Check your Understanding (Normal Distribution)

Section 1: Probability

1.	Drill & Practice
	Given that $X \sim \mathrm{N}\left(120, 2^2\right)$. Find (i) $\mathrm{P}\left(X > 119\right)$ (ii) $\mathrm{P}\left(X < 125\right)$ (iii) $\mathrm{P}(118 \le X < 121)$
	Answers (i) 0.691 (ii) 0.215 (iii) 0.533
(i)	$\mathrm{P}\left(X > 119\right) = 0.691$]
(ii)	$\mathrm{P}\left(X < 125\right) = 0.994$
(iii)	$\mathrm{P}(118 \le X < 121) = 0.533$

2.	YJC Promo 8865/2018/Q7
	In a national park, it is known that the lengths of garden snails are normally distributed with mean 3.42 cm and standard deviation 1.10 cm. It is found that 70% of them measure less than 4.0 cm and 80% measure at least 2.5 cm. (ii) Find the probability that the length of a randomly selected garden snail is within $\pm$ 0.3 cm of the mean length. [2]
	Answers (ii) 0.215
	YJC Promo 8865/2018/Q7
(ii)	$G \sim \mathrm{N}\left(3.42,\ 1.10^2\right)$ or $G \sim \mathrm{N}\left(3.42417,\ 1.09808^2\right)$ $\mathrm{P}\left(-0.3 < G - \mu < 0.3\right)$ $= \left(3.42 - 0.3 < G < 3.42 + 0.3\right)$ ≈ 0.214937 [or 0.21530] ≈ 0.215 (to 3 s.f.)

3. JPJC JC1 Promo 8865/2019/Q7

A butcher sells chicken in two different cuts, leg and wing. The masses, in kilograms, of these two cuts have independent normal distributions. The means and standard deviations of these distributions, and the selling prices, in $ per kilogram, are shown in the following table.

	Mean	Standard deviation	Selling price
Leg	0.35	0.010	12
Wing	0.09	0.003	9

Stating clearly the mean and variance of all distributions that you use, find the probability that

(i) the mass of a randomly chosen wing is within ± 0.005 kg of the mean mass of wings, [2]

(ii) out of 3 randomly chosen wings, exactly one is less than 0.088 kg, exactly one is more than 0.09 kg and exactly one is between 0.088 kg and 0.09 kg, [3]

JPJC JC1 Promo 8865/2019/Q7 (Solutions)

Let L and W denote the mass of a leg and wing respectively.

$L \sim N(0.35, 0.010^2)$

$W \sim N(0.09, 0.003^2)$

(i) $\mathrm{P}(0.085 < W < 0.095) = 0.90442 \approx 0.904$

(ii) Required probability

$= \mathrm{P}(W < 0.088) \times \mathrm{P}(W > 0.09) \times \mathrm{P}(0.088 < W < 0.09) \times 3!$

$= (0.25249) \times (0.5) \times (0.24751) \times 3!$

$= 0.18748 \approx 0.187$

4. MI Prelim 8865/2018/Q9

In this question you should state clearly the values of the parameters of any normal distribution you use.

In a particular stall, the masses, in kilograms, of a particular type of fish called groupers have a mean mass of 0.5 kg and standard deviation 0.0182 kg.

(ii) Find the probability that the mass of a randomly chosen grouper is within 0.01 kg of the mean mass of groupers in the stall. [2]

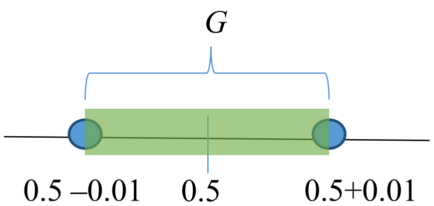
Answers

(ii) 0.417

MI Prelim 8865/2018/Q9

(ii) Let G be the random variable denoting the mass, in grams, of a randomly selected groupers.

$G \sim \mathrm{N}\left(0.5,\ 0.0182^2\right)$



$\mathrm{P}\left(0.5-0.01 < G < 0.5+0.01\right) = \mathrm{P}\left(0.49 < G < 0.51\right)$

$= 0.417$ (3 sf)

5. PJC Prelim 8865/2018/Q12

The masses of oranges sold by a supermarket have a normal distribution. The mean and standard deviation of the distribution is 0.175 kg and 0.09 kg respectively.

Find the probability that an orange chosen at random has mass

(i) at most 0.12 kg, [1]

(ii)	within 0.05 kg of the mean.	[2]

Answers
(i) 0.271 (ii) 0.421

PJC Prelim 8865/2018/Q12

Let R be the random variable denoting the mass, in grams, of a randomly selected orange.

$R \sim \mathrm{N}\left(0.175, 0.09^2\right)$

(ii) $\mathrm{P}\left(R \le 0.12\right) = 0.27056 \approx 0.271$

(ii) $\mathrm{P}\left(-0.05 < R - 0.175 < 0.05\right)$

$= \mathrm{P}\left(0.125 < R < 0.225\right)$

$= 0.42149 \approx 0.421$

6. CJC Prelim 8865/2018/Q9(i)

Kickers chocolates are sold in tins of 5 chocolates. The masses, in grams, of the individual Kickers chocolates and the empty tins have independent normal distributions with means and standard deviations as shown in the following table.

	Mean	Standard Deviation
Individual Kickers Chocolate	53	2.8
Empty Tin	15	0.4

(i) Find the probability that two randomly chosen Kickers chocolate each weigh more than 50 grams. [1]

Answers
(i) 0.736

CJC Prelim 8865/2018/Q9

(i) Let K be the random variable denoting the mass, in grams, of a randomly selected Kickers chocolate.

$\therefore K \sim \mathrm{N}\left(53,\ 2.8^2\right)$

$\mathrm{P}\left(K_1 > 50\right) \times \mathrm{P}\left(K_2 > 50\right) = 0.7361838758$

$\approx 0.736 \quad \left(\text{to 3 s.f.}\right)$

7. HCI Prelim 8865/2018/Q12

A supermarket sells two types of strawberries, A and B. Each type of strawberry comes in different sized packaging. The masses, in kg, of a packet of type A strawberries and a packet of type B strawberries are modelled as having independent normal distributions with means and standard deviations as shown in the table.

Strawberries	**Mean**	**Standard deviation**
Type *A*	1.2	0.2
Type *B*	1.1	0.1

Type A strawberries are sold at \$20 per kg and type B strawberries at \$26 per kg.

Audrey picks one packet of type *A* strawberries and one packet of type *B* strawberries.

(i) Find the probability that none of the two packets picked by Audrey weighs less than 1.2 kg. [2]

Answers
(i) 0.0793

HCI Prelim 8865/2018/Q12

Let X and Y be the weight (in kg) of a packet of type *A* and type *B* strawberries respectively.

$$X \sim \mathrm{N}\left(1.2, 0.2^2\right)$$

$$Y \sim \mathrm{N}\left(1.1, 0.1^2\right)$$

$\mathrm{P}\left(X \geq 1.2\right)\mathrm{P}\left(Y \geq 1.2\right) = 0.0793$

8. NJC Prelim 8865/2018/Q11

In this question you should state clearly the values of the parameters of any normal distribution you use.

A supermarket sells two types of durians, Red Prawn and Black Gold. The masses, in kilograms, of the durians each have independent normal distributions. The means and standard deviations of these distributions, and the selling prices, in \$ per kilogram, are shown in the following table.

	Mean mass (kg)	Standard deviation (kg)	Selling price ($ per kg)
Red Prawn	0.25	0.02	1.50
Black Gold	0.35	0.03	2.40

(ii) Three Red Prawn durians are randomly selected. Find the probability that exactly one of the durians has mass less than 0.24 kg and exactly one of the durians has mass more than 0.26 kg. [2]

Answers
(ii) 0.219

NJC Prelim 8865/2018/Q11

Let A be the random variable denoting the mass of a Red Prawn durian.

$A \sim \mathrm{N}\left(0.25, 0.02^2\right)$

(ii) $\mathrm{P}\left(A < 0.24\right)\mathrm{P}\left(0.24 \leq A \leq 0.26\right)\mathrm{P}\left(A \geq 0.26\right) \times 3! = 0.219$

Section 2: Use of Inverse Normal

1. Drill & Practice

Given that $X \sim \mathrm{N}\left(120, 2^2\right)$. Find

(i) $\mathrm{P}\left(X < x\right) = 0.5$

(ii) $\mathrm{P}\left(X > x\right) = 0.5$

	(iii) $\mathrm{P}\left(X > x\right) = 0.7$ (iv) $\mathrm{P}(x_1 < X \le x_2) = 0.77$, such that x_1 and x_2 are symmetrical about the mean. (v) $\mathrm{P}(x < X \le 120) = 0.2$

Answers
(i) 120 (ii) 120 (iii) 119 (iv) 117, 122 (v) 119

(i)	$\mathrm{P}\left(X < x\right) = 0.5$ Using GC, $x = 120$
(ii)	$\mathrm{P}\left(X > x\right) = 0.5$ Using GC, $x = 120$
(iii)	$\mathrm{P}\left(X > x\right) = 0.7$ Using GC, $x = 119$
(iv)	$\mathrm{P}(x_1 < X \le x_2) = 0.77$ Using GC, $x_1 = 117, x_2 = 122$
(v)	$\mathrm{P}(x < X \le 120) = 0.2$ $\mathrm{P}\left(X \le 120\right) - \mathrm{P}\left(X \le x\right) = 0.2$ $\mathrm{P}\left(X \le x\right) = \mathrm{P}\left(X \le 120\right) - 0.2$ $= 0.5 - 0.2 = 0.3$ Using GC, $x = 119$

2.	**AJC Prelim 8865/2018/Q12(i)**	
	The weights of the lemons sold on the market stall are normally distributed with mean weight 0.1 kg and standard deviation 0.05 kg. **(i)** Find the value that is exceeded by 75% of the weights of the lemons.	[1]

Answers
$a = 0.0663$

AJC Prelim 8865/2018/Q12

	Let L be the random variable denoting the weight, in kilograms, of a randomly selected lemon. $L \sim \mathrm{N}\left(0.1, 0.05^2\right)$ $\mathrm{P}\left(L > a\right) = 0.75 \Rightarrow a = 0.0663$	

3. CJC Prelim 8865/2018/Q9(iii)

The masses, in grams, of the individual Venus chocolates have a normal distribution with mean 35 grams and standard deviation σ grams. It is given that 85% of Venus chocolates weigh more than 34 grams.

(iii) Find σ, giving your answer correct to 4 decimal places. [3]

Answers
(iii) 0.9648

CJC Prelim 8865/2018/Q9

(iii)	Let V be the random variable denoting the mass, in grams, of a randomly selected Venus chocolate.

$\therefore V \sim \mathrm{N}\left(35,\ \sigma^2\right)$

$\mathrm{P}(V > 34) = 0.85$

$\mathrm{P}(V < 34) = 0.15$

$\mathrm{P}\left(Z < \frac{34-35}{\sigma}\right) = 0.15$

$\mathrm{P}\left(Z < \frac{-1}{\sigma}\right) = 0.15$

$\frac{-1}{\sigma} = -1.03643338$

$\sigma = 0.9648473501 \approx 0.9648 \ \left(\text{to 4 d.p.}\right)$

4. RVHS Prelim 8865/2018/Q11

Gary likes to take part in duathlons where there is a total running distance of 15 km and a total cycling distance of 36 km. His timings, in minutes, for running and cycling are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean	Standard Deviation
Timing for running (min)	100	15
Timing for cycling (min)	μ	σ

(i) The probability that he completes his cycling in less than an hour is equal to the probability that he completes his cycling after 2 hours. Write down the value of μ. [1]

(ii) The probability that he completes his cycling under 80 minutes is 0.158655. Show that σ = 10. [2]

Answers

(i) $\mu = 90$ minutes

RVHS Prelim 8865/2018/Q11

(i) $\mu = \frac{60+120}{2} = 90$ minutes

(ii) Let C be Gary's timing for cycling.

$C \sim N\left(90, \sigma^2\right)$

$Z = \frac{C-90}{\sigma} \sim N(0,1)$

$\mathrm{P}(C < 80) = 0.158655$

$\mathrm{P}\left(Z < \frac{80-90}{\sigma}\right) = 0.158655$

$\frac{80-90}{\sigma} = -1.000001057$

$\sigma = \frac{-10}{-1.000001057} \approx 10$

5.	**DHS Prelim 8865/2018/Q8**

An interactive simulation ride allows a group of 5 riders to take the ride at a time. The ride time, X minutes, follows a normal distribution with mean μ minutes and standard deviation 2 minutes.

(i) Show that $\mu = 14$, correct to the nearest integer, if $\mathrm{P}(\mu < X < 16) = 0.35$. [2]

Answers

DHS Prelim 8865/2018/Q8

(i)

$X \sim \mathrm{N}(\mu, 2^2)$

$\mathrm{P}(\mu < X < 16) = 0.35$

$\mathrm{P}(X < 16) - \mathrm{P}(X < \mu) = 0.35$

$\mathrm{P}(X < 16) = 0.85$

$\mathrm{P}\left(Z < \dfrac{16-\mu}{2}\right) = 0.85$

Using GC,

$\dfrac{16-\mu}{2} = 1.0364$

$\mu = 13.927 \approx 14$

6.	**HCI Prelim 8865/2018/Q12(ii)**

A supermarket sells two types of strawberries, A and B. Each type of strawberry comes in different sized packaging. The masses, in kg, of a packet of type A strawberries and a packet of type B strawberries are modelled as having independent normal distributions with means and standard deviations as shown in the table.

Strawberries	**Mean**	**Standard deviation**
Type *A*	1.2	0.2
Type *B*	1.1	0.1

Type A strawberries are sold at \$20 per kg and type B strawberries at \$26 per kg.

Audrey picks one packet of type A strawberries and one packet of type B strawberries.

(ii) The probability that the total weight of the two packets chosen by Audrey lies between 2.2 kg and m kg is 0.45, where $m > 2.2$. Find the value of m. [3]

Answers
(ii) 2.47

HCI Prelim 8865/2018/Q12

Let X and Y be the weight (in kg) of a packet of type A and type B strawberries respectively.

$X \sim \mathrm{N}(1.2, 0.2^2)$

$Y \sim \mathrm{N}(1.1, 0.1^2)$

	Let $T = X + Y \sim \mathrm{N}\left(1.2+1.1, 0.2^2+0.1^2\right)$ $T \sim \mathrm{N}\left(2.3, 0.05\right)$ $\mathrm{P}\left(T < 2.2\right) = 0.32736$ $\mathrm{P}\left(2.2 < T < m\right) = 0.45$ $\mathrm{P}\left(T < m\right) - \mathrm{P}\left(T < 2.2\right) = 0.45$ $\mathrm{P}\left(T < m\right) = 0.32736 + 0.45 = 0.77736$ $\therefore m = 2.47$

7.	**HCI Prelim 8865/2018/Q7**
	A factory manufactures paperweights consisting of glass mounted on a wooden base. The volume of glass, in cm^3, in a randomly chosen paperweight has a normal distribution with mean 56.5 and standard deviation 2.9 and the volume of wood, in cm^3, has an independent normal distribution with mean 38.4 and standard deviation σ. The probability that the total volume of a randomly chosen paperweight exceeds 100 cm^3 is 0.05. Find the value of σ. [4]
	Answers 1.10

HCI Prelim 8865/2018/Q7	
	Let X cm^3 and Y cm^3 be the volumes of glass and wood in a paperweight respectively. $X \sim \mathrm{N}(56.5, 2.9^2)$ $Y \sim \mathrm{N}(38.4, \sigma^2)$. $\therefore X + Y \sim \mathrm{N}(94.9, 2.9^2 + \sigma^2)$ $\mathrm{P}(X + Y > 100) = 0.05$ $\Rightarrow \mathrm{P}(X + Y \le 100) = 0.95$ $\Rightarrow \mathrm{P}\left(Z \le \dfrac{100 - 94.9}{\sqrt{2.9^2 + \sigma^2}}\right) = 0.95$ $\Rightarrow \dfrac{5.1}{\sqrt{2.9^2 + \sigma^2}} = 1.64485$ $\Rightarrow 2.9^2 + \sigma^2 = \left(\dfrac{5.1}{1.64485}\right)^2$ $\Rightarrow \sigma = \sqrt{1.20363} = 1.09710 = 1.10$ (to 3 sf)

Section 3: Use of standardisation

1.	**AJC Prelim 8865/2018/Q6**	
	A random variable X is normally distributed with a mean of μ and a variance of σ^2. Given that $\mathrm{P}\left(X > 2\right) = 0.1$ and that $\mathrm{P}\left(X < 1.75\right) = 0.18$, find $\mathrm{P}\left(X > 1.90\right)$.	[4]
		Answers

$\mathrm{P}(X > 1.90) = 0.344$

AJC Prelim 8865/2018/Q6

$X \sim \mathrm{N}(\mu, \sigma^2)$

Given $\mathrm{P}(X > 2) = 0.1$

Standardising, $\mathrm{P}\left(Z > \dfrac{2-\mu}{\sigma}\right) = 0.1$

$$\frac{2-\mu}{\sigma} = 1.2816 \Rightarrow \mu + 1.2816\sigma = 2 \quad (1)$$

$\mathrm{P}(X < 1.75) = 0.18$

Standardising, $\mathrm{P}\left(Z < \dfrac{1.75-\mu}{\sigma}\right) = 0.18$

$$\frac{1.75-\mu}{\sigma} = -0.91537 \Rightarrow \mu - 0.91537\sigma = 1.75 \quad (2)$$

Solving (1) & (2): $\mu = 1.8542$ and $\sigma = 0.11379$

$\mathrm{P}(X > 1.90) = 0.344$

2.	**ACJC Prelim 8865/2018/Q6**
	Over a long period of time, a slimming centre found that at the end of a slimming programme, 20% of their clients lose more than 20 kg and 5% of their clients lose less than 10 kg. By modelling the weight loss of the slimming centre's clients to be a normal distribution, find the mean and variance of the distribution. [4]

Answers
$\sigma = 4.02,\ \mu = 16.6$

ACJC Prelim 8865/2018/Q6

Let X be the random variable denoting amount of weight loss (in kg) of a customer at the end of a slimming programme.

Given $X \sim N(\mu, \sigma^2)$.

Given $P(X > 20) = 0.2$ and $P(X < 10) = 0.05$

$P(X > 20) = 0.2$

$\Leftrightarrow P(Z > \dfrac{20-\mu}{\sigma}) = 0.2$

$\Leftrightarrow \dfrac{20-\mu}{\sigma} = 0.8416$

$\Leftrightarrow 20 - \mu = 0.8416\sigma$

$\Leftrightarrow \mu + 0.8416\sigma = 20 \ \cdots (1)$

	$P(X<10)=0.05$ $\Leftrightarrow P(Z>\frac{10-\mu}{\sigma})=0.05$ $\Leftrightarrow \frac{10-\mu}{\sigma}=-1.645$ $\Leftrightarrow 10-\mu=-1.645\sigma$ $\Leftrightarrow \mu-1.645\sigma=10 \ \cdots(2)$ Solving (1) and (2), $\sigma=4.02$ (3 sig. fig.) $\mu=16.6$ (3 sig. fig.)

3. MJC Prelim 8865/2018/Q7

An egg wholesaler packs their chicken eggs according to their weights. The chicken eggs are weighed and classified according to the table below.

Criteria	Classification of egg
Weight of chicken egg more than 65 grams	Premium
Weight of chicken egg less than 48 grams	Small

A large batch of randomly chosen chicken eggs are weighed and classified accordingly and it is found that 12% are premium and 6% are small. Assuming a normal distribution, find the mean weight and standard deviation of a randomly chosen chicken egg. [5]

The wholesaler also distributes ostrich eggs for sale.
Explain whether or not a normal model is likely to be appropriate for the weight of an egg chosen at random from the combined group of chicken eggs and ostrich eggs. [1]

Answers
(i) $\sigma \approx 6.23$; $\mu \approx 57.7$

MJC Prelim 8865/2018/Q7

(i)	Let X be the random variable denoting the weight in grams of a randomly chosen egg. $X \sim \mathrm{N}\left(\mu,\sigma^2\right)$ $\mathrm{P}(X>65)=0.12 \Rightarrow \mathrm{P}\left(Z>\frac{65-\mu}{\sigma}\right)=0.12$ $\Rightarrow \frac{65-\mu}{\sigma}=1.174987$ $65-\mu=1.174987\sigma$ $\mu+1.174987\sigma=65 -------- (1)$ $\mathrm{P}(X<48)=0.06 \Rightarrow \mathrm{P}\left(Z<\frac{48-\mu}{\sigma}\right)=0.06$

	$\Rightarrow \dfrac{48-\mu}{\sigma} = -1.554774$ $48-\mu = -1.554774\sigma$ $\mu - 1.554774\sigma = 48 ------- (2)$ Solving using GC $\sigma = 6.2277 \approx 6.23$ $\mu = 57.683 \approx 57.7$
	A normal model is not appropriate because when we combine both groups with different distributions, it may result in a bi-modal distribution (i.e. with two modes) which contrasts with a normal distribution that has only one mode.

4.	**EJC Prelim 8865/2018/Q6**
	The Body Mass Index (BMI) is a measure of body fat based on height and weight that applies to men and women. In a certain country, a person with a BMI of 25 and above is deemed to be overweight. Anyone with a BMI above 30 is considered obese. The Health Promotion Board recorded the BMI of a large number of adult males. The results show that 36.2% of them are overweight and 5.5% of them are obese. Assuming that the BMI of adult males are normally distributed, find the mean and variance of the distribution. [4]
	Answers $\mu = 23.582 \approx 23.6, \quad \sigma = 4.0158 \approx 4.02 \quad \sigma^2 = 16.127 \approx 16.1$

EJC Prelim 8865/2018/Q6	
Let X be the random variable denoting the Body Mass Index of a randomly selected adult male $X \sim \mathrm{N}(\mu, \sigma^2)$. Given: $\mathrm{P}(X \geq 25) = 0.362$ $\mathrm{P}\left(Z \geq \dfrac{25-\mu}{\sigma}\right) = 0.362$ $\therefore \dfrac{25-\mu}{\sigma} \approx 0.353118$ $25 \approx \mu + 0.353118\sigma$ -----(1) Given: $\mathrm{P}(X > 30) = 0.055$ $\mathrm{P}\left(Z > \dfrac{30-\mu}{\sigma}\right) = 0.055$ $\therefore \dfrac{30-\mu}{\sigma} \approx 1.59819$ $30 \approx \mu + 1.59819\sigma$ ---(2) Solving (1) and (2): $\mu = 23.582 \approx 23.6$ $\sigma = 4.0158 \approx 4.02$ $\sigma^2 = 16.127 \approx 16.1$	

Section 4: Comparing Probability of Sum and Probability of Individual

1. JJC Prelim 8865/2018/Q11

(a) The masses, in kg, of red and green apples sold by a supermarket have independent normal distributions with means and standard deviations as shown in the following table.

	Mean	Standard Deviation
Red Apples	0.18	0.029
Green Apples	0.13	0.015

(i) Find the probability that the mass of a randomly chosen red apple is less than 0.2 kg. [1]

(ii) Two red apples are chosen at random. Find the probability that each one has a mass of more than 0.2 kg. [2]

(iii) Find the probability that the total mass of two randomly chosen red apples is more than 0.4 kg. Explain why this value is greater than your answer in part **(ii)**. [3]

Answers
(a)(i) 0.755 (ii) 00601 (iii) 0.165

JJC Prelim 8865/2018/Q11

(a)(i) Let X be the mass of a randomly chosen red apple sold by the supermarket in kg.

$$X \sim \mathrm{N}\left(0.18, 0.029^2\right)$$

$$\mathrm{P}\left(X < 0.2\right) = 0.75479 = 0.755 \text{ (3 s.f.)}$$

(ii)
Required probability

$$= \mathrm{P}\left(X_1 > 0.2\right)\mathrm{P}\left(X_2 > 0.2\right)$$

$$= \left[\mathrm{P}\left(X > 0.2\right)\right]^2$$

$$= \left(1 - 0.75479\right)^2$$

$$= 0.06128 = 0.0601 \text{ (3 s.f.)}$$

(iii) $X_1 + X_2 \sim \mathrm{N}\left(2\times 0.18, 2\times 0.029^2\right) = \mathrm{N}\left(0.36, 0.001682\right)$

$$\mathrm{P}\left(X_1 + X_2 > 0.4\right) = 0.16470 = 0.165 \text{ (3 s.f.)}$$

Possible explanation for answer in (iii) to be greater than the answer in (ii):

- Different Normal distribution in which combinations of masses adding to more than 0.4 are included, not just 0.2 twice.
- The event $\left\{X_1 > 0.2 \text{ and } X_2 > 0.2\right\}$ is a proper subset of $\left\{X_1 + X_2 > 0.4\right\}$
- The event depicted in (ii) is a subset of the event depicted in (iii).

2.	**MJC Prelim 8865/2018/Q12**

Brandon can choose to take the subway or taxi to and fro between his home and office. The one-way journey times, in minutes, by taxi and by subway have independent normal distributions. The means and standard deviations of these distributions are shown in the following table.

	Mean	Standard Deviation
Taxi	59	2
Subway	61	3

(i) Find the probability that a randomly chosen taxi journey takes less than an hour. [1]

(ii) Find the probability that two randomly chosen taxi journeys take more than an hour each. [2]

(iii) The probability that the total journey time taken for a randomly chosen trip to and fro between home and office by taxi is more than two hours in total is denoted by p. Without calculating its value, explain why p will be greater than your answer in part **(ii)**. [1]

Answers
(i) 0.691 (ii) 0.0952

MJC Prelim 8865/2018/Q12

(i)	Let X and Y be the journey times by a randomly chosen taxi and a randomly chosen subway respectively (in minutes). $X \sim \mathrm{N}\left(59, 2^2\right)$ $Y \sim \mathrm{N}\left(61, 3^2\right)$ $\mathrm{P}\left(X < 60\right) = 0.69146 \approx 0.691$ (3s.f.)
(ii)	$\left[\mathrm{P}\left(X > 60\right)\right]^2 = \left(1 - 0.69146\right)^2 = 0.095195 \approx 0.0952$ (3s.f.)
(iii)	The event in **(ii)** is a proper subset of the event in **(iii)**, thus p is greater than the answer in part **(ii)**.

3.	**RVHS Prelim 8865/2018/Q11**

Gary likes to take part in duathlons where there is a total running distance of 15 km and a total cycling distance of 36 km. His timings, in minutes, for running and cycling are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean	Standard Deviation
Timing for running (min)	100	15
Timing for cycling (min)	90	10

(iv) Find the probability that his timing for cycling is less than 80 minutes and his timing for running is less than 90 minutes. [2]

(v) Show that the probability of Gary finishing the duathlon under 170 minutes is 0.134. Give a reason why this probability is greater than the probability calculated in part **(iv)**. [2]

Answers
(iv) 0.0401 (v) 0.134

RVHS Prelim 8865/2018/Q11

	Let C be Gary's timing for cycling. $C \sim N\left(90, 10^2\right)$ Let R be Gary's timing for running. $R \sim N\left(100, 15^2\right)$

(iv)	Required probability $=\mathrm{P}(C<80)\times\mathrm{P}(R<90)$ $=0.0400592579$ ≈ 0.0401 (3 sig fig)
(v)	$C+R\sim N(190,325)$ $\mathrm{P}(C+R<170)=0.1336287896\approx 0.134$ (3 sig fig)(shown) The event in part (iv) is but a proper subset of that in this part, hence the probability is here is greater than that in (iv).

4.	**IJC Prelim 8865/2018/Q12**
	Potatoes and carrots are sold by weight in a market. The masses, in kilograms (kg), of potatoes and carrots are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean	Standard Deviation
Potatoes	0.165	0.015
Carrots	0.072	0.012

Potatoes cost \$2.50 per kilogram and carrots cost \$1.80 per kilogram.

(ii) Two potatoes are randomly chosen. Find the probability that each of them cost more than \$0.40. [3]

(iii) Without any calculation, explain why the probability that the total cost of two randomly chosen potatoes is more than \$0.80 will be greater than your answer to part **(ii)**. [1]

Answers
(ii) 0.398

	IJC Prelim 8865/2018/Q12
	Let X g be the mass of a randomly chosen potato. $X\sim\mathrm{N}(0.165,0.015^2)$ Y g be the mass of a randomly chosen carrot. $Y\sim\mathrm{N}(0.072,0.012^2)$
(ii)	Let A be the cost of a randomly chosen potato. $A=2.5X$ $\mathrm{E}(A)=2.5(0.165)=0.4125$ $\mathrm{Var}(A)=2.5^2(0.015^2)=0.00140625$ $A\sim\mathrm{N}(0.4125,0.00140625)$

	Required probability $=\left[\mathrm{P}\left(A>0.4\right)\right]^2=0.398$ (3 s.f) Or use mass to find the answer. $P\left(X>\dfrac{0.4}{2.5}\right)$
(iii)	Part **(iii)** answer is greater since **(ii)** is a subset of the possible cases in **(iii).** For eg, the total cost of 2 potatoes can still be more than \$0.80 when one of them is less than \$0.40 (eg. \$0.30) and the other is more than \$0.40 (eg. \$0.60).

5. EJC Prelim 8865/2018/Q11

A telecommunication company provides mobile services for its customers to make calls to City J and City C. The duration of calls, in minutes, made by its customers to City J and City C have independent normal distributions. The means and standard deviations of these distributions are shown in the following table.

	Mean	Standard deviation
City J	8	1.5
City C	10	1.8

(i) Find the probability that a randomly chosen call to City J takes more than 11 minutes. [1]

(ii) Find the probability that three randomly chosen calls to City J each takes less than 11 minutes. [2]

(iii) The probability that the total time for three randomly chosen calls to City J is less than 33 minutes is denoted by p. Without calculating its value, explain why p will be greater than your answer to part **(ii)**. [1]

Answers
0.0228 (ii) 0.933

EJC Prelim 8865/2018/Q11

Let X and Y denote the duration of a randomly chosen call to City J and City C respectively.

$$X \sim N\left(8, 1.5^2\right) \qquad Y \sim N\left(10, 1.8^2\right)$$

(i) $\mathrm{P}\left(X>11\right)=0.02275=0.0228$ (3sf)

(ii) Required Probability = $\left[\mathrm{P}\left(X<11\right)\right]^3=\left(1-0.02275\right)^3=0.933$ (3sf)

(iii) The cases in part (ii) is a subset of the cases in part (iii). Therefore, p is greater.

Section 5: Use of Linear Combinations of Normal Distributions

1. CJC Prelim 8865/2018/Q9

Kickers chocolates are sold in tins of 5 chocolates. The masses, in grams, of the individual Kickers chocolates and the empty tins have independent normal distributions with means and standard deviations as shown in the following table.

	Mean	Standard Deviation

Individual Kickers Chocolate	53	2.8
Empty Tin	15	0.4

(ii) Find the probability that the total mass of a tin containing 5 Kickers chocolates is less than 275 grams. State the mean and variance of the distribution that you use. [3]

The masses, in grams, of the individual Venus chocolates have a normal distribution with mean 35 grams and standard deviation 0.9648 grams.

The cost of producing Kickers and Venus chocolates is 2 cents per gram and 3 cents per gram respectively.

(iv) Find the probability that the cost of producing a Kickers chocolate is within 10 cents of the cost of producing a Venus chocolate. State an assumption needed for your calculation. [4]

Answers
(ii) 0.213 (iv) 0.883

CJC Prelim 8865/2018/Q9

(ii) Let T be the random variable denoting the mass, in grams, of a randomly selected empty tin.

$\therefore T \sim \mathrm{N}\left(15,\ 0.4^2\right)$

$K_1+K_2+K_3+K_4+K_5+T \sim \mathrm{N}\left(5\times 53+15,\ 5\times 2.8^2+0.4^2\right)$

$K_1+K_2+K_3+K_4+K_5+T \sim \mathrm{N}\left(280,\ 39.36\right)$

$\mathrm{P}\left(K_1+K_2+K_3+K_4+K_5+T<275\right)=0.2127339029$

$\approx 0.213 \quad \left(\text{to 3 s.f.}\right)$

(iv) $2K \sim \mathrm{N}\left(2\times 53,\ 2^2\times 2.8^2\right)$

$2K \sim \mathrm{N}\left(106,\ 31.36\right)$

$3V \sim \mathrm{N}\left(3\times 35,\ 3^2\times 0.9648473501^2\right)$

$3V \sim \mathrm{N}\left(105,\ 8.378373681\right)$

$2K-3V \sim \mathrm{N}\left(106-105,\ 31.36+8.378373681\right)$

$2K-3V \sim \mathrm{N}\left(1,\ 39.73837368\right)$

$\mathrm{P}\left(-10<2K-3V<10\right)=0.8828159632\approx 0.883 \quad \left(\text{to 3 s.f.}\right)$

The mass of the Kickers chocolates is independent of the mass of the Venus chocolates.

2. **AJC Prelim 8865/2018/Q12**

The weights of the oranges sold on a market stall are normally distributed with mean 0.4 kg and standard deviation 0.06 kg. The weights of the lemons sold on the market stall are normally distributed with mean weight 0.1 kg and standard deviation 0.05 kg.

State clearly the mean and variance of the distribution that you use in all the questions below.

(i) Ann buys 1 orange and 1 lemon. Calculate the probability that the weight of her orange is more than three times the weight of her lemon. [3]

The selling price of the oranges and lemons are \$2.40 per kilogram and \$1.50 per kilogram respectively.

Candy buys 7 oranges and 20 lemons.

(ii) Find the probability that the total weight of the oranges differ from the total weight of the lemons by less than 1 kg. [3]

(iii) Find the probability that Candy has to pay more than \$10 for her purchase. [3]

Answers
(i) 0.732 (ii) 0.767 (iii) 0.291

AJC Prelim 8865/2018/Q12

Let G = weight, in kg, of an orange $\sim \mathrm{N}\left(0.4, 0.06^2\right)$

L = weight, in kg, of a lemon $\sim \mathrm{N}\left(0.1, 0.05^2\right)$

(i) $3L \sim \mathrm{N}\left(3\times 0.1,\ 3^2\times 0.05^2\right)=\mathrm{N}\left(0.3,\ 0.0225\right)$

$\mathrm{P}\left(G>3L\right)=\mathrm{P}\left(G-3L>0\right)=0.732$ where $G-3L\sim \mathrm{N}\left(0.1,\ 0.0261\right)$

(ii) Let $X=\sum_{i=1}^{7}G_i\sim \mathrm{N}\left(7\times 0.4,\ 7\times 0.06^2\right)=\mathrm{N}\left(2.8,\ 0.0252\right)$

$Y=\sum_{i=1}^{20}L_i\sim \mathrm{N}\left(20\times 0.1,\ 20\times 0.05^2\right)=\mathrm{N}\left(2,\ 0.05\right)$

$\mathrm{P}\left(-1<X-Y<1\right)=0.767$

where $X-Y\sim \mathrm{N}\left(0.8,\ 0.0752\right)$

(iii) Let A = price of 7 oranges

$= 2.4X$

$\sim \mathrm{N}\left(2.4\times 2.8,\ 2.4^2\times 0.0252\right)=\mathrm{N}\left(6.72,\ 0.145152\right)$

Let B = price of 20 lemons

$= 1.5Y$

$\sim \mathrm{N}\left(1.5\times 2,\ 1.5^2\times 0.05\right)$ i.e. $\mathrm{N}\left(3,\ 0.1125\right)$

Therefore, $A+B\sim \mathrm{N}\left(9.72,\ 0.257652\right)$

$\mathrm{P}\left(A+B>10\right)=0.291$

3. TPJC Prelim 8865/2018/Q10

The masses, in kg, of two types of oranges, A and B, sold by a supermarket have independent normal distributions with means and standard deviations as shown in the following table.

	Mean	Standard deviation

Type A	0.26	0.04
Type B	0.16	0.02

It is found that 40% of oranges of type A have a mass less than 0.25 kg.
4 oranges of type A and 3 oranges of type B are chosen at random.

(iv) Find the probability that the total mass of 4 oranges of type A is at least 0.6 kg more than the total mass of 3 oranges of type B. [4]

Oranges of type A cost $4.50 per kg and oranges of type B cost $5 per kg.

(v) Find the probability that the total cost of 4 oranges of type A and 3 oranges of type B is between $6.50 and $7.50. State the mean and variance of the distribution that you use.[4]

Answers
(iv) 0.323 (v) 0.780

TPJC Prelim 8865/2018/Q10

(iv) Let A and B be the mass of oranges of type A and type B, in kg, respectively

$A \sim \mathrm{N}(0.26, 0.04^2)$

$B \sim \mathrm{N}(0.16, 0.02^2)$

$(A_1 + A_2 + A_3 + A_4) - (B_1 + B_2 + B_3) \sim \mathrm{N}(4(0.26) - 3(0.16), 4(0.04^2) + 3(0.02^2))$

$(A_1 + A_2 + A_3 + A_4) - (B_1 + B_2 + B_3) \sim \mathrm{N}(0.56, 0.0076)$

$\mathrm{P}((A_1 + A_2 + A_3 + A_4) \geq (B_1 + B_2 + B_3) + 0.6)$

$= \mathrm{P}((A_1 + A_2 + A_3 + A_4) - (B_1 + B_2 + B_3) \geq 0.6)$

= 0.323 (to 3 s.f)

(v) $4.5(A_1 + A_2 + A_3 + A_4) + 5(B_1 + B_2 + B_3) \sim \mathrm{N}(7.08, 0.1596)$

$\mathrm{P}(6.50 < 4.5(A_1 + A_2 + A_3 + A_4) + 5(B_1 + B_2 + B_3) < 7.50)$

= 0.780 (to 3 s.f)

4. PJC Prelim 8865/2018/Q12

The masses of oranges sold by a supermarket have a normal distribution. The mean and standard deviation of the distribution is 0.175 kg and 0.09 kg respectively.
The oranges are randomly packed into “small” and “large” bags. Each small bag contains 5 oranges and each large bag contains 15 oranges.

(iii) Find the probability that the mass of a randomly chosen large bag exceeds the mass of a randomly chosen small bag by less than 2 kg. [5]

The supermarket sells the oranges in small bags and large bags at $4.10 per kg and $3.80 per kg respectively. Annie buys 4 small bags and 2 large bags of oranges.

(iv) Find the least amount of money, to the nearest dollar, that Annie has to bring to the supermarket so that she is at least 99% sure that she has enough money to pay for the oranges. [5]

Answers
(iii) 0.733 (iv) $41

PJC Prelim 8865/2018/Q12

R ~ mass of a randomly chosen orange

$R \sim \mathrm{N}(0.175, 0.09^2)$

(iii) $S = R_1 + R_2 + R_3 + R_4 + R_5$

$\mathrm{E}(S) = \mathrm{E}(R_1 + R_2 + R_3 + R_4 + R_5) = 5\mathrm{E}(R) = 5(0.175) = 0.875$

$\mathrm{Var}(S) = \mathrm{Var}(R_1 + R_2 + R_3 + R_4 + R_5) = 5\mathrm{Var}(R) = 5(0.09^2) = 0.0405$

$S \sim \mathrm{N}(0.875, 0.0405)$

$L = R_1 + R_2 + ... + R_{15}$

$\mathrm{E}(L) = \mathrm{E}(R_1 + R_2 + ... + R_{15}) = 15\mathrm{E}(R) = 15(0.175) = 2.625$

$\mathrm{Var}(L) = \mathrm{Var}(R_1 + R_2 + ... + R_{15}) = 15\mathrm{Var}(R) = 15(0.09^2) = 0.1215$

$L \sim \mathrm{N}(2.625, 0.1215)$

$X = L - S$

$\mathrm{E}(X) = \mathrm{E}(L - S) = \mathrm{E}(L) - \mathrm{E}(S) = 2.625 - 0.875 = 1.75$

$\mathrm{Var}(X) = \mathrm{Var}(L - S) = \mathrm{Var}(L) + \mathrm{Var}(S) = 0.1215 + 0.0405 = 0.162$

$X \sim \mathrm{N}(1.75, 0.162)$

$\mathrm{P}(X < 2) = 0.73274 \approx 0.733$

(iv) Let $C = 4.10(S_1 + S_2 + S_3 + S_4) + 3.80(L_1 + L_2)$

$$\begin{aligned}\mathrm{E}(C) &= \mathrm{E}\left(4.10(S_1 + S_2 + S_3 + S_4) + 3.80(L_1 + L_2)\right) \\ &= 4.10(4)\mathrm{E}(S) + 3.80(2)\mathrm{E}(L) \\ &= 4.10(4)(0.875) + 3.80(2)(2.625) \\ &= 34.3\end{aligned}$$

$$\begin{aligned}\mathrm{Var}(C) &= \mathrm{Var}\left(4.10(S_1 + S_2 + S_3 + S_4) + 3.80(L_1 + L_2)\right) \\ &= 4.10^2(4)\mathrm{Var}(S) + 3.80^2(2)\mathrm{Var}(L) \\ &= 4.10^2(4)(0.0405) + 3.80^2(2)(0.1215) \\ &= 6.2321\end{aligned}$$

$C \sim \mathrm{N}(34.3, 6.2321)$

Let a be the amount of money that Annie brings to the supermarket

Given

$\mathrm{P}(C \le a) \ge 0.99$

$a \ge 40.108$

Therefore, the least amount of money Annie has to bring is \$41.

5.	**MI Prelim 8865/2018/Q9 (iii)**
	In this question you should state clearly the values of the parameters of any normal distribution you use.

In a particular stall, the masses, in kilograms, of a particular type of fish called groupers have a mean mass of 0.5 kg and standard deviation 0.0182 kg.

It is also given that the masses, in kilograms, of mud crabs have the distribution $\mathrm{N}\left(0.35,\ 0.01^2\right)$.

Groupers and mud crabs are sold by weight in the stall. Groupers are sold at \$24 per kilogram and mud crabs are sold at \$28 per kilogram.

(iii) Find the probability that the total cost of two randomly chosen groupers and three randomly chosen mud crabs is at most \$55.
State an assumption needed for your calculations. [5]

Answers
(iii) 0.979

MI Prelim 8865/2018/Q9

Let G be the mass, in kg, of a randomly chosen grouper.

$G \sim \mathrm{N}\left(0.5,\ 0.0182^2\right)$

Let M be the mass, in kg, of a randomly chosen mud crab.

$M \sim \mathrm{N}\left(0.35,\ 0.01^2\right)$

Let $T = 24\left(G_1 + G_2\right) + 28\left(M_1 + M_2 + M_3\right)$

$\mathrm{E}\left(T\right) = 24\left(0.5\right)\left(2\right) + 28\left(0.35\right)\left(3\right) = 53.4$

$\mathrm{Var}\left(T\right) = 24^2\left(0.0182^2\right)\left(2\right) + 28^2\left(0.01^2\right)\left(3\right) = 0.61678$

$T \sim \mathrm{N}\left(53.4, 0.61678\right)$

$\mathrm{P}\left(T \le 55\right) = 0.979$ (3 sf)

Assumption:
The masses of both types of seafood, crabs and groupers, are independent within and across each type.
Alternatively, the masses of all crabs and groupers are independent of each other. (Among each type and across each type)

6. IJC Prelim 8865/2018/Q12

Potatoes and carrots are sold by weight in a market. The masses, in kilograms (kg), of potatoes and carrots are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean	Standard Deviation
Potatoes	0.165	0.015
Carrots	0.072	0.012

(i) Stating clearly the mean and variance of the distribution that you use, find the probability that the total mass of 2 randomly chosen carrots is within 20 g of the mass of a randomly chosen potato. [4]

Potatoes cost \$2.50 per kilogram and carrots cost \$1.80 per kilogram.

Mathew buys 8 potatoes and 6 carrots as part of his ingredients to cook a vegetable stew.

	(iv)	Stating clearly the mean and variance of the distribution that you use, find the probability that Matthew spends between \$3.80 and \$4.20. [4]

Answers
(i) 0.447 (iv) 0.840

IJC Prelim 8865/2018/Q12

(i) Let X be the random variable denoting mass in kilograms of a randomly chosen potato.

$X \sim \mathrm{N}\left(0.165, 0.015^2\right)$

Let Y be the random variable denoting the mass in kilograms of a randomly chosen carrot.

$Y \sim \mathrm{N}\left(0.072, 0.012^2\right)$

Let $T = Y_1 + Y_2 - X$

$\mathrm{E}(T) = 2(0.072) - 0.165 = -0.021$

$\mathrm{Var}(T) = 2\left(0.012^2\right) + 0.015^2 = 0.000513$

$T \sim \mathrm{N}(-0.021, 0.000513)$

Required probability

$= \mathrm{P}(-0.02 < T < 0.02)$

$= 0.447 (3 \text{ s.f})$

(iv) Let W be the cost of 8 potatoes and 6 carrots.

Let $W = 2.5(X_1 + X_2 + ... + X_8) + 1.8(Y_1 + ... + Y_4 + Y_5 + Y_6)$

$\mathrm{E}(W) = 2.5(8 \times 0.165) + 1.8(6 \times 0.072)$

$= 4.0776$

$\mathrm{Var}(W) = 2.5^2\left(8 \times 0.015^2\right) + 1.8^2\left(6 \times 0.012^2\right)$

$= 0.014049$

$W \sim N(4.0776, 0.014049)$

Required probability

$= \mathrm{P}(3.8 < W < 4.2) = 0.840 (3 \text{ s.f})$

7. NJC Prelim 8865/2018/Q11

In this question you should state clearly the values of the parameters of any normal distribution you use.

A supermarket sells two types of durians, Red Prawn and Black Gold. The masses, in kilograms, of the durians each have independent normal distributions. The means and standard deviations of these distributions, and the selling prices, in \$ per kilogram, are shown in the following table.

	Mean mass (kg)	Standard deviation (kg)	Selling price (\$ per kg)
Red Prawn	0.25	0.02	1.50
Black Gold	0.35	0.03	2.40

	(iii) Find the probability that the total mass of six randomly chosen Red Prawns durians is within 0.2 kg of the total mass of five randomly chosen Black Gold durians. [4] Mr Phang buys three Red Prawn durians and three Black Gold durians. Mr Fong buys ten Red Prawn durians. **(iv)** Find the probability that Mr Phang pays more than Mr Fong. [4]
	Answers (iii) 0.273 (iv) 0.262
NJC Prelim 8865/2018/Q11	
	Let A be the random variable denoting the mass of a Red Prawn durian. $A \sim \mathrm{N}\left(0.25, 0.02^2\right)$
(iii)	Let B be the random variable denoting the mass of a Black Gold durian. Let $T = \left(A_1 + A_2 + ... + A_6\right) - \left(B_1 + B_2 + ... + B_5\right)$ $\mathrm{E}(T) = 6\mathrm{E}(A) - 5\mathrm{E}(B) = 6(0.25) - 5(0.35) = -0.25$ $\mathrm{Var}(T) = 6\mathrm{Var}(A) + 5\mathrm{Var}(B) = 6(0.02)^2 + 5(0.03)^2 = 0.0069$ $\mathrm{P}(-0.2 < T < 0.2) = 0.274$
(iv)	Let \$$V$ and \$$W$ be the amount that Mr Phang and Mr Fong pay respectively. $V = 1.5\left(A_{V1} + A_{V2} + A_{V3}\right) + 2.4\left(B_1 + B_2 + B_3\right)$ $W = 1.5\left(A_{W1} + A_{W2} + ... + A_{W10}\right)$ Need to compute $\mathrm{P}(V > W)$ or $\mathrm{P}(V - W > 0)$ $\mathrm{E}(V - W) = \mathrm{E}(V) - \mathrm{E}(W)$ $= (1.5)(3)\mathrm{E}(A) + (2.4)(3)\mathrm{E}(B) - \left[(1.5)(10)\mathrm{E}(A)\right]$ $= 3.645 - 3.75$ $= -0.105$ $\mathrm{Var}(V - W) = \mathrm{Var}(V) + \mathrm{Var}(W)$ $= (1.5)^2(3)\mathrm{Var}(A) + (2.4)^2(3)\mathrm{Var}(B) + (1.5)^2(10)\mathrm{Var}(A)$ $= 0.018252 + 0.009$ $= 0.027252$ $V - W \sim \mathrm{N}(-0.105, 0.027252)$ $\mathrm{P}(V - W > 0) = 0.262$

Section 6: Explanation Type

1.	MJC Prelim 8865/2018/Q12

Brandon can choose to take the subway or taxi to and fro between his home and office. The one-way journey times, in minutes, by taxi and by subway have independent normal distributions. The means and standard deviations of these distributions are shown in the following table.

	Mean	Standard Deviation
Taxi	59	2
Subway	61	3

Journeys are charged by the time taken. For the taxi journey, the charge is \$0.69 per minute and for the subway journey, the charge is \$0.13 per minute.

Let A represent the cost of the taxi journey from Brandon's home to work.

Let B represent the cost of the subway journey from Brandon's home to work.

(v) Find $\mathrm{P}\left(2A-\left(B_1+B_2\right)<68\right)$ and explain, in the context of the question, what your answer represents. [5]

Answers

(v) 0.807

MJC Prelim 8865/2018/Q12

Let X and Y be the journey times by a randomly chosen taxi and a randomly chosen subway respectively (in minutes).

$X \sim \mathrm{N}\left(59, 2^2\right)$

$Y \sim \mathrm{N}\left(61, 3^2\right)$

(v)

$A = 0.69X \sim \mathrm{N}\left(40.71, 1.9044\right)$

$B = 0.13Y \sim \mathrm{N}\left(7.93, 0.1521\right)$

$2A-\left(B_1+B_2\right) \sim \mathrm{N}\left(65.56, 7.9218\right)$

$\mathrm{P}\left(2A-\left(B_1+B_2\right)<68\right) = 0.80701 \approx 0.807$

The answer represents the probability that 2 times the cost of a randomly chosen taxi journey exceeds the cost of the sum of 2 randomly chosen subway journeys by less than \$68.

2.	ACJC Prelim 8865/2018/Q9(ii)

The masses, in grammes, of oranges and tangerines are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean mass	**Standard deviation**
Oranges	181 g	4.77 g
Tangerines	165.8 g	23.07 g

Let X represent the mass (in grammes) of a randomly selected orange.

Let Y represent the mass (in grammes) of a randomly selected tangerine.

(ii) Juices obtained from squeezing an orange is only 40% of the weight of the orange squeezed while juices obtained from squeezing a tangerine is only 35% of the weight of the tangerine squeezed.

Find $\mathrm{P}\left(0.4\left[X_1+X_2+X_3+X_4\right]>0.35\left[Y_1+Y_2+Y_3+Y_4+Y_5\right]\right)$, stating the mean and variance of the distribution. Explain, in the context of this question, what your answer represents.[5]

Answers
(i) 0.338; (ii) $N(-0.55, 340.55)$, 0.488; (iii) 0.0459

ACJC Prelim 8865/2018/Q9

Let random variable X be the mass (in grammes) of an orange.
Let random variable Y be the mass (in grammes) of an orange.
Given $X \sim (181, 77^2)$ and $Y \sim (165.8, 23.07^2)$

(ii) $P(0.4(X_1 + \ldots + X_4) > 0.35(Y_1 + \ldots + Y_5))$
$= P(0.4(X_1 + \ldots + X_4) - 0.35(Y_1 + \ldots + Y_5) > 0)$

$E(0.4(X_1 + \ldots + X_4) - 0.35(Y_1 + \ldots + Y_5))$
$= (0.4)(4)E(X) - (0.35)(5)E(Y)$
$= (0.4)(4)(181) - (0.35)(5)(165.8)$
$= -0.55$

$Var(0.4(X_1 + \ldots + X_4) - 0.35(Y_1 + \ldots + Y_5))$
$= (0.4^2)(4)Var(X) - (0.35^2)(5)Var(Y)$
$= (0.4^2)(4)(4.77^2) - (0.35^2)(5)(23.07^2)$
$= 340.5496073$
$0.4(X_1 + \ldots + X_4) - 0.35(Y_1 + \ldots + Y_5) \sim N(-0.55, 340.55)$
$P(0.4(X_1 + \ldots + X_4) - 0.35(Y_1 + \ldots + Y_5) > 0) = 0.488$

The answer is the probability that the total weight of orange juice squeezed from 4 oranges is more than the total weight of tangerine juice squeezed from 5 tangerines.

3. EJC Prelim 8865/2018/Q11(v)

A telecommunication company provides mobile services for its customers to make calls to City J and City C. The duration of calls, in minutes, made by its customers to City J and City C have independent normal distributions. The means and standard deviations of these distributions are shown in the following table.

	Mean	Standard deviation
City J	8	1.5
City C	10	1.8

A customer, Ashton, makes calls to City J and City C for work purposes. The telecommunication company charges a rate of \$0.20 per minute for every call to City J and a rate of \$0.30 per minute for every call to City C.

Let J represent the cost of one call from Ashton to City J.

Let C represent the cost of one call from Ashton to City C.

(v) Find $\mathrm{P}\left(J_1 + J_2 + J_3 - 2C > 1.50\right)$ and explain, in the context of this question, what your answer represents. [4]

Answers (v) 0.0121

EJC Prelim 8865/2018/Q11

(iv) $J \sim N\left(0.20\times 8, 0.20^2\times 1.5^2\right)$, $C \sim N\left(0.30\times 10, 0.30^2\times 1.8^2\right)$

$J \sim N\left(1.6, 0.09\right)$ $C \sim N\left(3, 0.2916\right)$

$J_1+J_2+J_3-2C \sim N\left(1.6\times 3-2\times 3, 3\times 0.09+2^2\times 0.2916\right)$

$J_1+J_2+J_3-2C \sim N\left(-1.2, 1.4364\right)$

$\mathrm{P}\left(J_1+J_2+J_3-2C>1.50\right)=0.0121$ (3 s.f)

This is the probability that Ashton will pay more than \$1.50 for the total costs of 3 calls to City *J* to exceed twice that the cost of one call to City *C*.

4. YJC Prelim 8865/2018/Q12

Leon Fish Farm breeds only salmon and scallops. The salmon are packed in boxes of half a dozen each while the scallops are packed in boxes of 100 each. The masses, in kilograms, of salmon and scallops sold by the fish farm have independent normal distributions. The means and standard deviations of these distributions, and the selling prices, in \$ per kilogram, are shown in the following table.

	Mean (kg)	Standard deviation (kg)	Selling price (\$ per kg)
Salmon	5	1.5	30
Scallops	0.2	0.05	16

Let V be the selling price of a box of salmon.
Let W be the selling price of a box of scallops.

(iv) Find $\mathrm{P}\left(-150<V_1+V_2-6W<150\right)$ and explain, in the context of this question, what your answer represents. [5]

Answers (iv) 0.524

YJC Prelim 8865/2018/Q12

Let S be the mass (in kg) of a salmon,
Let A be the mass (in kg) of a scallop.
$S \sim \mathrm{N}(5, 1.5^2)$, $A \sim \mathrm{N}(0.2, 0.05^2)$

$V = 30(S_1 + S_2 + \ldots + S_6) \sim \mathrm{N}(30(6\times 5), 30^2(1.5)^2\times 6)$
i.e. $V \sim \mathrm{N}(900, 12150)$
$W = 16(A_1 + A_2 + \ldots + A_{100}) \sim \mathrm{N}(16(100\times 0.2), 16^2(0.05)^2\times 100)$
i.e. $W \sim \mathrm{N}(320, 64)$
$V_1 + V_2 - 6W \sim \mathrm{N}(2(900) - 6\times 320, 2(12150)+6^2(64))$
i.e. $V_1 + V_2 - 6W \sim \mathrm{N}(-120, 26604)$

$\mathrm{P}\left(-150<V_1+V_2-6W<150\right) = 0.524$

	It is the probability that the selling price of 2 boxes of salmon differs from six times the selling price of a box of scallops by less than \$150. Or It is the probability of the 'the difference between the selling price of 2 boxes of salmon and six times the selling price of a box of scallops is within ± 150.

Section 7: Average Type

1. HCI Prelim 8865/2018/Q12

A supermarket sells two types of strawberries, *A* and *B*. Each type of strawberry comes in different sized packaging. The masses, in kg, of a packet of type *A* strawberries and a packet of type *B* strawberries are modelled as having independent normal distributions with means and standard deviations as shown in the table.

Strawberries	Mean	Standard deviation
Type *A*	1.2	0.2
Type *B*	1.1	0.1

Type *A* strawberries are sold at \$20 per kg and type *B* strawberries at \$26 per kg.

Audrey picks one packet of type *A* strawberries and one packet of type *B* strawberries.

(iv) Jackson buys one packet of type *A* strawberries and three packets of type *B* strawberries. Find the probability that the packet of type *A* strawberries weighs more than the average weight of the three packets of type *B* strawberries by less than 0.2 kg. [4]

Answers
(iv) 0.369

HCI Prelim 8865/2018/Q12

Let *X* and *Y* be the weight (in kg) of a packet of type *A* and type *B* strawberries respectively.

$$X \sim \mathrm{N}\left(1.2, 0.2^2\right)$$

$$Y \sim \mathrm{N}\left(1.1, 0.1^2\right)$$

Let *M* be the average weight of the three packets of type *B* strawberries bought by Jackson.

$$M = \frac{Y_1 + Y_2 + Y_3}{3}$$

$$M \sim \mathrm{N}\left(\frac{3(1.1)}{3}, \frac{3\left(0.1^2\right)}{3^2}\right) = \mathrm{N}\left(1.1, \frac{0.1^2}{3}\right)$$

$$X - M \sim \mathrm{N}\left(1.2 - 1.1, 0.2^2 + \frac{0.1^2}{3}\right) = \mathrm{N}\left(0.1, \frac{0.13}{3}\right)$$

$$\mathrm{P}\left(0 \le X - M < 0.2\right) = 0.36905 = 0.369 \text{(to 3 s.f)}$$

2. JJC Prelim 8865/2018/Q11(aiii)

The masses, in kg, of red and green apples sold by a supermarket have independent normal distributions with means and standard deviations as shown in the following table.

	Mean	Standard Deviation
Red Apples	0.18	0.029
Green Apples	0.13	0.015

(iv) Find the probability that the average mass of 5 randomly chosen red apples is greater than twice the average mass of 3 randomly chosen green apples. [3]

Answers
(aiii) 0.165

JJC Prelim 8865/2018/Q11

(iv) Average mass of 5 red apples, $\bar{X} \sim \mathrm{N}\left(0.18, \frac{0.029^2}{5}\right)$

Average mass of 3 green apples, $\bar{Y} \sim \mathrm{N}\left(0.13, \frac{0.015^2}{3}\right)$

$\bar{X} - 2\bar{Y} \sim \mathrm{N}\left(0.18 - 2\times 0.13, \frac{0.029^2}{5} + 2^2 \times \frac{0.015^2}{3}\right)$

$\bar{X} - 2\bar{Y} \sim \mathrm{N}\left(-0.08, 0.0004682\right)$

$\mathrm{P}\left(\bar{X} > 2\bar{Y}\right) = \mathrm{P}\left(\bar{X} - 2\bar{Y} > 0\right) = 0.00010902 = 0.000109$ (3 s.f.)

3. VJC Prelim 8865/2018/Q11

In a high school, the times taken, in minutes, for boys and girls to complete their 2.4 km test run have independent normal distributions with means and standard deviations as shown in the following table.

	Mean	Standard deviation
Boys	11.51	0.72
Girls	13.17	0.99

(iv) the average time taken by the two boys and two girls is less than the mean time taken by the boys in the high school. [4]

Answers
(v) 0.276

VJC Prelim 8865/2018/Q11

Let X mins be the time taken by a boy and Y mins be the time taken by a girl.

$X \sim \mathrm{N}\left(11.51, 0.72^2\right)$

$Y \sim \mathrm{N}\left(13.17, 0.99^2\right)$

$$\bar{W} = \frac{X_1 + X_2 + Y_1 + Y_2}{4}$$

$$\mathrm{E}\left(\bar{W}\right) = \frac{1}{4}\left[2\mathrm{E}\left(X\right) + 2\mathrm{E}\left(Y\right)\right] = 12.34$$

$$\mathrm{Var}\left(\bar{W}\right) = \frac{1}{16}\left[2\mathrm{Var}\left(X\right) + 2\mathrm{Var}\left(Y\right)\right] = 0.1873125$$

$$\bar{W} \sim \mathrm{N}\left(12.34, 0.1873125\right)$$

$$\mathrm{P}\left(\bar{W} < 11.51\right) = 0.0276$$

4. NYJC Prelim 8865/2018/Q11

Every morning, a student needs to reach the bus stop at 7:30am to catch a bus to school. If he reaches school after 8:00am, he will be considered late. Assume that the waiting times for a bus is normally distributed with mean 8 minutes and variance 5 minutes ^{2}, and the duration of the bus journey is normally distributed with mean 20 minutes and variance 4 minutes ^{2}.

(v) Find the probability that the mean time taken to travel from the bus stop (including waiting for the bus) to school in 40 days is between 28 and 29 minutes. [2]

Answers
(iv) 0.482

NYJC Prelim 8865/2018/Q11(iv)

Let W and J denote the average waiting time and journey time

$W \sim \mathrm{N}(8, 5)$

$J \sim \mathrm{N}(20, 4)$

Let T denote the sum of the waiting and journey time

$T \sim \mathrm{N}(28, 9)$

(iv) Let $\bar{T}$ be the mean traveling time for 40 days

$\bar{T} \sim \mathrm{N}(28, \frac{9}{40})$

$\mathrm{P}(28 < \bar{T} < 29) = 0.48249 = 0.482$ (to 3 sig fig)