1}{\alpha^2\beta^2} = \frac{1}{9}$ $\alpha\beta = 3$ $\alpha^2 + \beta^2 = k$ $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$ $= k + 6$ $\alpha + \beta = \sqrt{k + 6}$
11a	$y - 1 = 2(x + 1)$ $y = 2x + 3$
11b	Midpoint of $AC = \left(\frac{-1+5}{2}, \frac{1+3}{2}\right) = (2, 2)$ Perpendicular gradient $-\frac{1}{2}$

	$y-2=-\frac{1}{2}(x-2)$ $y=-\frac{1}{2}x+3$
11c	$-\frac{1}{2}x+3=2x+3$ $x=0$ $y=3$ $\left(\frac{-1+b_x}{2},\frac{1+b_y}{2}\right)=(0,3)$ $B(1,5)$
11d	$\left(\frac{1+d_x}{2},\frac{5+d_y}{2}\right)=(2,2)$ $D(3,-1)$
11e	$\frac{1}{2}\begin{vmatrix} -1 & 1 & 5 & 3 & -1 \\ 1 & 5 & 3 & -1 & 1 \end{vmatrix}$ $=\frac{1}{2}\lvert -5+3-5+3-1-25-9-1 \rvert$ $= 20 \text{ units}^2$