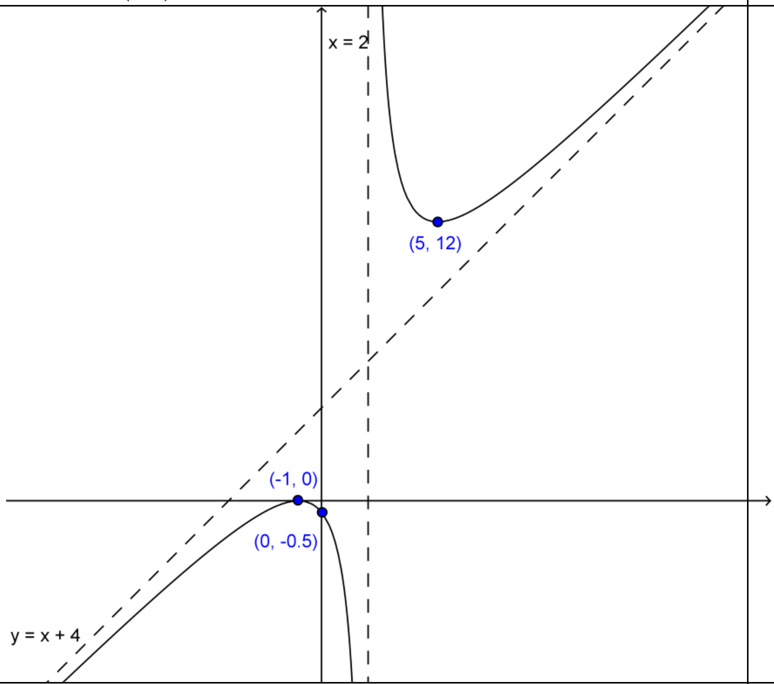
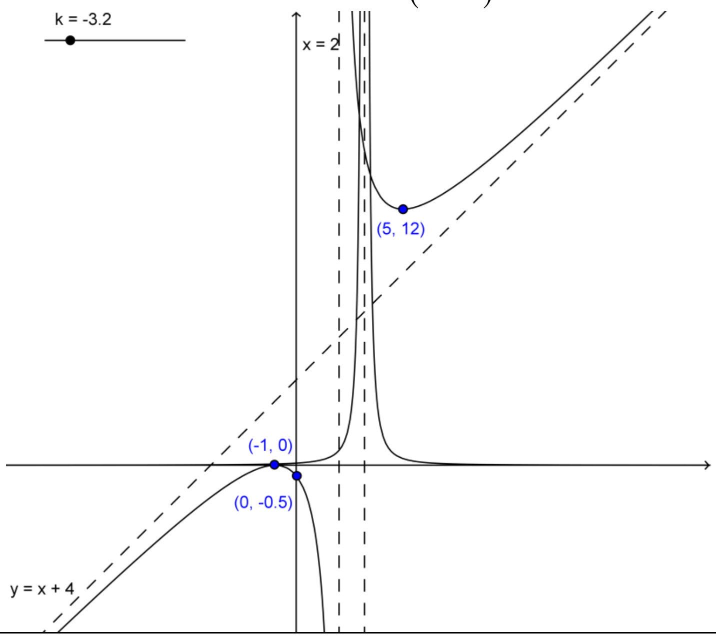
RVHS 2015 Y6 H2 MA Prelim Paper 2 (Solutions)

Section A: Pure Mathematics [40 marks]

<table>
<tr><td colspan="3">Question 1 [8 Marks]</td></tr>
<tr><td>(i)</td><td>Vertical asymptote when $x = 2$
$2 + b = 0 \rightarrow b = -2$

Substitute $x = -1$, $y = 0$ into the equation for C:
$0 = \dfrac{(-1)^2 + a(-1) + 1}{(-1) + b} \rightarrow a = 2$</td><td></td></tr>
<tr><td>(ii)</td><td>
</td><td></td></tr>
<tr><td>(iii)</td><td>$\left[(x+1)(x+k)\right]^2 = x - 2$
$\dfrac{(x+1)^2}{x-2} = \dfrac{1}{(x+k)^2}, \quad x \neq 2$

Therefore, draw the graph $y = \dfrac{1}{(x+k)^2}$

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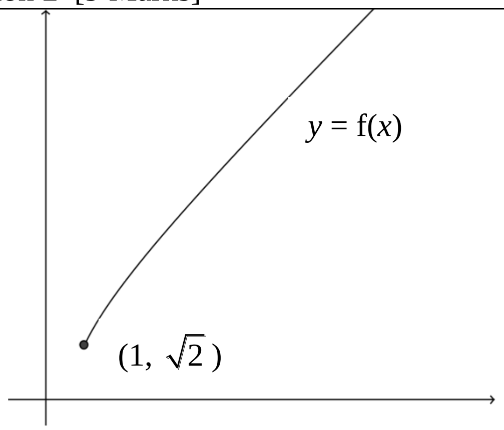
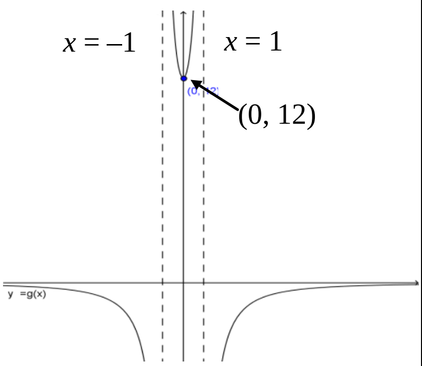
	By observation, the graphs intersect twice when $k < -2$. Furthermore, when $k = -2$, equation becomes: $(x+1)^2(x-2)^2 = x-2$ $(x-2)[(x+1)^2(x-2)-1]=0$ $x=2$ or $x=2.10380$ $\therefore k=-2$ is an answer as well. Ans: $k \le -2,\ k \in \mathbb{R}$	

Question 2 [9 Marks]		
(i)	 Any horizontal line $y = k$ will cut the graph of y = f(x) at most once. Therefore, f is one-one.	
(ii)	Let y = f(x). Then: $y=\sqrt{x^2+4x-3}$ $y^2=(x+2)^2-7$ $x=-2\pm\sqrt{y^2+7}$ (Rej negative $\because x \ge 1$) Thus, $\mathrm{f}^{-1}: x \mapsto -2+\sqrt{x^2+7},\quad x \ge \sqrt{2}$	
(iii)	fg exists iff $R_{\mathrm{g}} \subseteq D_{\mathrm{f}}$ i.e. $R_{\mathrm{g}} \subseteq [1,\infty)$ So, $R_{\mathrm{g}} = [12,\infty)$ $\Rightarrow D_{\mathrm{g}} = (-1,1)$  	

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<tr>
<td>(iv)</td>
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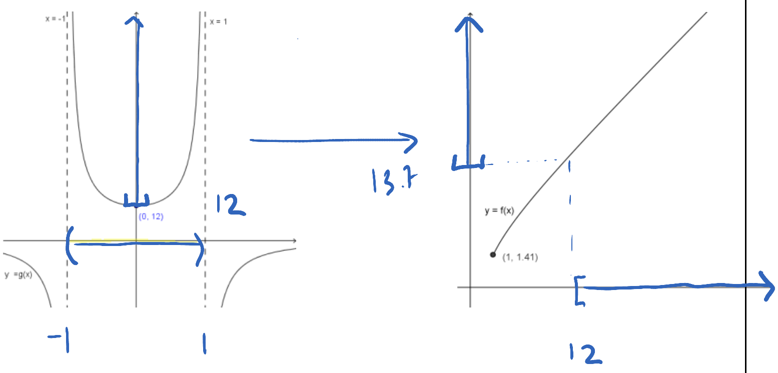
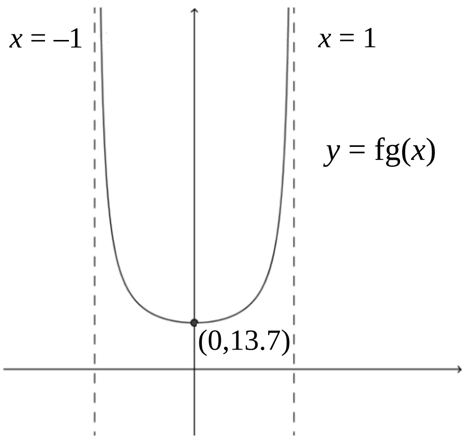
$D_{\mathrm{g}} = (-1,1) \xrightarrow{\mathrm{g}} [12,\infty) \xrightarrow{\mathrm{f}} [\sqrt{189},\infty)$ (OR $[13.7,\infty)$)



Therefore, $R_{\mathrm{fg}} = [13.7,\infty)$

<u>Alternatively:</u>

$$\begin{aligned}&\mathrm{fg}(x)\\&=\mathrm{f}(\mathrm{g}(x))\\&=\mathrm{f}\left(\frac{12}{1-x^2}\right)\\&=\sqrt{\left(\frac{12}{1-x^2}\right)^2+4\left(\frac{12}{1-x^2}\right)-3}\end{aligned}$$



From the graph,
$R_{\mathrm{fg}} = [13.7,\infty)$

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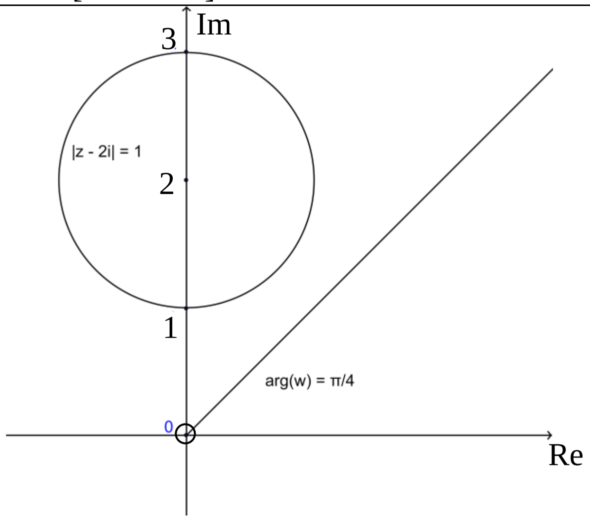
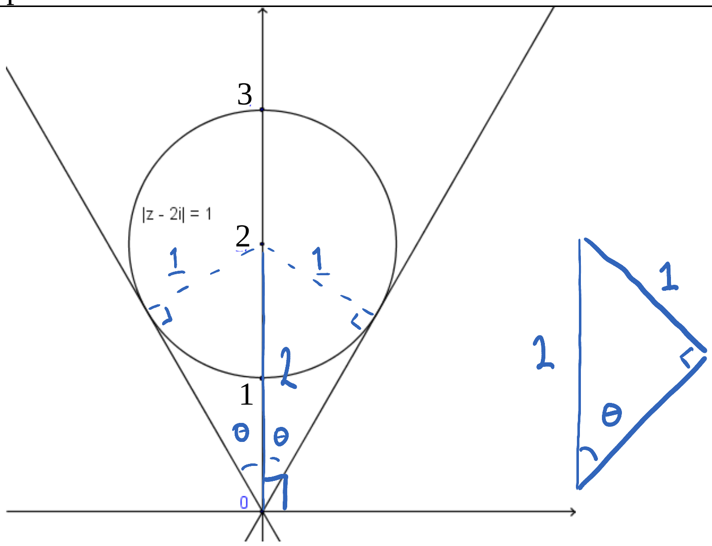
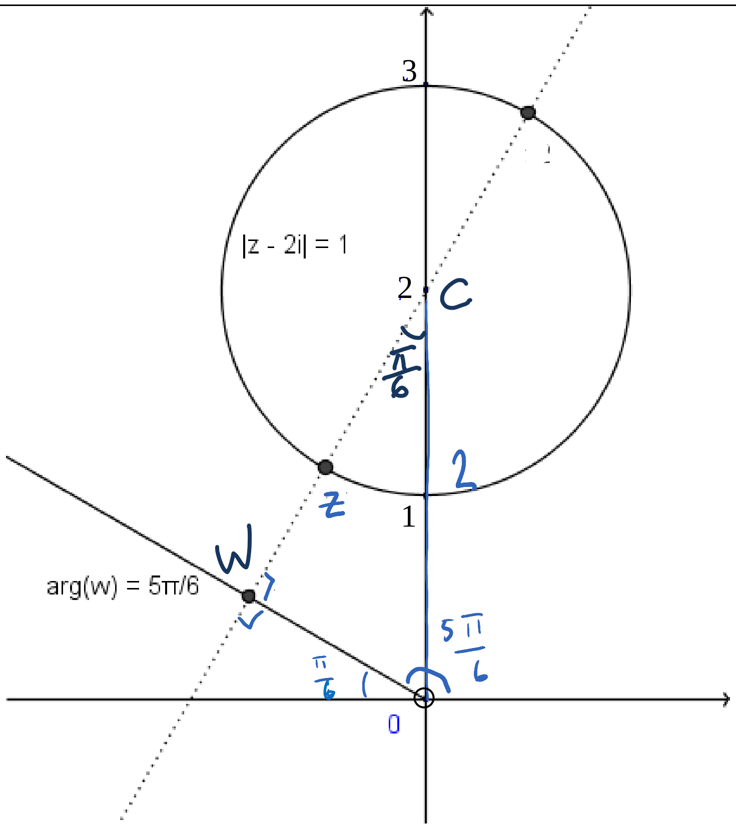
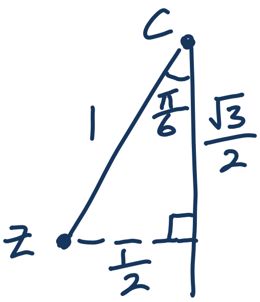
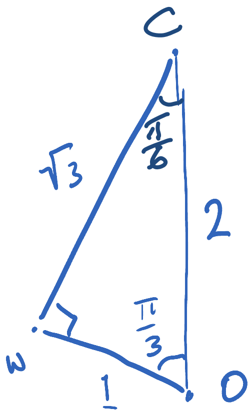
Question 3 [11 Marks]		
(i)	 $\lvert z-2i \rvert=1$ is a circle centred on (0, 2) with radius 1. $\arg(w)=\dfrac{\pi}{4}$ is the half line from the origin (excluding the origin), which makes an angle of $\dfrac{\pi}{4}$ with the positive real-axis.	
(ii)	 Intersect at one point implies the half line is a tangent to the circle. $\sin\theta=\dfrac{1}{2}\Rightarrow\theta=\dfrac{\pi}{6}$ Therefore, $k=\dfrac{\pi}{2}-\dfrac{\pi}{6}=\dfrac{\pi}{3}$ or $k=\dfrac{\pi}{2}+\dfrac{\pi}{6}=\dfrac{2\pi}{3}$	

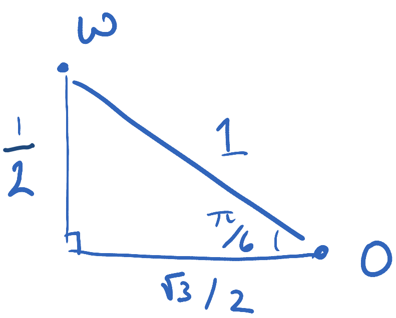
(iii)



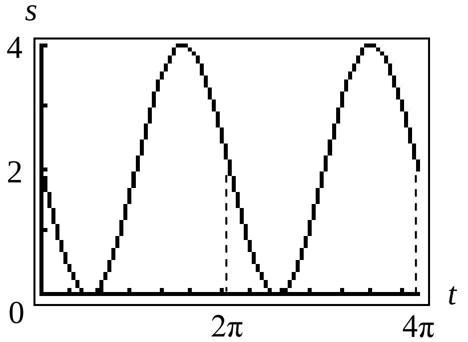


$$z = 2i - \frac{\sqrt{3}}{2}i - \frac{1}{2} = \frac{-1}{2} + i\frac{4-\sqrt{3}}{2}$$





$$w = \frac{-\sqrt{3}}{2} + \frac{i}{2}$$

Question 4 [12 Marks]		
(a)	$y = ux + 2 \Rightarrow \frac{dy}{dx} = x\frac{du}{dx} + u$ Substituting into DE: $\frac{dy}{dx} - \sqrt{4-(y-2)^2} = 0$ $\frac{dy}{dx} - \sqrt{4-(y^2-4y+4)} = 0$ $\frac{dy}{dx} = \sqrt{4y-y^2}$ (shown) $\therefore \int \frac{1}{\sqrt{4y-y^2}}\,dy = \int 1\,dx$ $\int \frac{1}{\sqrt{4-(y-2)^2}}\,dy = \int 1\,dx$ $\sin^{-1}\left(\frac{y-2}{2}\right) = x + c$ $y - 2 = 2\sin(x+c)$ $u = \frac{2\sin(x+c)}{x}$	
(b) (i)	From part (i), $s = 2\sin(t+c) + 2$ when $t = \frac{5\pi}{6}$, $s = 1$: $\sin\left(\frac{5\pi}{6} + c\right) = -\frac{1}{2}$ $\frac{5\pi}{6} + c = -\frac{\pi}{6}$ $c = -\pi$ $\therefore s = 2\sin(t-\pi) + 2$ 	
(ii)	The object oscillates about the starting point which is 2m from *O*, with an amplitude of 2m. The motion assumes the absence of resistance whereby the amplitude remains constant which is unrealistic in real-life.	

Section B: Statistics [60 marks]

<table>
<tr><td colspan="3">Question 5 [3 Marks]</td></tr>
<tr><td>(i)</td><td>No. of ways = $\left[(i+j+k)-1\right]!$</td><td></td></tr>
<tr><td>(ii)</td><td>No. of ways = $3!i!j!k! = 6i!j!k!$</td><td></td></tr>
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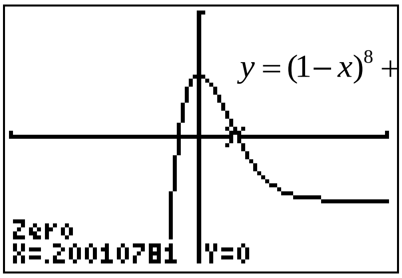
<table>
<tr><td colspan="3">Question 6 [6 Marks]</td></tr>
<tr><td>(a)</td><td>A population is the <u>entire collection</u> of data that we want to study while a sample is a <u>subset of</u> the units in <u>the population</u> having the same characteristics that we want to measure.</td><td></td></tr>
<tr><td>(b)
(i)</td><td>This procedure will not result in a random sample as each of the 120 teachers does not have an equal chance of being selected, e.g. P(a Humanities teacher chosen) = $\frac{10}{21}$ while P(a Language teacher chosen) = $\frac{10}{33}$.</td><td></td></tr>
<tr><td>(ii)</td><td>We calculate the number of teachers from each department based on simple ratio and proportion:
<table>
<tr><th>Department</th><th>Number of teachers selected</th></tr>
<tr><td>Humanities</td><td>$\frac{21}{120}\times 40 = 7$</td></tr>
<tr><td>Language</td><td>$\frac{33}{120}\times 40 = 11$</td></tr>
<tr><td>Mathematics</td><td>$\frac{27}{120}\times 40 = 9$</td></tr>
<tr><td>Science</td><td>$\frac{39}{120}\times 40 = 13$</td></tr>
</table>
7 Humanities teachers, 11 Language teachers, 9 Mathematics teachers and 13 Science teachers are chosen randomly from each department.</td><td></td></tr>
</table>

Question 7 [7 Marks]		
	Let X denote the number of correct answers gotten by someone totally clueless of Singapore's history. Then $X \sim \mathrm{B}(8, p)$, where p is the probability of getting a correct answer by mere guessing. Given $\mathrm{P}(X \le 1) = 0.50331648$ $\Rightarrow (1-p)^8 + \binom{8}{1} p(1-p)^7 = 0.503$ $(1-p)^8 + 8p(1-p)^7 - 0.503 = 0$  From the graph, $p \approx 0.200 = \frac{1}{5}$. Therefore, there are 5 options for each of the question. (shown)	
	So $X \sim \mathrm{B}(8, 0.2)$ $\mathrm{P}(X = 0) = 0.1678$ $\mathrm{P}(X = 1) = 0.3355$ $\mathrm{P}(X = 2) = 0.2936$ The mode is 1.	
	$\overline{X} \sim \mathrm{N}\left(1.6, \frac{1.28}{50}\right)$ approx. by CLT, since n = 50 is large. $\therefore \mathrm{P}(\overline{X} \ge 2) = 0.00621$ (to 3 s.f.)	

Question 8 [7 Marks]		
(i)	<u>The average number of calls</u> requiring ambulance assistance <u>is constant throughout</u> the operating hours of the hospital in <u>a day</u>. OR <u>For any time interval within</u> the operating hours of the hospital in <u>a day</u>, <u>the mean number of calls</u> requiring ambulance <u>assistance is proportional to the time interval</u>.	
(ii)	Let X denote the number of calls requiring ambulance assistance in 1 week. Then $X \sim \mathrm{Po}(7)$. Required probability $= \mathrm{P}(X < 5) = \mathrm{P}(X \le 4) = 0.172992 = 0.173$ (to 3sf)	

(iii)	Let Y denote the number of weeks (out of 52) with more than 4 calls requiring ambulance assistance. Then $Y \sim \mathrm{B}(52, 1-0.172992)$ i.e. $Y \sim \mathrm{B}(52, 0.827008)$ Since n = 52 is large, $np = 43.004416 > 5$ and $nq = 8.995584 > 5$, $\therefore Y \sim \mathrm{N}(43.004416, 7.439419933)$ approx. Required probability $= \mathrm{P}(40 < Y \le 45)$ $= \mathrm{P}(40.5 < Y < 45.5)$ C.C. $= 0.641$ (to 3sf)	

Question 9 [8 Marks]		
(i)	Let X denote the time required by the machine to complete a task and μ be the mean time. Test $\mathrm{H}_0 : \mu = 47.0$ against $\mathrm{H}_1 : \mu > 47.0$	
(ii)	$\bar{x} = 48.1$ $s^2 = \frac{n}{n-1}\sigma_n{}^2 = \frac{12}{11}(1.9)^2 = 3.938181818$ Test statistic: Under H_0, $T = \frac{\bar{X} - 47.0}{\sqrt{\frac{3.938181818}{12}}} \sim t(11)$ Using GC, p-value = 0.0405659745 > 0.03 (level of significance) $\therefore$ do not rej. H_0. There is insufficient evidence at 3% level of significance that the mean time required by the machine to complete the task is understated.	
(iii)	Given: σ = 2.1 Test statistic: Under H_0, $Z = \frac{\bar{X} - 47.0}{2.1/\sqrt{12}} \sim \mathrm{N}(0,1)$ For H_0 to be rejected, $Z_{\text{cal}} > Z_{\text{critical}}$ i.e. $\frac{\bar{x} - 47.0}{2.1/\sqrt{12}} > 1.88079361$ $\Rightarrow \bar{x} > 48.14017053$ $\therefore \bar{x} > 48.1$ (or $\bar{x} \ge 48.2$)	

Question 10 [8 Marks]		
(i)	P(all 3 balls have different colours) $= {}^4P_3 \times \left(\frac{1}{4}\right)^3$ $= \frac{3}{8}$ or 0.375	
(ii)	$\dfrac{\text{P(the 3 balls have diff. colours and nos '0,0,0' or '0,1,1')}}{\text{P(all 3 balls have different colours)}}$ $= \dfrac{{}^4P_3 \times \left(\frac{1}{8}\right)^3 + {}^4P_3 \times {}^3C_2 \times \left(\frac{1}{8}\right)^3}{\frac{3}{8}}$ $= \frac{1}{2}$	
	$\mathrm{P}(A \mid B) = \frac{1}{2}$ (from part (ii)) $\mathrm{P}(A) = \mathrm{P}(0,0,0) + \mathrm{P}(0,1,1)$ $= \left(\frac{1}{2}\right)^3 + {}^3C_1 \times \left(\frac{1}{2}\right)^3$ $= \frac{1}{2} = \mathrm{P}(A \mid B)$ $\therefore$ A and B are independent. <u>ALTERNATIVE METHOD</u> $\mathrm{P}(B) = \frac{3}{8}$ (from part (i)) $\mathrm{P}(A) = \mathrm{P}(0,0,0) + \mathrm{P}(0,1,1)$ $= \left(\frac{1}{2}\right)^3 + {}^3C_1 \times \left(\frac{1}{2}\right)^3$ $= \frac{1}{2}$ $\therefore \mathrm{P}(A) \cdot \mathrm{P}(B) = \frac{1}{2} \times \frac{3}{8} = \frac{3}{16}$ In addition, $\mathrm{P}(A \cap B) = \frac{3}{16}$ (from (ii)) $= \mathrm{P}(A) \cdot \mathrm{P}(B)$ Hence, A and B are independent.	

Question 11 [9 Marks]

(i) Let A and B denote the masses of a random mobile phone sold by Company A and B respectively.

$B \sim \mathrm{N}\left(\mu,\ \sigma^2\right)$

Given $\mathrm{P}(B < 134) = \mathrm{P}(B > 146)$

By symmetry,

$$\mu = \frac{134+146}{2} = 140$$

<u>Method 1</u>

$\mathrm{P}(B < 134) = 0.234$

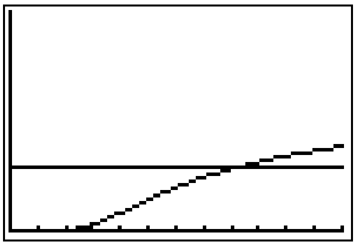
$$\mathrm{P}\left(Z < \frac{134-140}{\sigma}\right) = 0.234$$

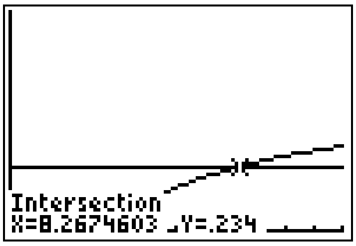
$$\frac{-6}{\sigma} = -0.7257370278$$

$\sigma = 8.267457453 \approx 8.27$

<u>Method 2</u>

Sketch graph of $y = normalcdf\,(-1E99, 134, 140, x)$ and $y = 0.234$, and find intersection:





<u>Method 3</u>

$\mathrm{P}(B > 146) = 0.234$

$$\mathrm{P}\left(Z < \frac{146-140}{\sigma}\right) = 0.766$$

$$\frac{6}{\sigma} = 0.7257370278$$

$\sigma = 8.267457453 \approx 8.27$

(ii) $A \sim \mathrm{N}(130, 6^2)$ $\qquad$ $B \sim \mathrm{N}(136, 8^2)$

$2B \sim \mathrm{N}(272, 256)$

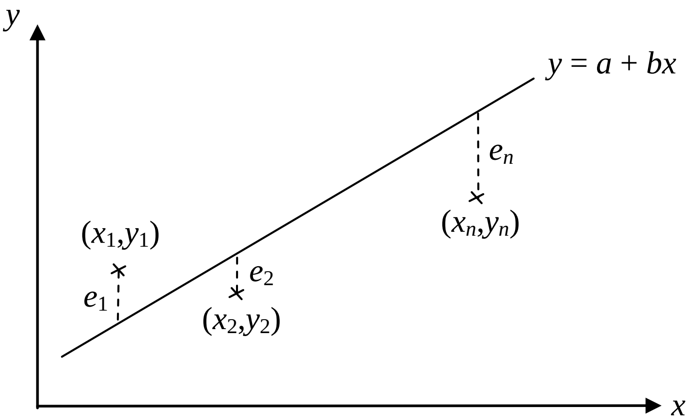
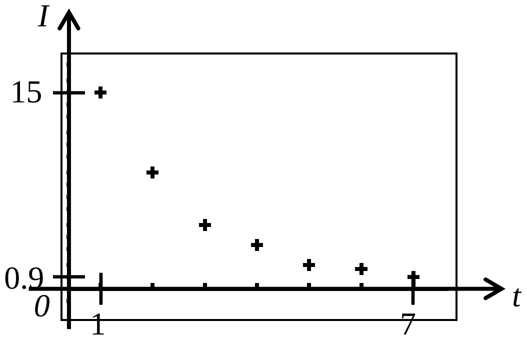
$A - 2B \sim \mathrm{N}(-142, 292)$

$$\therefore \mathrm{P}\left(\left|A-2B\right| < 150\right) = \mathrm{P}\left(-150 < A-2B < 150\right)$$

$$= 0.6801665426$$

$$\approx 0.680$$

(iii)	Required probability $=\mathrm{P}(A_1 \le 135)\times\mathrm{P}(A_2 \le 135)\times\ldots\times\mathrm{P}(A_{10} \le 135)$ $=\left[\mathrm{P}(A \le 135)\right]^{10}$ $=0.1042897699$ ≈ 0.104	
(iv)	The masses of the mobile phones sold by Company A are all independent of each other.	

Question 12 [12 Marks]		
(a)	Let the sample of bivariate data be (x_i, y_i) where $i = 1, 2, \ldots n$. Let $e_i = y_i - (a + bx_i)$ be the vertical deviation between the point (x_i, y_i) and the line $y = a + bx$.  The line $y = a + bx$ is the least square regression line for the sample of bivariate data if the sum of the squares of the vertical deviations, i.e. $\sum_{i=1}^{n}(e_i)^2$, is the minimum.	
(b) (i)	 	
(ii)	$r = -0.901356207 \approx -0.901$ Although $\lvert r \rvert$ is close to 1, suggesting a linear relationship between I and t, from the scatter diagram, it seems that a curvilinear relationship is a better fit.	

(iii)	We can determine which of these 2 formulae is a better model by calculating the *r*-value of each of these 2 formulae and then choose the formula whose $\lvert r \rvert$ is closer to 1.	
(iv)	For $I = c + dt^2$, $r = -0.7940683332 \approx -0.794$ For $I = e + f \ln t$, $r = -0.9838669707 \approx -0.984$ Hence, $I = 14.06964212 - 7.341502661 \ln t$ is used. When $I = 2.5$, $t = 4.835201938 \approx 4.84$ The estimate is reliable because: • $I = 2.5$ is within the data range provided • $\lvert r \rvert \approx 0.984$ is close to 1 suggesting a strong linear correlation • The appropriate regression line (*I* on *t*) is used, since *t* is the independent/ fixed variable	