Anglo-Chinese School

(Independent)

THE BEST IS YET TO BE

PRELIMINARY EXAMINATION 2022

YEAR 6 IB DIPLOMA PROGRAMME

MATHEMATICS: ANALYSIS AND APPROACHES

HIGHER LEVEL

PAPER 2

Wednesday, 14 September 2022

2 hours

Candidate Session Number

0	0	2	3	2	9				

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions on the writing paper provided. Fill in your session number on each answer sheet, and attach them to this examination paper using the string provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics: Analysis and Approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**
- Questions with an asterisk (*) are common to both HL and SL papers

Section A (55 Marks)		Section B (55 Marks)	
Question	**Marks**	**Question**	**Marks**
1		10	
2			
3			
4		11	
5			
6			
7		12	
8			
9			
Subtotal		**Subtotal**	
TOTAL	/ 110		

This question paper consists of 14 printed pages including this cover page.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. *[Maximum mark: 4]

(a) In a circle centre O, radius r, AB is any chord and $\angle AOB = \theta$ where $0 < \theta < \pi$. Find an expression for the area of the minor segment cut off by AB in terms of r and θ. [1]

(b) If this area is one quarter of the area of the circle, show that $2\theta - 2\sin\theta = \pi$. [1]

(c) Find the value of $\angle AOB$. [2]

2. *[Maximum mark: 4]

The derivative of a function f is given by $f'(x) = \sin x\sqrt{\cos x}$, where $x \in \mathbb{R}$. The graph of f passes through the point $\left(\frac{\pi}{3}, 0\right)$. Find $f(x)$ in exact form.

3. *[Maximum mark: 6]

Two events X and Y are not independent and their respective probabilities of occurring are given by $P(X)=0.6$ and $P(Y)=0.7$. It is given that $P(X \cup Y)=0.95$.

Calculate

a) $P(X \cap Y)$ [2]

b) $P(Y \mid X)$ [2]

c) $P(X \cup Y')$ [2]

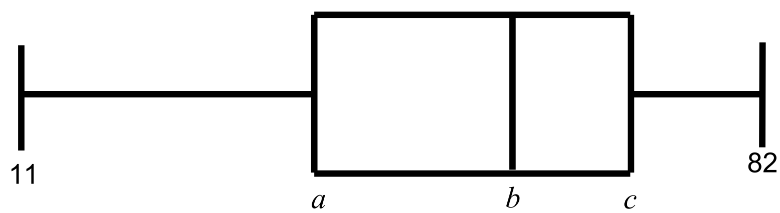
4. *[Maximum mark: 7]

The table below shows the number of hours spent on the computer in a week for 10 adults.

Adult	A	B	C	D	E	F	G	H	I	J
No. of hours	71	21	82	35	11	75	43	73	59	33

(a) Find the mean hours and variance for these ten adults. [2]

(b) A box and whisker plot for the data is drawn. Write down the values of a, b and c. [3]



(c) An error was spotted in the data recorded and q hours are subtracted from each timing recorded. State the new mean and standard deviation of the weekly hours spent on the computer in terms of q. [2]

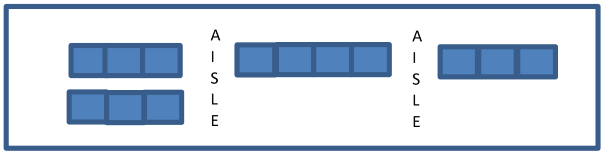
5. *[Maximum mark: 7]

A particle is moving along a straight line, and t seconds after it passes the point O of the line, its velocity is v m/s where $v = A - \ln(t+B)$ and A and B are constants. It is also known that when $t = 10$, the acceleration of the particle is $-\dfrac{1}{20}$ m/s^2 and when $t = 100$, the particle comes to rest.

(a) Find the **exact** values of B and of A. [4]

(b) Find the distance travelled in the 4th second. [3]

6. [Maximum mark: 5]



There are 13 people in a tour group and they are assigned the above seats on the plane they will be taking. Find the number of ways the tour group can be seated on the plane if

(a) there are no restrictions; [2]

(b) the Lee family (comprising Mr & Mrs Lee and their 3 children) is to be seated in a row next to one another (having aisles in between them is fine) with the 3 children between Mr & Mrs Lee. [3]

7. [Maximum mark: 9]

Consider the function $f\left(x\right)=\dfrac{\ln(\sec x)}{x}$ for $x\geq 0$.

(a) Use l'Hopital's rule to evaluate $\lim\limits_{x\to 0} f\left(x\right)$. [2]

(b) Show that the Maclaurin series for $\ln(\sec x)$, as far as the term in x^4 is $\dfrac{1}{2}x^2+\dfrac{1}{12}x^4$. [4]

(c) The region R is bounded by the $x-$axis, the curve $f\left(x\right)=\dfrac{\ln(\sec x)}{x}$ and the line $y=\dfrac{\pi}{2}-x$. Find the volume of the solid of revolution formed by rotating R completely about the $x-$axis. [3]

Q7 continues here.

8. [Maximum mark: 6]

Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 e^y$ with boundary condition $x = 1,\ y = 0.5$.

(a) By Euler's method with step size $h = 0.1$, find an approximate value of y when $x = 1.3$. [3]

(b) Find the particular solution of the differential equation. [3]

9. [Maximum mark: 7]

The pineapples harvested from Oldham Farm have masses X in kilograms with a normal distribution $X \sim N\left(\mu,\ \sigma^2\right)$. Pineapples with a mass exceeding 1.61 kg are rejected as too large and those with a mass less than 1.42 kg are rejected as too small. A crate of pineapples is graded and it is found that, on average, 10% are rejected as too large and 9% are rejected as too small.

(a) Find the values of μ and σ. [4]

(b) From this crate, n pineapples are chosen. Find the largest value of n such that the probability of rejecting at most 2 pineapples exceeds 80%. [3]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer sheets provided. Please start each question on a new page.

10.* [Maximum mark:15]

At a theme park, the designers are trying out two new proposed rollercoaster rides. The functions $f(t)$ and $g(t)$ model the heights of Rollercoasters A and B in the first 4 seconds of each ride, respectively.

The height of Rollercoaster A at time t in seconds can be modelled by the function $f(t)=3\sin(2t-10)+4t+19$ for $0\le t\le 4$.

The height of Rollercoaster B at time t in seconds can be modelled by the function $g(t)=\cos(5t)+3t+20$ for $0\le t\le 4$.

(a) Using the two models, find the initial height of

(i) Rollercoaster A;

(ii) Rollercoaster B. [2]

(b) Find the values of t when $f(t)=g(t)$. [3]

(c) For $1.5\le t\le 3$, find the range of values of t when $g(t)$ is an increasing function. [3]

(d) Find the $t-$coordinates of the points of inflexion of $f(t)$ in the interval $0\le t\le 4$, with justification as to why they are points of inflexion. [4]

(e) Find the area bounded by the 2 graphs between the 2 points of intersection in the interval $0\le t\le 4$. [3]

11. [Maximum mark: 21]

The plane Π has vector equation $\underset{\sim}{r} = \underset{\sim}{i} + 7\underset{\sim}{j} - 3\underset{\sim}{k} + \lambda\left(\underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}\right) + \mu\left(2\underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k}\right),\ \ \lambda, \mu \in \mathbb{R}$.

The points P and Q, which are not in Π, have position vectors $4\underset{\sim}{i} + 5\underset{\sim}{j} + 7\underset{\sim}{k}$ and $10\underset{\sim}{i} + 8\underset{\sim}{j} + \underset{\sim}{k}$ respectively.

(a) Show that the equation of the line l which passes through the points P and Q is

$$l : \underset{\sim}{r} = \begin{pmatrix} 4 \\ 5 \\ 7 \end{pmatrix} + \alpha \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix},\ \alpha \in \mathbb{R}.$$ [2]

(b) Find the acute angle between the line l and the plane Π. [4]

(c) The line l meets the plane Π at point R. Find the coordinates of R. [4]

The foot of the perpendicular from point P to Π is F.

(d) (i) Find the position vector $\overrightarrow{OF}$.

(ii) Hence, or otherwise, find the shortest distance from P to Π. [7]

(e) The line l is reflected in the plane Π. Show that the equation of the reflected line, l', can be expressed in the form $\underset{\sim}{r} = -2\underset{\sim}{i} + 2\underset{\sim}{j} + 13\underset{\sim}{k} + \beta\left(x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}\right),\ \ \beta \in \mathbb{R}$ where $x,\ y,\ z$ are integers to be determined. [4]

12. [Maximum mark: 19]

It is given that the complex number $z = e^{i\theta}$.

(a) Show that $1 + z^n = 2\cos\frac{n\theta}{2} e^{i\frac{n\theta}{2}}$ where $n \in \mathbb{Z}^+$. [3]

(b) (i) Show that for $z \neq -1$, we have $z - z^2 + z^3 - ... + z^7 = \frac{z + z^8}{1 + z}$.

(ii) Hence, or otherwise, show that when θ is not an odd multiple of π, we have

$$\sum_{k=1}^{7} (-1)^{k-1} \cos(k\theta) = \frac{\cos 4\theta \cos\frac{7\theta}{2}}{\cos\frac{\theta}{2}}.$$ [6]

(c) Given that $1 - z^n = -2i\sin\frac{n\theta}{2} e^{i\frac{n\theta}{2}}$, show that $\frac{1 + z^n}{1 - z^n} = i\cot\frac{n\theta}{2}$, where $n \in \mathbb{Z}^+$. [2]

(d) Solve $z^3 = -\frac{1}{\sqrt{2}}(1 + i)$, giving each root in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$ [4]

(e) Hence, using the results from (c) and (d), deduce the **exact** values of the roots of the equation $\left(\frac{w-1}{w+1}\right)^3 = -\frac{1}{\sqrt{2}}(1 + i)$. [4]

END OF PAPER