Quantum Physics I

1 Historical Background

Hundreds of years back, scientists were speculating about whether light was made up of particles or waves. By 1800s, it was generally accepted that light was a wave. The definitive feature of waves is their ability to diffract, suppose and produce interference patterns. Particles on the other hand, cannot superpose. Instead, they collide and bounce off each other.

In 1887 Hertz discovered the *photoelectric effect*. It was found that electrons, called *photoelectrons*, were emitted from a clean metal surface when high frequency light such as ultraviolet light fell on it. The experimental data could not be explained by treating light as a wave but they could be explained by treating light as particles.

Particles of light or packets of electromagnetic radiation are called *photons*. Each packet of energy or radiation is called a quantum. Since then, many quantities other than energy have been found to be quantised under certain conditions. This quantisation of physical quantities and the probabilistic nature of their measurements are two of the features of Quantum Physics that we will study.

Photon is a packet of electromagnetic radiation energy.

In Quantum Physics, many phenomena seem very bizarre. This is because quantum behaviour is mostly encountered in the microscopic realm, of which we would normally have very little experience of.

2 Photoelectric Effect

Photoelectric effect is the emission of electrons from *metals* by electromagnetic radiation of sufficiently high frequency.

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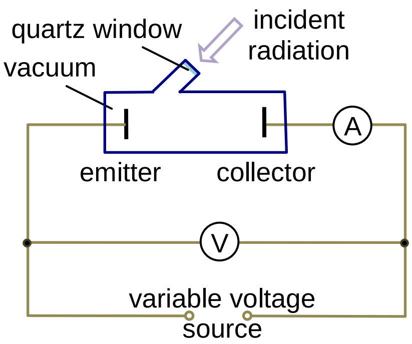

Fig. 2.1

The incident light falls on the emitter metal, ejecting electrons (*photoelectrons*), some of which are collected by the collector, giving rise to a current (*photoelectric current*).

The vacuum inside is needed to prevent formation of oxide layer on metal and to facilitate the movement of emitted electrons from emitter to collector.

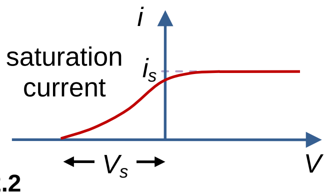

Fig. 2.2

A positive p.d is applied when collector is at higher potential while negative p.d. is when collector is at lower potential than emitter. The data in Fig. 2.2 is obtained when p.d. V is varied while light intensity is kept fixed.

A *positive* p.d creates an E-field that accelerates the electrons towards the collector. It also steers some electrons that would otherwise have collided with the casing and be absorbed towards the collector. When all the electrons reach the collector, the current is called *saturation current.*

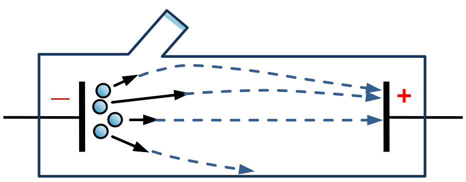

Fig. 2.3

Saturation current is the maximum photoelectric current which occurs when all the emitted electrons are able to reach the collector.

A *negative* p.d. causes some emitted electrons to slow down and even turn back to the emitter. Some fast enough electrons may still be able to reach the collector. A *more* negative p.d. will cause more electrons to get turned back and thus the current to decrease.

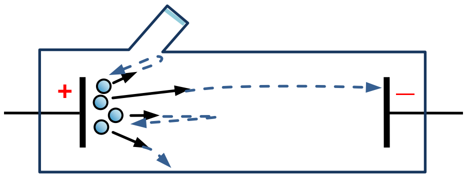

Fig. 2.4

Eventually, when p.d. is $-V_s$, the fastest electrons just fail to reach the collector and the current goes to zero. These fastest electrons lost all their KE just before reaching the collector and the lost KE becomes electric PE (from 'Electric Field', ΔEPE or $\Delta U_E = Q\Delta V$ where ΔV is p.d.). Thus

$$\frac{1}{2}m_e v_{max}^2 = eV_S$$

where m_e & e are electron mass and charge respectively, v_{max} is speed of the fastest electrons and V_s is the magnitude of the negative p.d. known as *stopping potential*.

Stopping potential is the *minimum* magnitude of the p.d. required to bring the photoelectric current to zero by *just* stopping all electrons from reaching the collector.

When electrons just fail to reach collector:

$$\frac{1}{2}m_e v_{max}^2 = eV_S$$

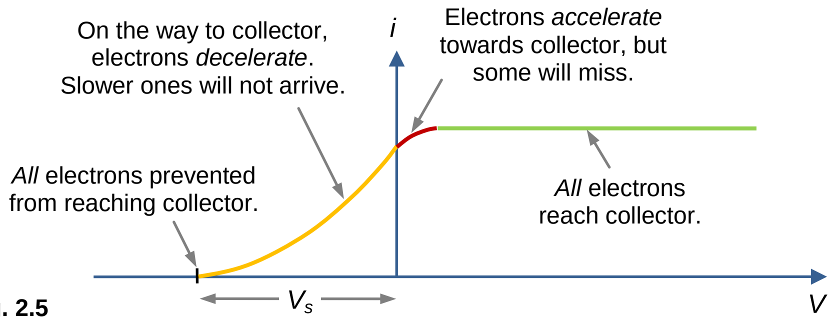

Fig. 2.5

More Data and Comparison of Wave and Particle Theories

1. The photoelectric current *i* is proportional to the intensity I of the light at a *fixed frequency* (Fig. 2.6).

Data 1
Photoelectric current $i \propto$ light intensity I at a *fixed frequency*.

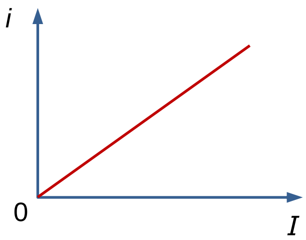

Fig. 2.6

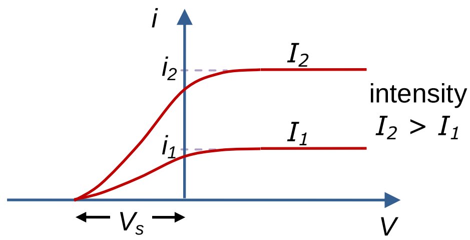

Fig. 2.7

Intensity is fundamentally the 'rate of energy transfer' per unit area. In our situation, it would be the rate of arrival of energy per unit metal surface area.

Intensity for light as wave is $I \propto a^2$

Intensity for light as photons is $I = E_p\left(\frac{N_p}{t}\right)\frac{1}{A}$ where $E_p = hf$

Wave Theory	Einstein's Particle Theory
For electromagnetic waves, intensity is proportional to the squared of the wave amplitude *a*. Greater intensity means greater oscillating E and B-field amplitudes. $I \propto a^2$	For a stream of particles or photons, intensity is logically proportional to the 'energy in each packet' E_p, and the 'rate of arrival of packets' N_p/t, per unit area: $I = E_p\left(\frac{N_p}{t}\right)\frac{1}{A}$
Both theories expect that if I increases, the rate of arrival of energy on metal increases. Hence more electrons per unit time can gain energy to escape. Thus current should increase as in Fig. 2.6 and 2.7.	

Both theories can explain Data 1

2. There is a *threshold frequency* or minimum frequency f_{min} of incident light for electrons to be emitted, regardless of intensity (Fig. 2.8).

3. Electrons are emitted with a range of KE and the maximum KE($=eV_s$) is dependent on the frequency but not intensity.

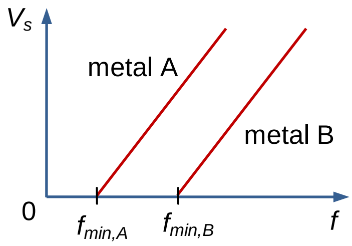

Fig. 2.8

For a given metal, regardless of intensity used, data in Fig. 2.8 show that when frequency of light is below f_{min}, no electrons are emitted hence $V_s = 0$.

Earlier, we saw that when electrons are emitted, they emerged with a range of KE and $KE_{max} = eV_s$. In Fig. 2.8, we see that for f *above* f_{min}, as f increases, the electrons emitted have greater KE_{max}. This should mean that the average KE of emitted electrons is greater.

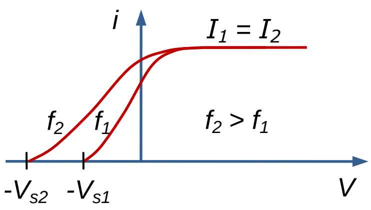

Fig. 2.9

The dependence of $KE_{max}(=eV_s)$ on f can also be seen in Fig. 2.9. The independence of V_s on intensity is shown in Fig. 2.7.

In short, whether electrons are emitted and also KE_{max} are determined by frequency of light.

Data 2
There is a *threshold frequency* or minimum frequency f_{min} of incident radiation for electrons to be emitted, regardless of the intensity of radiation.

Data 3
Electrons are emitted with a range of KE and the maximum KE($=eV_s$) is dependent on the frequency but not intensity.

Wave Theory	Einstein's Particle Theory
No matter how low I is, with enough time for electrons to absorb the wave energy, they should be ejected eventually. Hence wave theory expects electron emission to depend on I only. However, for electromagnetic waves, *I does not depend on frequency*. Thus, wave theory *cannot* explain why there exists a threshold frequency f_{min}. As for dependence of KE_{max} on frequency, wave theory similarly cannot account for it.	Einstein postulated that: 1 a photon's energy, $E_p = hf$ where Plank constant $h = 6.63 \times 10^{-34}$ J s. 2 an electron either absorb *all* the energy of a photon or *not at all*. 3 probability of electron absorbing more than one photon is negligible. 4 each metal has a *minimum* energy ϕ needed for electrons to escape from the metal's attraction. ϕ is called *work function* and it is *characteristic* of the metal.

Wave theory cannot but particle theory can explain Data 2 & 3.

A model and explanation based on Einstein's postulates:

Electrons in a metal are like tennis balls in a depression in the ground (Fig. 2.10). For a tennis ball to get out of the depression and thus further from Earth, it needs to overcome gravity and gain gravitational PE. Similarly, an escaping electron must overcome the metal's attraction and gain electric PE.

Using imaginary numbers, the topmost electron needs the least energy (work function ϕ) of **6 J** to get out. After absorbing **10 J** from a photon, 6 J is converted to electric PE, the rest becomes KE upon escape, hence:

$$KE_{max} = hf - \phi = 10 - 6 = 4 \text{ J}$$

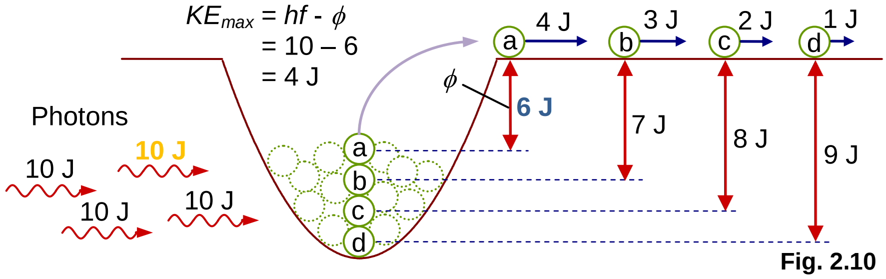

Fig. 2.10

Work function ϕ is the *minimum* energy required to remove an electron from a metal's surface. It is a characteristic property of a metal.

Einstein's Photoelectric Equation: $KE_{max} = hf - \phi$

It is important to note that electrons are emitted with a range of KE and Einstein's equation only deals with the maximum KE.

$$KE_{max} = hf - \phi$$

$$eV_s = hf - \phi$$

$$V_s = (h/e)f - \phi/e$$

As e, h and ϕ are constant, a graph of V_s against f will look like Fig. 2.11 where the y-intercepts are given by ϕ/e and the gradient is h/e.

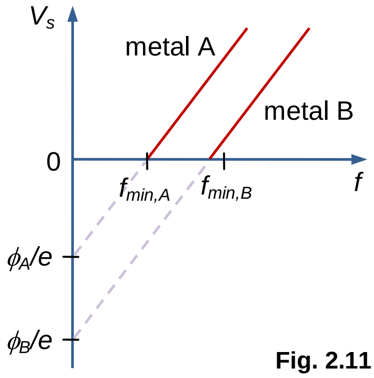

Fig. 2.11

Einstein's Photoelectric Equation:
$KE_{max} = hf - \phi$

4. When intensity is very low the emission occurs with no time delay provided frequency is above threshold value.

Wave Theory	Einstein's Particle Theory
Wave theory expects that when intensity is very low, it might take longer for electrons to gain enough energy to escape. But data shows that emission is immediate, whether the intensity is high or low, provided the frequency is above the threshold.	Intensity is $I = E_p\left(\frac{N_p}{t}\right)\frac{1}{A} = hf\left(\frac{N_p}{t}\right)\frac{1}{A}$ For a metal with given area A, the intensity is determined by the rate of arrival of photons N_p/t and the frequency f.

In section 4 later, we will see that lamps can produce light which has only a few discrete frequencies or a wide continuous range of frequencies, depending on the nature of the light producing material. Hence to change the frequency or frequencies of a source would normally mean changing the entire lamp. The way intensity is usually adjusted is by changing the voltage or current supplied to the lamp. This would change the rate of emission of photons but not change the frequencies produced by the lamp. If a lamp produces many frequencies, a filter is used to block the unwanted frequencies.

When intensity is reduced, the photons heading towards the metal are just as energetic as before ($E_p = hf$) but the rate of arrival N_p/t is reduced. Thus, an electron can still be emitted immediately provided that the packet of energy absorbed is enough for its escape. On the other hand, if the frequency is lower than f_{min}, a higher intensity is useless because each packet of energy is too little and the electrons have negligible chance of absorbing more than one photon simultaneously. If the energy absorbed is insufficient for electron escape, the energy will eventually be lost through collisions or emission.

$$\mathbf{hf} = \boldsymbol{\phi} + \mathbf{KE_{max}}$$

$$hf = \frac{hc}{\lambda}, \quad \phi = hf_{min} = \frac{hc}{\lambda_{max}}, \quad KE_{max} = \frac{1}{2}m_e v^2_{max} = eV_s$$

$$I = E_p\left(\frac{N_p}{t}\right)\frac{1}{A} = hf\left(\frac{N_p}{t}\right)\frac{1}{A}$$

Data 4
When frequency is above threshold, current is detected the moment radiation falls on the emitter, even when intensity is very low.

Wave theory expects a time delay before emission when intensity gets low but particle theory easily explains why no such delay is expected.

When intensity of a lamp is changed, there is no change to the frequencies emitted. Instead, the rate of emission and arrival of photons N_p/t is changed.

3 Wave-particle Duality

Electron Diffraction

Since electromagnetic waves can behave like particles, can the reverse be true? In 1924, Louis de Broglie (pronounced "de Broy") hypothesised that wavelength of particles is given by

$$\lambda = \frac{h}{p}$$

where λ is the *de Broglie wavelength* associated with the particle
p is the momentum mv of the particle
& h is the Planck constant

Two key properties of waves are diffraction and superposition. The proof that particles have wave properties should be found in diffraction or interference patterns. Indeed a few years after de Broglie's hypothesis, experiments done by Davisson, Germer and G.P. Thomson showed that electrons can produce diffraction patterns after passing through a thin film of material. Different patterns are formed for crystalline and polycrystalline materials.

Distinctive features of waves are 1) diffraction, 2) interference by supposition

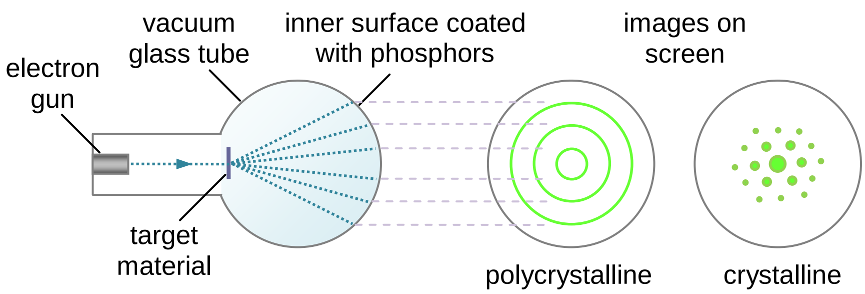

Fig. 3.1

Electron diffraction is evidence that particles have wave properties.

Before such electron diffraction patterns were observed, the same patterns had already been observed using X-rays, thus confirming that those electrons were behaving like waves as they pass through the target material.

Photoelectric effect is evidence that electromagnetic waves have particle properties.

The action of the target materials was similar to that of diffraction gratings. If a calculation were made for the de Broglie's wavelength of the electrons in the beam, you would get a value of ~10^{-10} m which is the typical spacing between atoms in crystals. This closeness of the wavelength to the atomic spacing is important for diffraction to be observable.

Wave-particle Duality

We see that *photoelectric effect* led to the conclusion that electromagnetic waves can behave like particles while *electron diffraction* showed that electrons can have wave properties. Since those experiments, more experiments have confirmed that all matter have both wave and particle properties. Furthermore, in any setting, you will only observe one of the two natures but never both *simultaneously*. It is rather like the two sides of a coin; you can only see one side at a time and whichever side you look at, it is the same coin. This dual nature of light is known as a 'wave particle duality'.

Wave-particle duality is the concept that *all matter* & *EM waves* have both wave and particle properties. But only one property can be observed at a time.

De Broglie's equation is not just for finding the *wavelength* of a particle of *known momentum mv*. The same equation is also used to find the *momentum* of a packet of wave or photon of *known wavelength* λ. However, in the latter case, it is important to note that p of a photon *cannot* be written as mc because photons are massless. These massless photons when absorbed or reflected by a surface can exert a force on the surface just like balls hitting the surface.

$\lambda = \frac{h}{p}$	gives wavelength λ of a particle of momentum mv

$p = \frac{h}{\lambda}$	gives momentum p ($\neq mc$) of a photon of wavelength λ

$\lambda = \frac{h}{p}$ gives wavelength of particles with mass while $p = \frac{h}{\lambda}$ gives momentum of waves.

Another experiment that will reveal the particle nature of light is to have a very sensitive light sensor detect the light from a source. In Fig. 3.2, when intensity is high, the reading displayed by the sensor is a fairly steady value but when the intensity is turned all the way down, the sensor starts to pick up pulses corresponding to individual photons.

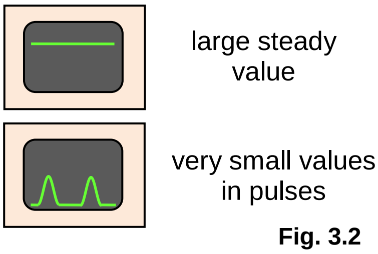

Fig. 3.2

Further evidence for photons is the detection of a single photon by a sensor.

Another experiment that reveals the wave property of particles such as protons and atoms is the double slit set-up. When these particles are fired towards the double slit one at a time, and the positions of impact are recorded on a sensor (Fig. 3.3a), the final pattern after a long time is as shown in Fig. 3.3b.

Further evidence of wave property of particles is the formation of interference pattern in a double-slit set-up even when particles are fired one at a time.

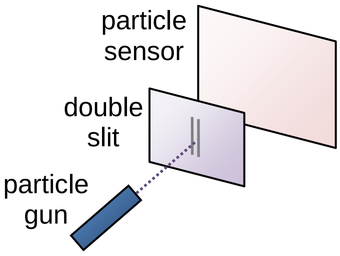

Fig. 3.3a

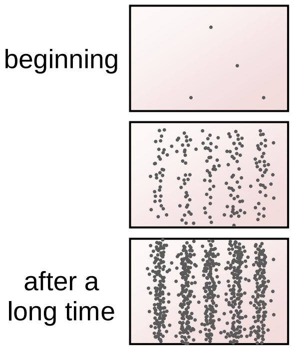

Fig. 3.3b

The result shows that the wave of each particle interferes with itself after passing through the double slit.

4 Spectra

What is a Spectrum?

An electromagnetic(EM) *spectrum* (plural: spectra) is an arrangement of EM radiation separated according to wavelength or frequency. Common set-ups to separate the different wavelengths involve the use of a slit with prism (Fig. 4.1) or diffraction grating (Fig. 4.2).

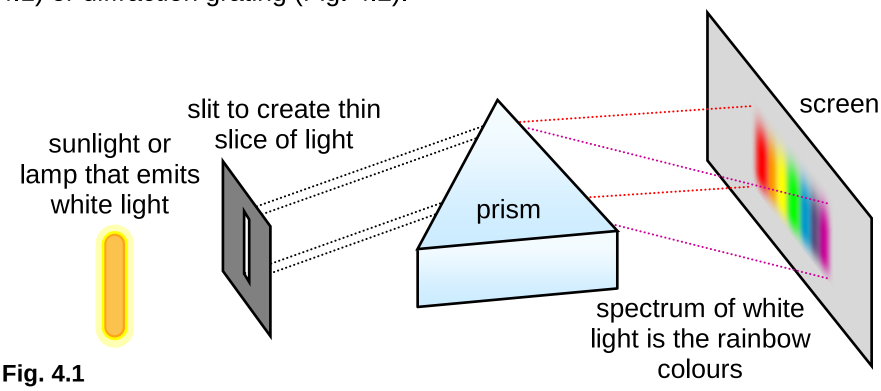

Fig. 4.1

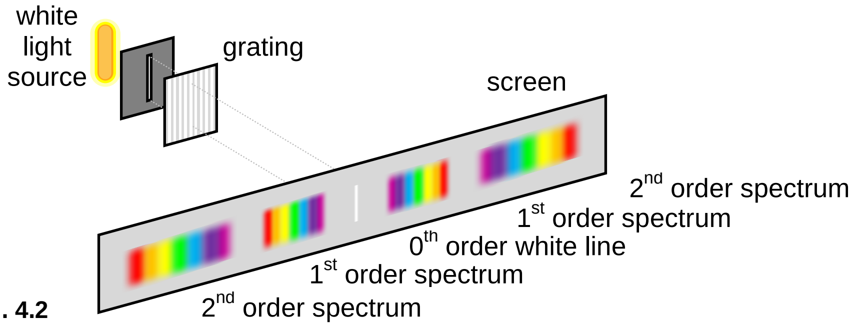

Fig. 4.2

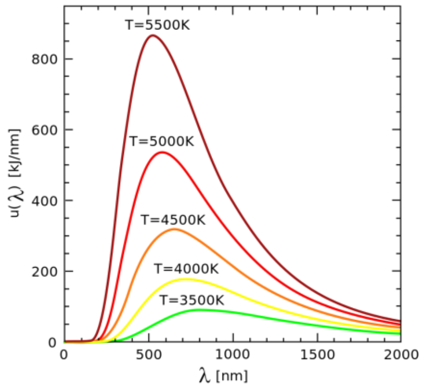

Fig. 4.3 © 4C / Wikimedia Commons / CC-BY-SA-3.0 / GFDL

A spectrum need not be visible like the examples above. The whole EM spectrum consists of many invisible wavelengths like those in the ultra-violet and radio wave regions. Hence sensors that can measure the intensity of the invisible EM waves might be used to measure the amount of radiation at each wavelength and the result plotted on a graph like in Fig. 4.3. Such a graph is also called a spectrum.

Emission Spectrum

All objects with temperatures above absolute zero emit EM waves but we typically do not see most of it. Fig. 4.3 actually shows that as the temperature of an object increases from 3 500 K to 5 500 K, the dominant wavelength in the radiation shifts from about 800 nm(red-yellow hot) to about 500 nm(blue-white hot). Also, generally *solids* and *liquids* emit EM radiations which span a continuous range or spectrum like in Fig. 4.3. Such spectra are called *continuous emission spectra*.

Emission spectra are produced when emitted light is separated according to λ or *f*.

Emission line spectrum is produced when a hot gas emits light with a finite set of frequencies which get separated according to λ or *f*.

Hot gases emit radiations with a finite number of wavelengths, including invisible ones. The set-up in Fig. 4.4 can reveal the visible part of the discrete spectrum. If the slit were replaced by a hole, the screen would have 3 dots instead of 3 lines.

Emission spectra can be *continuous*(Fig. 4.1 & 4.2) or *discrete*(Fig. 4.4 & 4.5).

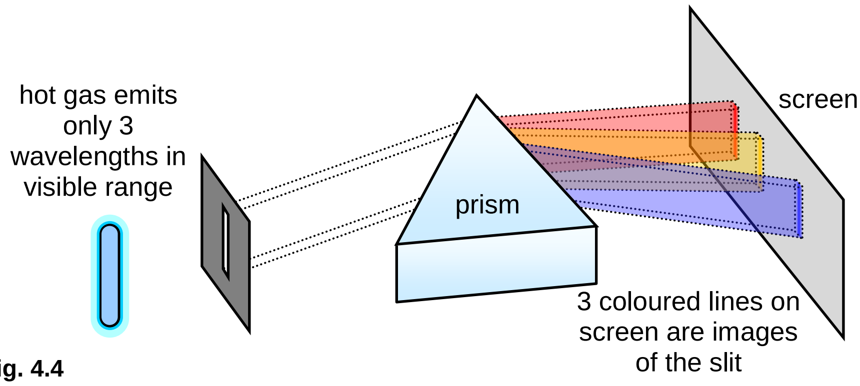

Fig. 4.4

Other than directly heating the gas, it can also be made to emit light by trapping it in a glass discharge tube and then applying a high voltage to it.

The key to how a spectrum is formed is the grating which separates light into different angles of deviation according to λ or *f*. Greater λ leads to greater θ.

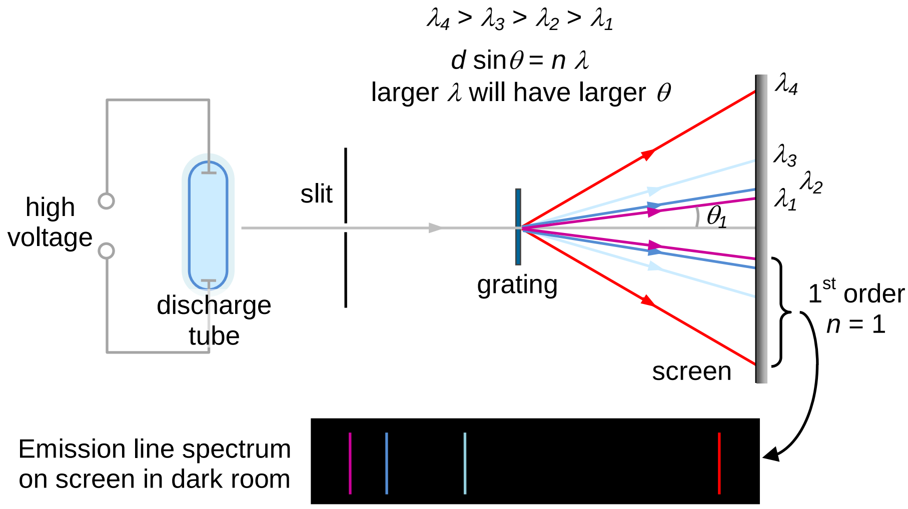

Fig. 4.5

Emission line spectra from different gases are different in terms of the set of frequencies and the line separations. Each spectrum is like a unique *fingerprint* of the gas. This uniqueness is used to identify the kind of atoms or molecules, and the process is called spectroscopy. Spectroscopy allows us to determine the composition of distant stars, including the sun.

Absorption Spectrum

Absorption spectrum is formed when *white* light passes through a *cool* gas and the transmitted light is separated according to wavelength or frequency. *White* light is not light that looks white but rather it contains all the visible wavelengths. Hot solids and liquids produce white light. An example is the light from a hot tungsten filament. The light looks orange-yellow because it has more radiation of that colour compared to other colours. A mixture of red, green and blue light will appear white to our eyes but a grating can be used to show that the light has only 3 and not a continuous range of colours.

Fig. 4.6 shows a set-up for absorption spectra. White light passes through the cool gas and some frequencies are absorbed. Hence light reaching the screen has all colours except those absorbed. Different cool gases absorb different frequencies just as different hot gases emit different frequencies.

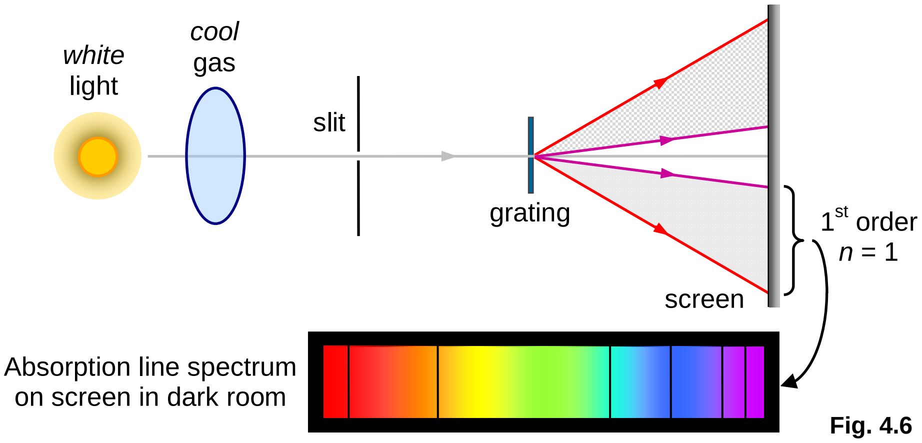

Fig. 4.6

Absorption line spectrum is produced when *white* light passing through a *cool* gas has some of its frequencies absorbed before being separated according to λ or f.

Both emission line spectra and absorption line spectra have *unique* fingerprint like patterns which correspond to *specific* kinds of gas atoms or molecules.

Comparison of 3 Types of Spectrum

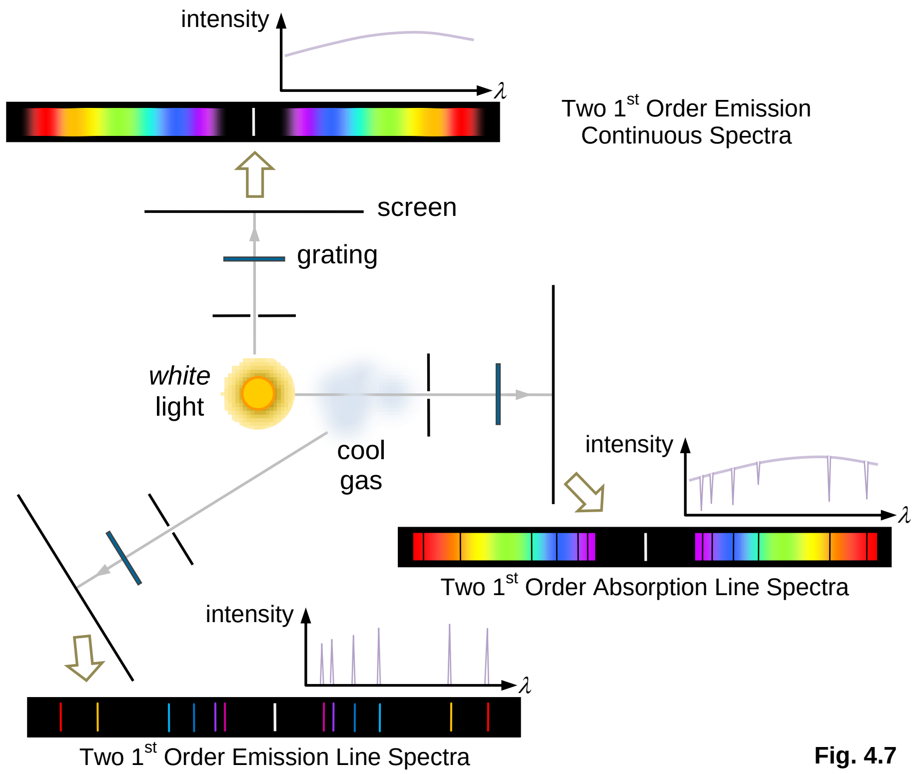

Fig. 4.7

Notice that the absorption lines are not completely dark because the absorbed photons are re-emitted and some of them will reach the screen on the right.

5 Energy Levels of Atoms

The existence of a unique spectrum for each gas could not be explained by classical physics (in contrast to modern physics which includes relativity and quantum theory). In 1913, Niels Bohr proposed a model for the hydrogen atom that not only accounted for the presence of spectral lines, but predicted their wavelengths to a great degree of accuracy.

Bohr's Theory

Bohr's Theory was based on the following 2 ideas:

1 Electrons in an atom can only move in certain allowed orbits or radii.
2 Energy is radiated or absorbed when an electron switch orbits.

An electron orbiting a positive nucleus will have KE and electric PE. The total energy would get greater when the electron is found in orbits further away from the nucleus in a similar way that satellites have greater total energy when orbiting further from Earth.

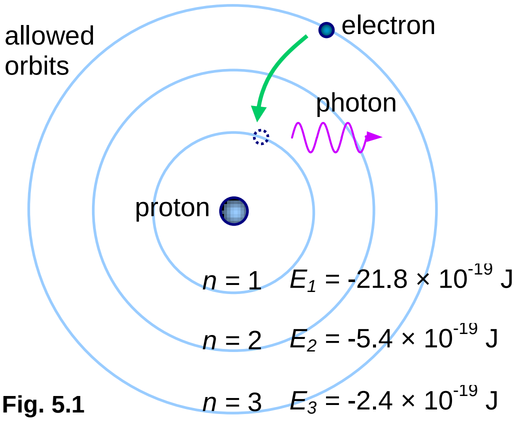

Fig. 5.1

Consider the hydrogen atom in Fig. 5.1. As the electron can only revolve the nucleus in allowed orbits, it must make a so called quantum jump to get to another orbit. When it jumps to an orbit nearer the nucleus, it loses energy in the form of a photon. It can also absorb energy to jump from a lower energy orbit to a higher energy orbit.

In Bohr's theory, electrons in an atom have only certain allowed orbits that corresponded to discrete atomic energy levels. For a photon to be emitted or absorbed, its energy must be equal to $| E_f - E_i |$.

The lowest energy level is assigned the quantum number n = 1. When the atom is in this lowest energy state, the atom or the electron is said to be in a *ground state*. By the principle of conservation of energy, the energy of the emitted or absorbed photon E_p (= $hf = hc/\lambda$) must be given by

$$E_p = | E_f - E_i |$$

where E_f is the final energy level while E_i is the initial energy level. It is important to note that the discrete energy *levels* are characteristic of atoms which are isolated. When the atoms are squeezed closely in a liquid or solid, they no longer have discrete energy levels. Later in 'Lasers and Semiconductors', you will see that solids have energy *bands* instead.

There are more details to Bohr's atomic model but it was incomplete. Further development of quantum theory revealed that orbiting electrons behave more like waves than particles and they do not follow precise orbits.

Excitation of Atom

The energy levels mentioned are actually allowed energies of the electron *and* nucleus. However, it is common to see the electron as the one changing orbits or states while the nucleus stays put. Hence, it is common to associate the different energy values with the electron only and speak as if the energies are the electron's.

The atom's energy is often considered as the electron's. *Excitation* refers to an atom or electron gaining energy to reach a higher energy level.

Normally, the atom exists in the ground state as higher energy states are unstable. When an atom absorbs energy and the electron jumps to a higher energy level, we say that the atom or electron has been '*excited* to a higher energy level'. If the electron jumps to the zero energy level, it means it has effectively been removed from the nucleus (recall a similar situation in gravitational systems) where the electric attraction is negligible. The energy needed to remove the electron is called the *ionisation energy*.

An electron or atom will only remain in an excited state for a very short time (in the order of 10^{-10} s). Then it will spontaneously fall back to the ground state either in a single jump or multiple jumps giving off one or more photons.

Generally, an atom can gain energy in 2 ways –

1. Absorb a photon of *almost exactly* the right amount of energy to jump to a higher level. Hence if the photon has energy slightly less or more than the exact amount needed, it will not be absorbed. Also, the probability of the atom absorbing 2 photons to make the jump is negligible.
2. Through collision with another particle. In this case, the other colliding particle transfers a portion of its KE to the atom to allow it to jump to a higher energy state.

Excited states are usually very unstable and so are followed by de-excitation.

Two ways to gain energy:
1. Absorption of photon whose energy *matches* difference in energy levels.
2. By collision with another particle which transfers *part* of its energy.

From Energy Levels to Spectrum

Hydrogen atom is the simplest atom as it is made up of a single electron orbiting a single proton. As such, for a basic understanding, we always start with the hydrogen atom. We will see that the *unique finite set of discrete energy levels* determine the *unique positioning* and *spacing of spectral lines*.

The energy levels of an *isolated* hydrogen atom are shown below. As the energies are rather small in joules, it is more common to use the unit eV. 1 eV is the KE that an electron gains when it is accelerated across a p.d. of 1 V. As $U_E = QV$, $\Delta KE = \Delta U_E = e\Delta V = (1.6 \times 10^{-19}\text{ C})(1\text{ V}) = 1.6 \times 10^{-19}$ J.

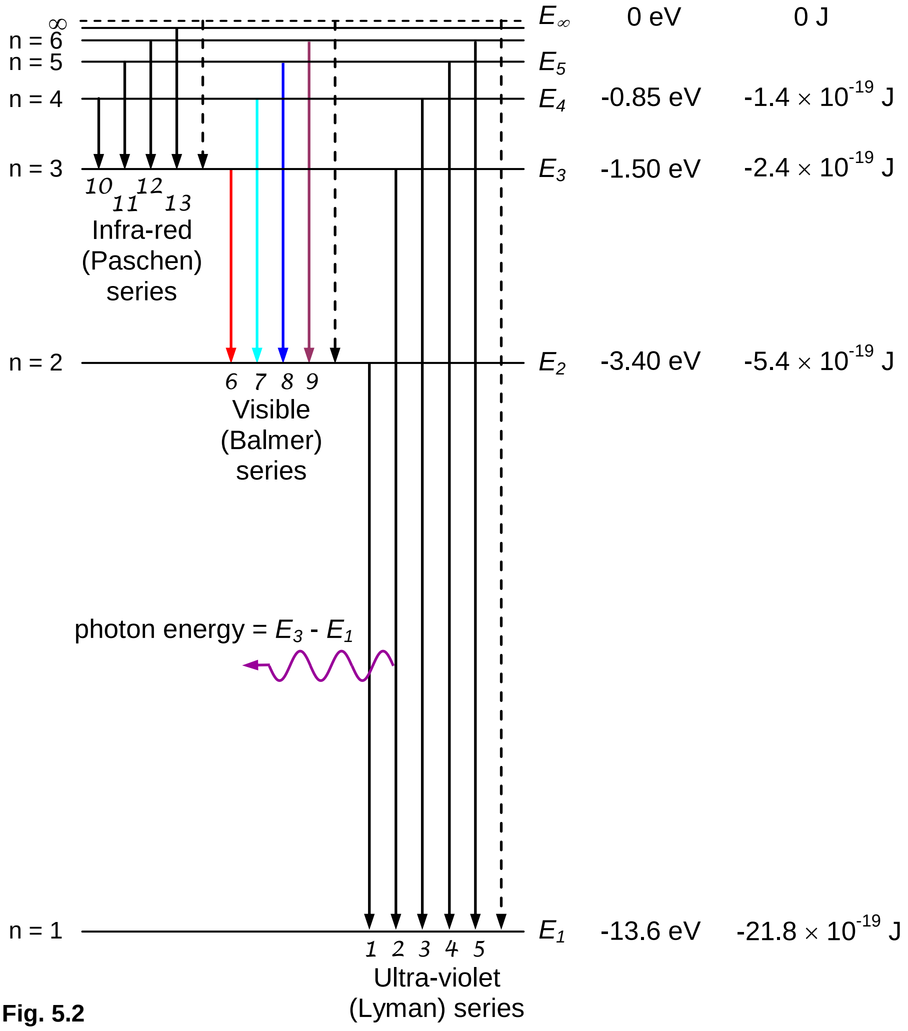

Fig. 5.2

Ground state is the lowest energy level where n = 1**.**

Excited states are all energy states higher than ground state.

Transitions are jumps in the energy of the electron. Jumps to a lower energy level or orbit nearer to the proton are represented by downward arrows. Upward arrows represent jumps to higher energy levels.

A *series* is a group of transitions between a given level and all higher levels.

Ionisation energy or energy to remove an electron from an atom is 13.6 eV for hydrogen.

In Fig. 5.3, with all the arrows lined up, we can more easily compare the photon energies($E_p = hf$) and hence their frequencies. On the screen where frequency increases to the right, each arrow's length gives an *approximate* indication of its position on the screen.

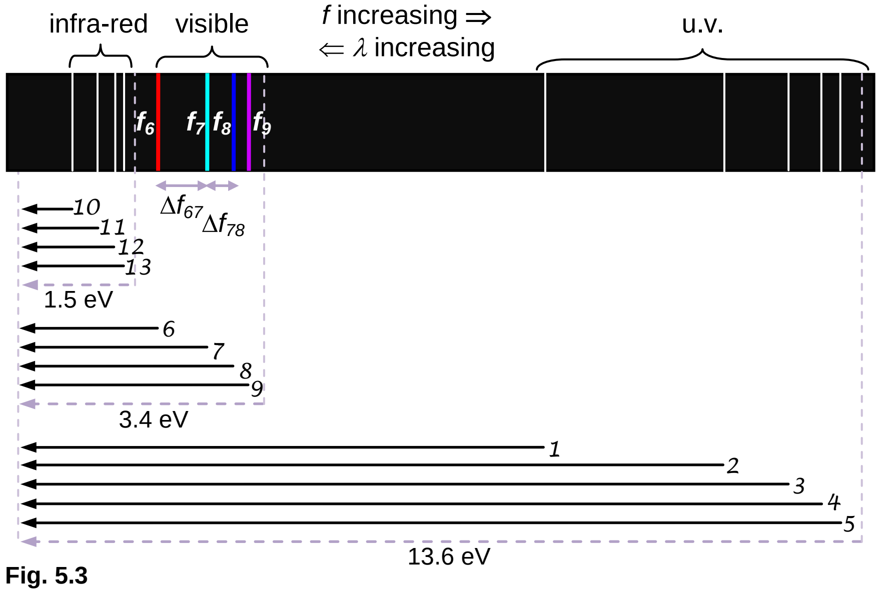

Fig. 5.3

Within each series, the spacing of spectral lines gets smaller towards the higher frequency end.

Quantisation of Atom's Energy

An electron orbiting a nucleus can be treated as a wave along the orbit. An allowed orbit must have a certain length that supports the formation of a standing wave by constructive interference. Disallowed orbits are those where the electron's wave will interfere destructively. For an orbit to support a standing wave, the orbital length must be an integer number n of wavelengths i.e. $2\pi r = n\lambda$. This condition leads to quantisation of the atom's energy.

The reason for Bohr's *allowed orbits* is due to the requirement that in such orbits, electron's wave must interfere constructively to give a standing wave.

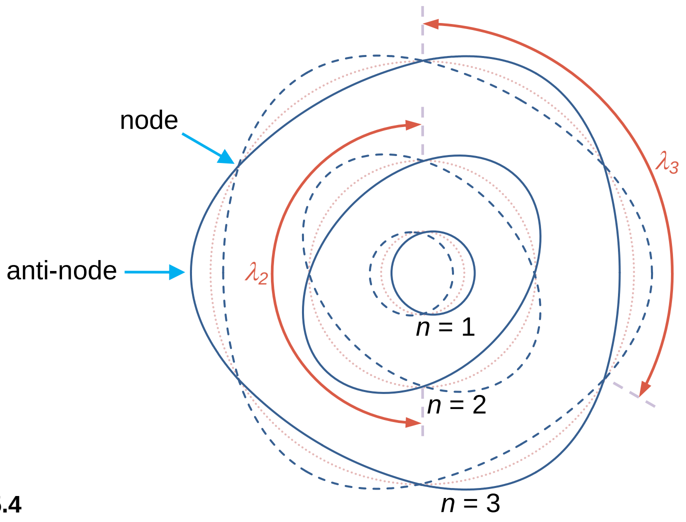

Fig. 5.4

An excellent flash animation of Fig. 5.4 is available at:
http://www.upscale.utoronto.ca/PVB/Harrison/Flash/QuantumMechanics/CircularStandWaves/CircularStandWaves.html