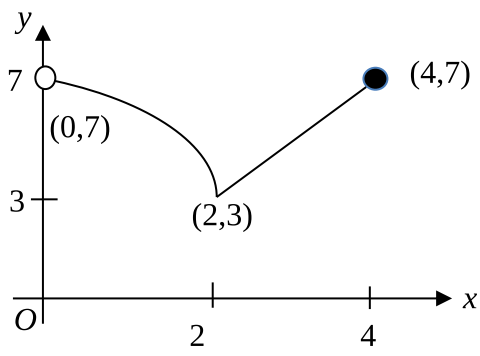
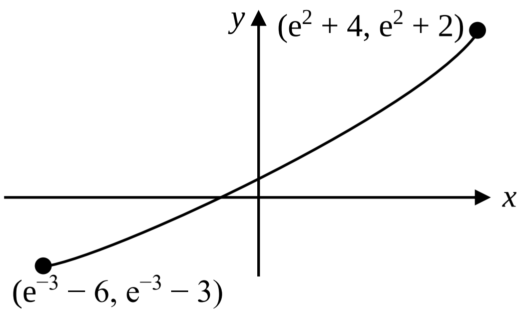
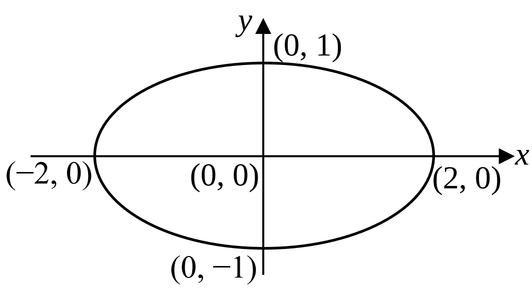
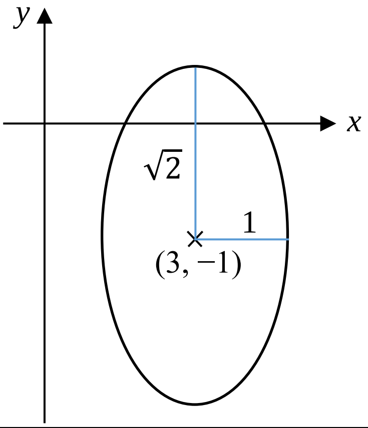
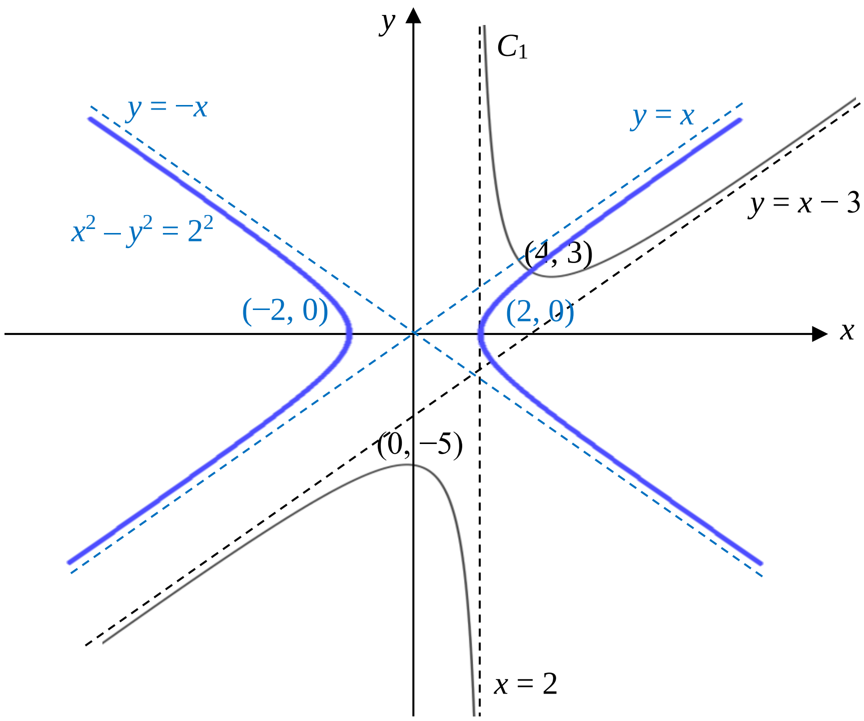
H2 Mathematics (9758)
Chapter 1 Graphing Techniques
Extra Practice Solutions

Q1	Solution
	

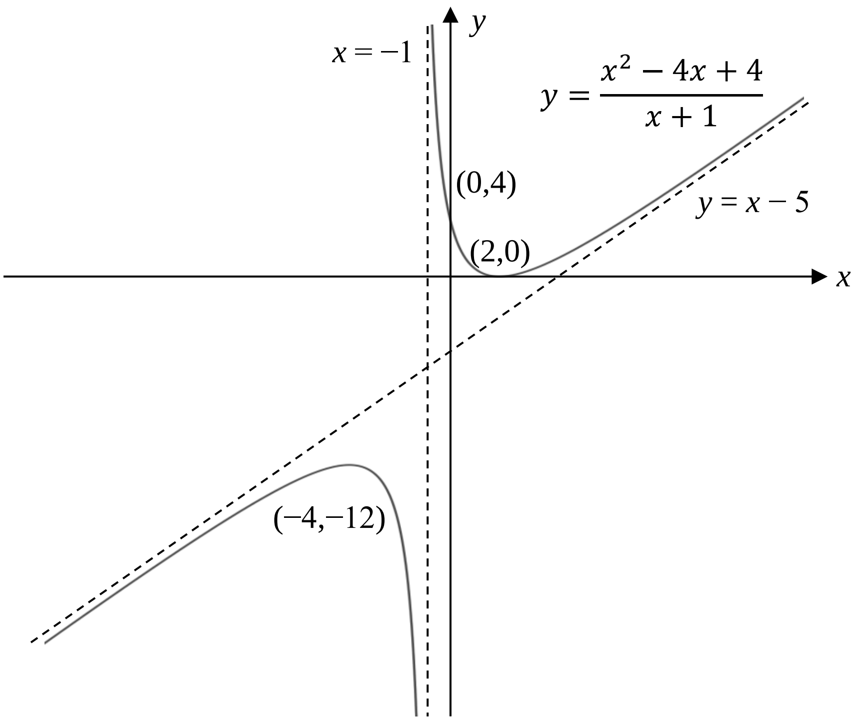
Q2	Solution
	
	Note: There is no easy way to solve for the intercepts in this question hence you are not required to label the intercepts to obtain the 1 mark. **Note:** Although the graph looks like a straight line, it is actually not a straight line (it is a curve), and your graph needs to showcase this feature

Q3	Solution
(i)	$x = 2\cos t$ and $y = \sin t$ $\cos t = \dfrac{x}{2}$ Since $\sin^2 t + \cos^2 t = 1,$ $\Rightarrow y^2 + \left(\dfrac{x}{2}\right)^2 = 1$
(ii)	

Q4	**Solutions**
	$2(x-3)^2-18+(y+1)^2-1+17=0$ $2(x-3)^2+(y+1)^2=2$ $(x-3)^2+\frac{(y+1)^2}{2}=1$ or $\frac{(x-3)^2}{1^2}+\frac{(y-(-1))^2}{\left(\sqrt{2}\right)^2}=1$ 

Q5	**Solution**
(i)	$C_1:\ y=\frac{x^2-5x+10}{x-2}=x-3+\frac{4}{x-2}$ $C_2:\ x^2-y^2=4$ 
(ii)	Point of intersection satisfy both equations. Solving simultaneous equations give $x^2-\left(\frac{x^2-5x+10}{x-2}\right)^2=4$

	$\left(x^2-4\right)=\dfrac{\left(x^2-5x+10\right)^2}{\left(x-2\right)^2}$ $\left(x^2-4\right)\left(x-2\right)^2=\left(x^2-5x+10\right)^2$
(iii)	Use GC, draw $y=\left(x^2-4\right)\left(x-2\right)^2-\left(x^2-5x+10\right)^2$ When $y = 0$, $x = 3.66$

Q6	**Solution**
(i)	Since C has a vertical asymptote at $x=-1$, thus $a=1$.
(ii)	$y=\dfrac{x^2-4x+4}{x+1}=x-5+\dfrac{9}{x+1}$ The oblique asymptote is $y=x-5$.
(iii)	$y=\dfrac{x^2-4x+4}{x+1}$ $y\left(x+1\right)=x^2-4x+4$ $yx+y=x^2-4x+4$ $x^2+\left(-4-y\right)x+4-y=0$ For real roots, discriminant $\geq$ 0. $\left(-4-y\right)^2-4\left(1\right)\left(4-y\right)\geq 0$ $y^2+8y+16-16+4y\geq 0$ $y^2+12y\geq 0$ $y\left(y+12\right)\geq 0$ $y\leq -12$ or $y\geq 0$ So C cannot lie between −12 and 0.
(iv)	

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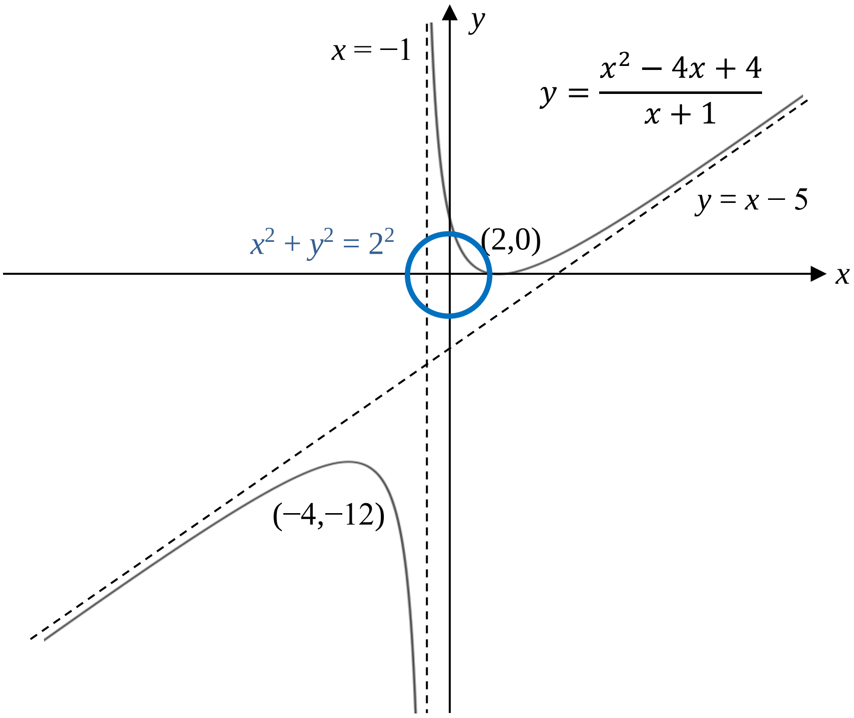
$\left(4-x^2\right)\left(x+1\right)^2=\left(x^2-4x+4\right)^2 \Rightarrow \left(\frac{x^2-4x+4}{x+1}\right)^2=4-x^2 \Rightarrow y^2=4-x^2$

Need to sketch $x^2+y^2=2^2$, a circle centred (0, 0) and radius 2.



Since there are 2 intersections between the graphs,
$\left(4-x^2\right)\left(x+1\right)^2=\left(x^2-4x+4\right)^2$ has 2 real roots.

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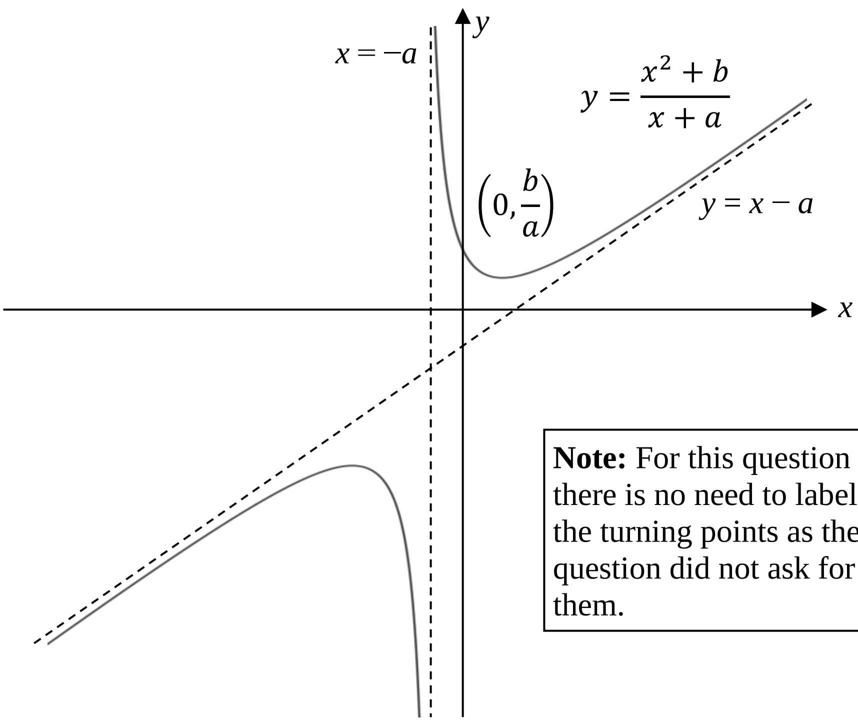
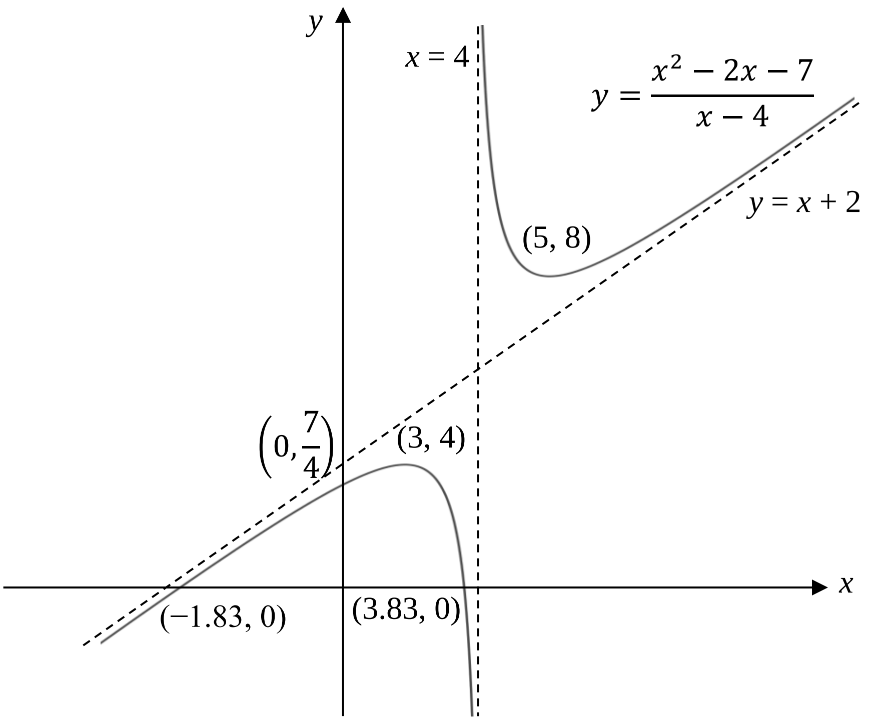
Q7	Solution
(i)	$(0, \frac{b}{a})$
(ii)	$y = \frac{x^2 + b}{x + a}$ $= (x - a) + \frac{b + a^2}{x + a}$ $x = -a$; $y = x - a$ are equations of the asymptotes.
(ii)	 **Note:** For this question there is no need to label the turning points as the question did not ask for them.
(iv)	$x^2 + b = (x + a)(kx - a)$ $\frac{x^2 + b}{x + a} = kx - a$ $\Rightarrow y = kx - a$ **Note:** $y = kx - a$ corresponds to the asymptote $y = x - a$ of the original curve with k changing the gradient of the line (y-intercept acts like a pivot) For no real roots, from graph, $0 < k \le 1$

Q8	Solution
(i)	Vertical asymptote: $x = 4 \Rightarrow c = 4$ When $x = 4$, $y = 6$, $6 = 4d + 2$ $d = 1 \Rightarrow a = 1$ $x + 2 + \frac{h}{x - 4} = \frac{x^2 + bx - 7}{x - 4}$ $\frac{(x + 2)(x - 4) + h}{x - 4} = \frac{x^2 + bx - 7}{x - 4}$ $x^2 + 2x - 4x - 8 + h = x^2 + bx - 7$ By comparing coefficient of x, $b = -2$

(ii)



(iii) $4 < k < 8$

$\{k \in \mathbb{R} : 4 < k < 8\}$

9	**Suggested Solutions**

(i)

$y^2 - 9(x-1)^2 = 9$

$\dfrac{y^2}{3^2} - \dfrac{(x-1)^2}{1^2} = 1$

Equations of asymptotes:

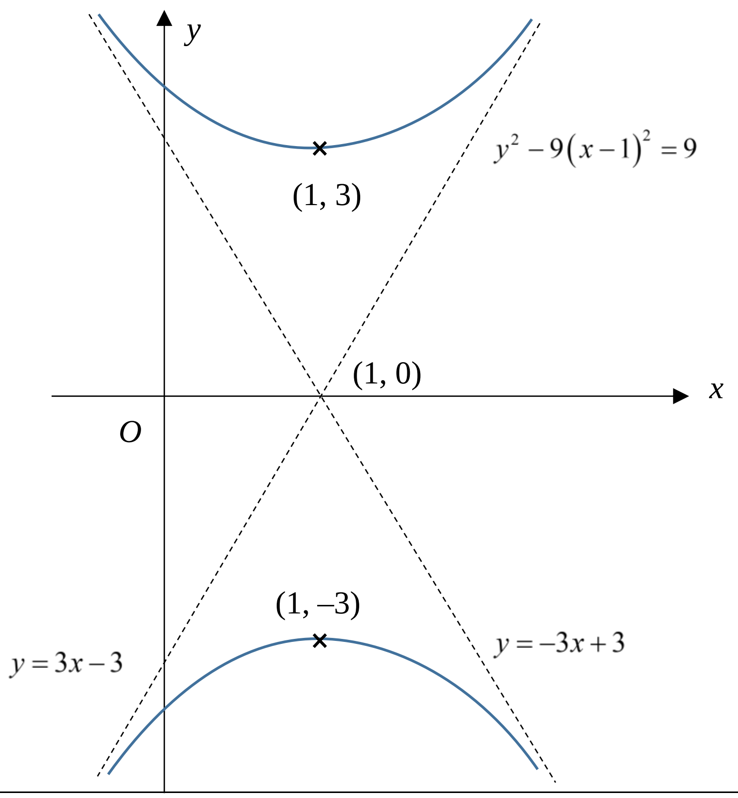
$\dfrac{y^2}{3^2} - (x-1)^2 = 0$

$\dfrac{y^2}{3^2} = (x-1)^2$

$y = \pm 3(x-1)$

$y = 3x - 3$

or $y = -3x + 3$



(ii) Since the gradients of the asymptotes are 3 and –3 respectively and the line $y = k(x-1)$ passes through the point (1,0), for $y = k(x-1)$ to intersect C at 2 points, set of values of k = $\{k \in \mathbb{R} : k > 3 \text{ or } k < -3\}$

(iii)	$\frac{(x-1)^2}{(2r)^2}+\frac{y^2}{r^2}=1$ is an ellipse centred at (1,0) with the top and bottom vertices at (1, r) and (1, – r) respectively. Hence, for it to intersect C at 4 distinct points, $r > 3$. **<u>Note:</u>** Students should think of **how to use the previous part** i.e. the sketch. The technique is to think of how to manipulate the expression to involve two graphs one of which has been sketched and the other to be inserted and conclude that we are looking at intersection points between the two graphs.