X JUNIOR COLLEGE
YEAR 6 PRELIMINARY EXAMINATION
Mock Arrangement
in preparation for Candidates' Examination
Higher (than) 2

CANDIDATE NAME

MATHEMATICS 9758/01

Paper 2 **Set I**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Write your answers in the space provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need of clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

ABOUT THIS PAPER

X Junior College (XJC) is an unofficial initiative aimed at preparing pre-university and/or junior college students in Singapore for school-level and/or national-level H2 Mathematics examinations through self-prepared mock papers. It has no affiliation with any existing institution in Singapore or worldwide.

This mock paper follows closely the 9758 H2 Mathematics GCE Advanced Level syllabus, most suitable for preparation towards preliminary examinations and A–Levels. The paper intends to explore the unconventional ways and/or applications in which topics within the syllabus can be tested, which may affect the difficulty of this paper to varying degrees. **While it is ideal to attempt this paper under examination constraints, prospective candidates are reminded not to use this potentially non-conforming paper as a definitive gauge for actual performance.**

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This document consists of **24** printed pages and **4** blank pages.

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Section A: Pure Mathematics [40 marks]

1 When the complex polynomial $\mathrm{P}(z)$ is divided by $(z+\mathrm{i})$, $(z-\mathrm{i})$ and (z^2+1), the remainders are $1+\mathrm{i}$, $1-\mathrm{i}$ and $Az+B$ respectively. Find A and B. [3]

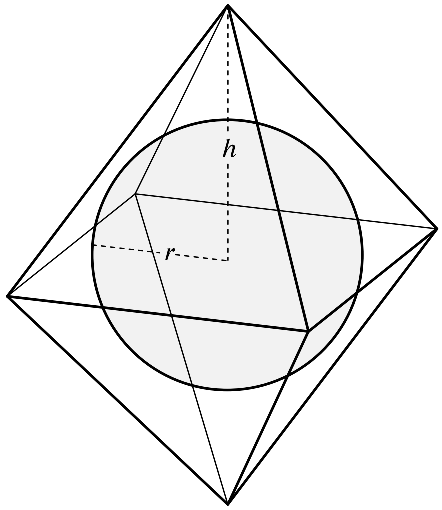
2 A complex number z can be expressed as $x + \mathrm{i}y$, where x and y are real, and satisfies $|z + 3| = 2\mathrm{Re}(z)$.

(i) Show that x and y are related by the equation of a hyperbola. State the equations of its asymptotes. [3]

(ii) Sketch the part of the hyperbola on which any point (x, y) satisfies the above equation in z, including its asymptotes. On your sketch, indicate the values of the axial intercepts of these asymptotes. [2]

(iii) Deduce the exact range of $\arg(z - 1)$. [2]

3 [The volume of a pyramid is $\frac{1}{3} \times$ base area $\times$ height.]



A sphere with fixed radius r is inscribed in an octahedron shape formed by joining two square-based right pyramids of height h base-to-base (see diagram). Find, in terms of r, the value of h which minimises the volume of the inscribing octahedron. Justify that the resulting volume is minimum and find its value exactly in terms of r. [8]

3 [Continued]

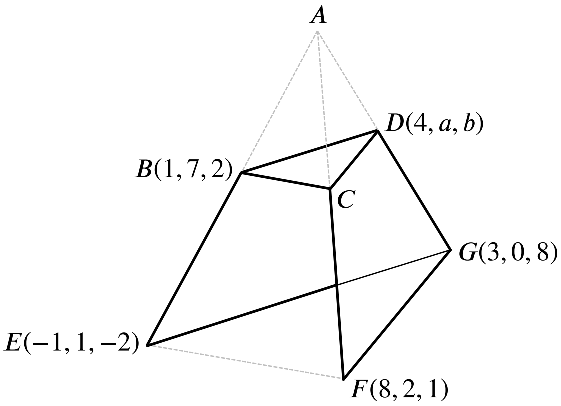
4 It is given that $y = e^{2x} \cos ax$, where a is a real constant.

(i) Show that $\dfrac{d^2y}{dx^2} = 4\dfrac{dy}{dx} - (a^2 + 4)y$. [3]

(ii) Using the result in **(i)**, find the first four terms of the Maclaurin expansion of y. [4]

(iii) Hence, find the Maclaurin expansion of $\mathrm{e}^{2x} \sin ax$ as far as the term in x^2. [2]

5



A cardboard is cut and folded into the shape following the surface of a truncated tetrahedron *BCDEFG*, which is formed by removing the tetrahedron *ABCD* from the larger tetrahedron *AEFG*. It is known that the faces *BCFE* and *EFG* are hollow, and that the plane *BCD* is parallel to plane *EFG*. The coordinates of *B*, *D*, *E*, *F* and *G* are $(1, 7, 2)$, $(4, a, b)$, $(-1, 1, 2)$, $(8, 2, 1)$ and $(3, 0, 8)$ respectively (see diagram).

(i) Show that $a = 6.25$ and find b. [2]

(ii) The area of the cardboard used to create the folded shape *BCDEFG* can be expressed in the form

$$p\left|\overrightarrow{DG} \times \overrightarrow{EG}\right| + q\left|\overrightarrow{DG} \times \overrightarrow{FG}\right| + r\left|\overrightarrow{EG} \times \overrightarrow{FG}\right|.$$

Find the exact value of p, q and r. Hence find this area numerically. [6]

[The area of a trapezium is $\frac{1}{2} \times$ height $\times$ sum of parallel sides.]

5 [Continued]

(iii) The folded shape is detached along *DG* and flattened such that *BCD* remains attached to *BDGE* at *BD* and to *CDGF* at *CD*. Find the shortest distance between *C* and *E* on this flat surface. [5]

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[Question 6 starts on the next page.]

Section B: Probability and Statistics [60 marks]

6 A continuous random variable X has the distribution $\mathrm{N}(\mu, \sigma^2)$. It is known that $\mathrm{P}(X \leq 6a) = 0.15866$, and that the average of four independent observations of X lies between $6a$ and $7a$ with a probability of 0.81859.

(i) Find μ and σ in terms a, giving any numerical constants up to 3 decimal places. [5]

(ii) Sketch a graph of $\mathrm{P}(X \leq x)$ against x, indicating any important features of the graph and the coordinates of the point where $x = 6a$ and $x = \mu$. [2]

7 A computer simulates a repeated summation. Starting from an initial sum 0, the computer does either one of two actions: add 1 to the sum with probability p, where $0 < p < 1$, or subtract 1 from the sum with probability q, where $q = 1 - p$. Each action is done independently of all other actions.

(i) Show that the probability that the sum is 3 after 9 actions is $84p^6q^3$. [1]

(ii) If it is equally likely that the sum is 3 after 9 actions as it is after 5 actions, find the possible values of p. [3]

(iii) Find the range of values of p such that, for integers $k > 1$, the computer is more likely to end up with a positive sum after $(2k + 1)$ actions than it is after $(2k - 1)$ actions. [5]

8 Box A and Box B each contain N balls labelled 1, 2, 3, …, N. Each ball is equally likely to be picked from their respective box. A person picks a ball each from Box A and Box B, and the label on each ball is denoted by X_A and X_B respectively. The random variable X is defined by

$$X = \begin{cases} X_A, & \text{if } X_A \geq X_B, \\ X_B, & \text{if } X_B \geq X_A. \end{cases}$$

(i) Show that $\mathrm{P}(X = r) = \dfrac{2r-1}{N^2}$ for $r = 1, 2, 3, \ldots, N$. [2]

(ii) Find an expression for $\mathrm{E}(X)$ in terms of N, leaving your answer as a simplified fraction. [2]

[You may use the result $\displaystyle\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ without proof.]

The *median* of X is defined to be the integer m such that $\mathrm{P}(X \geq m) \geq \frac{1}{2}$ and $\mathrm{P}(X \leq m) \geq \frac{1}{2}$.

(iii) Show that $\frac{\sqrt{2}}{2}N \leq m \leq \frac{\sqrt{2}}{2}N + 1$. Hence find the median of X when $\mathrm{E}(X) = \frac{40299}{600}$. [5]

9 The events A and B are such that $\mathrm{P}(A) = \frac{1}{3}$, $\mathrm{P}(A \cup B) = \frac{1}{2}$ and $\mathrm{P}(A|B') = \frac{1}{6}$.

(i) Find $\mathrm{P}(B)$ and $\mathrm{P}(A \cap B)$. [3]

Another event C is such that $\mathrm{P}(C) = \frac{1}{4}$. It is given that A is independent of C, and B is independent of C.

(ii) Using the result $\mathrm{P}\big(A \cap (B \cup C)\big) = \mathrm{P}\big((A \cap B) \cup (A \cap C)\big)$, show that

$$\mathrm{P}(A \cup B \cup C) = \mathrm{P}(A) + \mathrm{P}(B) + \mathrm{P}(C) - \mathrm{P}(A \cap B) - \mathrm{P}(A \cap C) - \mathrm{P}(B \cap C) + \mathrm{P}(A \cap B \cap C).$$ [1]

(iii) Given that $\text{P}(A' \cap B' \cap C') = \frac{11}{30}$, find $\text{P}(A \cap B \cap C)$. [2]

(iv) Find exactly the maximum and minimum possible values of $\text{P}(A' \cap B' \cap C')$. [4]

10 An electronics company produces chips for devices. Since its establishment, the company has produced 100 000 chips, 1 000 of which are of the newest model. The electronics company takes a sample of chips and is then able to carry out a z-test to determine the effect of the newest chips on device performance.

(i) State **three** details about the way the sample of chips are taken which make it appropriate for testing. [3]

The sample chips are given to the quality assurance team for testing. For each chip, the time taken t, in seconds, for a device to carry out tasks with the chip installed is recorded. The results are summarised as follows.

Time, t	$10 \le t < 20$	$20 \le t < 30$	$30 \le t < 40$	$40 \le t < 50$	$50 \le t < 60$
Number of devices	4	14	37	33	12

Using the results above, a z-test is carried out at $\alpha\%$ significance level against the null hypothesis that the average time taken for the same device to carry out the same tasks using a chip of the newest model is 40 seconds.

(ii) Show that $\mathrm{E}(t) = 38.5$ and hence find $\mathrm{Var}(t)$. [2]

(iii) It is given that the test conclusion is to reject the null hypothesis, and that only one unique alternative hypothesis gives this conclusion at the given significance level. By considering the test statistic and possible critical regions, find the alternative hypothesis and the corresponding range of values of α. [7]

11 The study of calorimetry in Chemistry concerns the measure of temperature change during a chemical reaction. In a calorimetry experiment, two substances are mixed in a polystyrene cup, and the temperature T (in degrees Celsius) of the mixture is recorded after time t (in minute) has passed since the initial temperature reading. During the reaction, heat may be lost to, or gained from, the environment depending on the type of reaction, and as such the extreme temperature recorded may not yield the desired accuracy of temperature change.

The following 16 pairs of data are obtained for the reaction between substances A and B. The first few readings of T are taken when A is already in the cup. At $t = 1.5$, substance B is inserted into the cup and reaction occurs.

t	0	0.5	1	1.5	2	2.5	3	3.5
T	20.4	20.4	20.4	20.4	22.0	23.6	25.0	26.4

t	4	4.5	5	5.5	6	6.5	7	7.5
T	27.2	27.6	27.8	27.8	27.6	27.6	27.4	27.4

(i) On the grid provided, draw a scatter diagram of the data. [1]

The line L is the least squares regression line of t on T for the data where $5 \le t \le 7.5$. The ‘true’ maximum temperature change of the reaction is the difference between the initial temperature recorded and the temperature based on L at the time the reaction occurs.

(ii) Draw, by eye, a best-fit line for L on your diagram in **(i)**. Explain how you ensure that the line is best-fit. [2]

(iii) Use your diagram to find the ‘true’ maximum temperature change. Explain whether the value you found is reliable. [3]

(iv) Use your calculator to find the equation of L for the case where the temperature readings T have been converted to degrees Rankine. State the corresponding product moment correlation coefficient. [4]

[T degrees Celsius = $\left(\frac{9}{5}T + 491.67\right)$ degrees Rankine]

11 [Continued]

When A reacts with B, temperature increases at a rate of k degrees Celsius per minute.

(v) Using data in an appropriate range, find k. Explain how you ensure that the selected data gives a reliable value of k. [3]

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