DYNAMICS

Content

- Newton's Law of motion
- Linear momentum and its conservation

Learning Outcomes

Candidates should be able to:

(a) state each of Newton's laws of motion.

(b) show an understanding that mass is the property of a body which resists change in motion (inertia).

(c) describe and use the concept of weight as the effect of a gravitational field on a mass.

(d) define and use linear momentum as the product of mass and velocity.

(e) define and use impulse as the product of force and time of impact.

(f) Relate resultant force to the rate of change of momentum.

(g) recall and solve problems using the relationship F = ma, appreciating that resultant force and acceleration are always in the same direction.

(h) state the principle of conservation of momentum.

(i) apply the principle of conservation of momentum to solve simple problems including inelastic and (perfectly) elastic interactions between two bodies in one dimension. (Knowledge of the concept of coefficient of restitution is not required.)

(j) Show an understanding that, for a (perfectly) elastic collision between two bodies, the relative speed of approach is equal to the relative speed of separation.

(k) show an understanding that, whilst the momentum of a closed system is always conserved in interactions between bodies, some change in kinetic energy usually takes place.

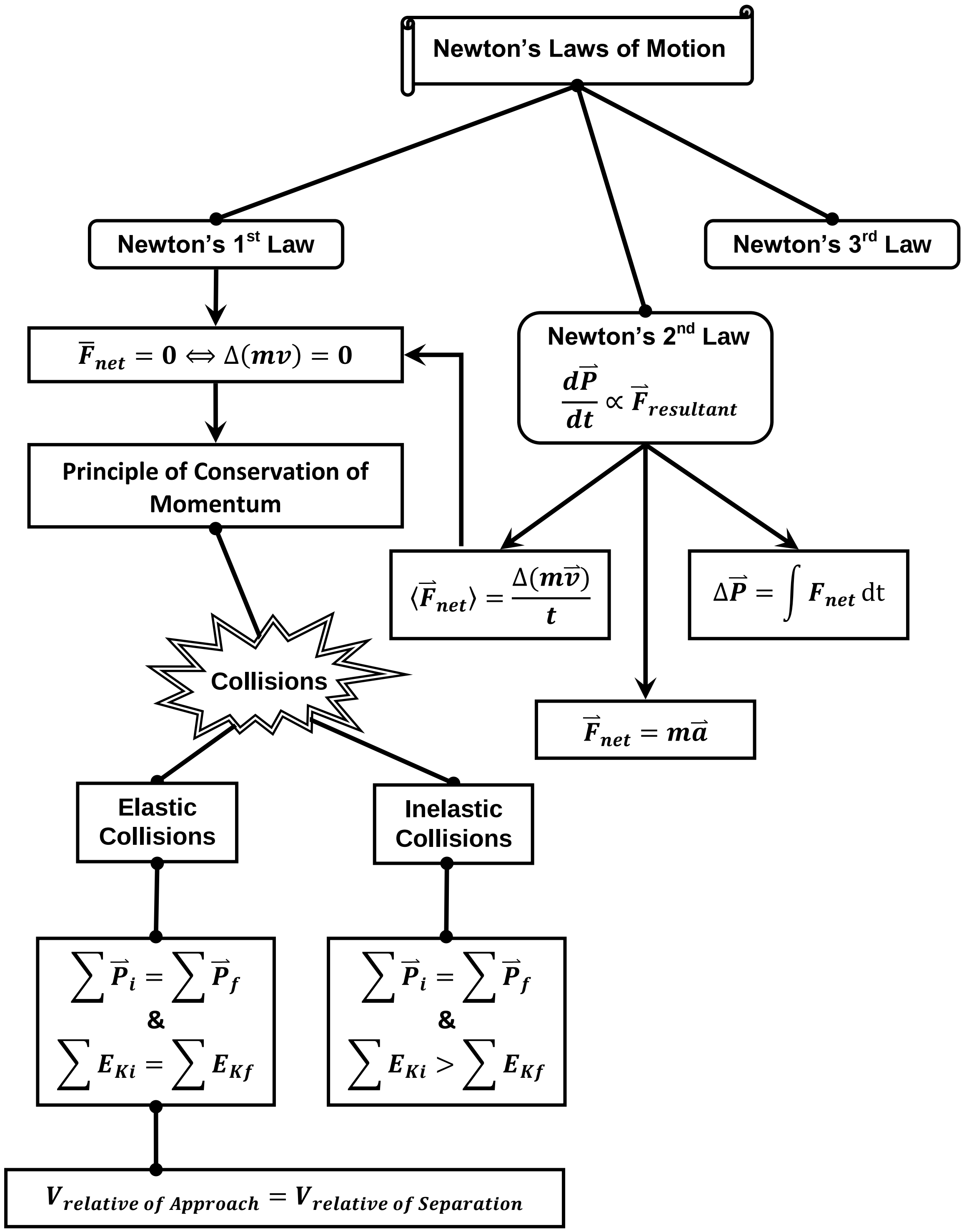
References:
Physics for Scientists and Engineers. Serway.
College Physics. Sears and Zemansky.

Useful applet:
Url :
http://iwant2study.org/lookangejss/02_newtonianmechanics_3dynamics/ejss_model_Momentum1D01/Momentum1D01_Simulation.xhtml

Look under Collision Carts, Atwood Machines, Newton Cradle

Or https://phet.colorado.edu/en/simulation/legacy/collision-lab

Concept Map


Newton's Laws of Motion
Newton's 1st Law
Newton's 3rd Law
Newton's 2nd Law
$\frac{d\vec{P}}{dt} \propto \vec{F}_{resultant}$
$\bar{F}_{net} = 0 \Leftrightarrow \Delta(mv) = 0$
Principle of Conservation of Momentum
$\langle \vec{F}_{net} \rangle = \frac{\Delta(m\vec{v})}{t}$
$\Delta\vec{P} = \int F_{net}\, \mathrm{dt}$
$\vec{F}_{net} = m\vec{a}$
Collisions
Elastic Collisions
Inelastic Collisions
$\sum \vec{P}_i = \sum \vec{P}_f$ & $\sum E_{Ki} = \sum E_{Kf}$
$\sum \vec{P}_i = \sum \vec{P}_f$ & $\sum E_{Ki} > \sum E_{Kf}$
$V_{relative\ of\ Approach} = V_{relative\ of\ Separation}$

3.0 Introduction

Dynamics is the branch of mechanics where the forces that act on a body are **not in equilibrium**. The vector sum of the forces gives a resultant force that causes the body to accelerate. This resultant force causes change in motion. In Kinematics, we learned how a body would move under constant acceleration, if we combine the knowledge from Dynamics and Kinematics, we will be able to predict the motion of a body when we know the forces acting on the body. In the seventeenth century, Sir Isaac Newton formulated the three Newton's laws of motion and it is the basis behind Newtonian Mechanics. Today, Newtonian mechanics is useful for many engineering efforts in our everyday scale, like how an artillery shell travels in air, and it explains many phenomena observed.

3.1 Newton's 3rd Law of Motion

Newton's Third Law of Motion states that <u>when body A exerts a force on body B, body B will exert an oppositely directed force of equal magnitude on body A</u>.

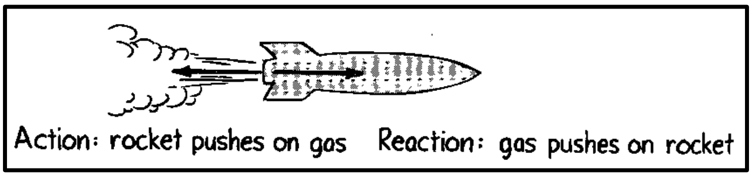
It should be noted that:

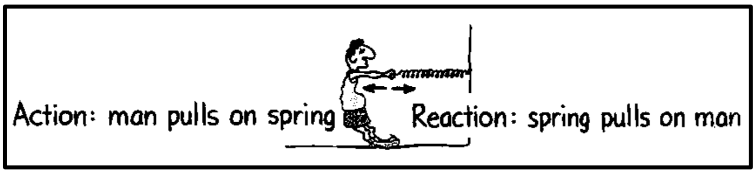
1. Newton's Third Law of Motion involves two different bodies; the 'action and reaction' pair of forces as stated within the law acts on **separate bodies** (if the 'action' acts on body A, then the 'reaction' must act on body B).
2. The pair of forces is equal in magnitude and opposite in direction; they must also be of the same type/nature. In other words, if the force that A exerts on B is a gravitational force, then the equal and opposite force exerted by B on A is also a gravitational force.

In summary:
The action and reaction pair must

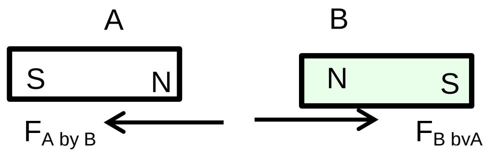
- act on different bodies
- be of the same type/nature
- have equal magnitude and act in opposite direction to one another

Examples of Newton's Third Law 'Action and Reaction' Pair of Forces





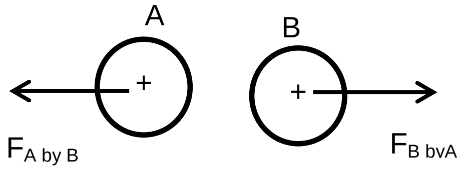
Magnetic Force



$F_{A\ by\ B}$: Magnetic force on magnet A due to magnet B.

$F_{B\ by\ A}$: Magnetic force on magnet B due to magnet A.

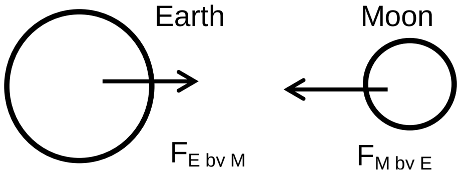
Electrostatic force



$F_{A\ by\ B}$: electrostatic force on electric charge A due to electric charge B.

$F_{B\ by\ A}$: electrostatic force on electric charge B due to electric charge A.

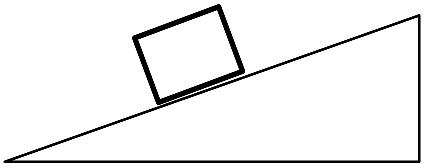
Gravitational force



$F_{E\ by\ M}$, Gravitational force on Earth due to Moon.

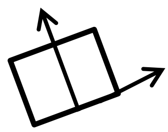
$F_{M\ by\ E}$, Gravitational force on Moon due to Earth.

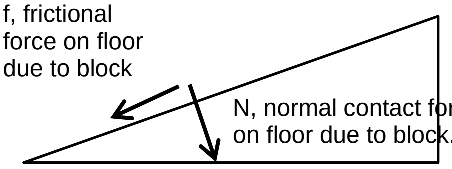
Contact force



Block resting on a slope.

Consider contact forces only:

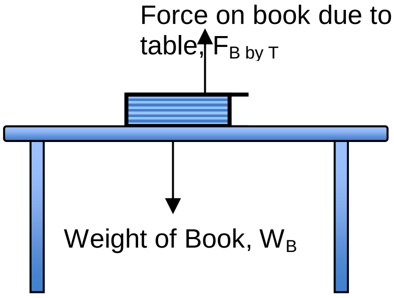




The resultant of the frictional force and the normal contact force acting on a body is known as **contact force**.

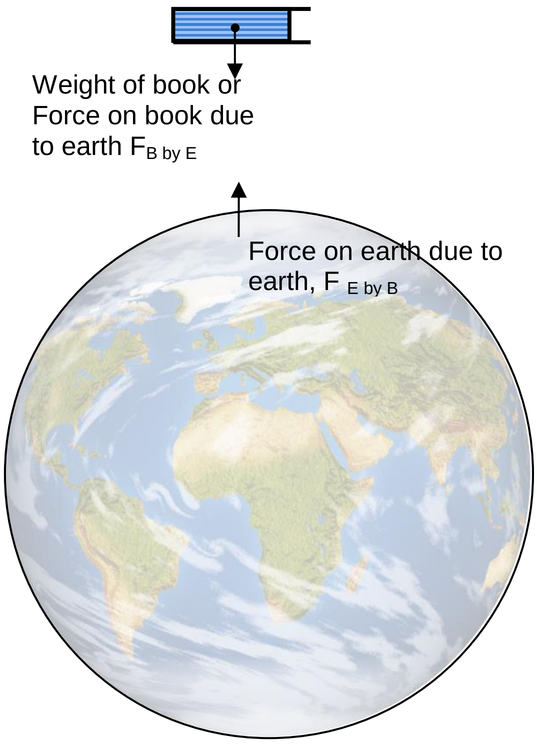
3.1.1 Identifying Action-Reaction Forces

For a book which is lying flat on a table, is the weight of the book and the force which the table acts on the book an action-reaction pair, and why?



Ans: No
1) W_B and $F_{B\ by\ T}$ are both acting on the same single body, i.e. the book itself! (Not on 2 different bodies)
2) W_B = gravitational force, and $F_{B\ by\ T}$ = contact force. (Not same type of force)

So, what is the action-reaction pair for weight of the book?



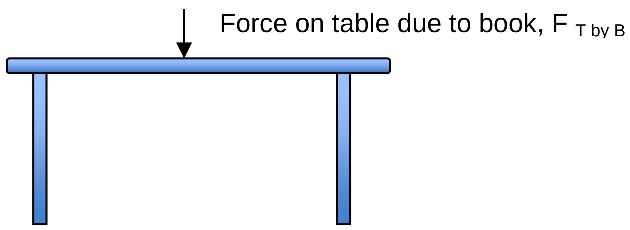
By Newton's 3[rd] law, the action-reaction pair must be a *gravitational force also*,

Force on earth by book, $F_{E\ by\ B}$

Action-Reaction Pair

What is the action-reaction pair for force on book due to the table, $F_{B\ by\ T}$?

In this context, by Newton's 3[rd] law, the action and reaction pair must also be a *contact force*, that is the Force on table by book, $F_{T\ by\ B}$.



Key Question

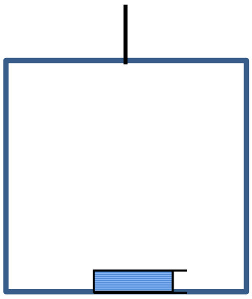
Although weight of the book is not force by book on table, can their magnitudes be the same? Why?

Yes: From FBD of book, when the book is in equilibrium, $W_B = F_{B\ by\ T}$.

No: When you throw the book onto the table, the force by the book on table (on impact) is more than the weight of the book,
i.e. $F_{BT} > W_B$!

Thinking question

The magnitude of the force by the table on book may not always be the same as the weight of the book. Imagine a book placed on the floor of a lift that is accelerating.



3.2 Newton's First Law of Motion

Newton's First Law of Motion states that an object continues to be in a state of rest or in motion with a constant velocity, unless acted upon by a net external force.

This law gives rise to the idea ***inertia*** as the resistance to change in the condition of rest or motion of a body. The inertia of a body can be considered as the reluctance of the body to start moving, as well as its reluctance to stop once it has begun moving. The ***mass*** of a body, quantified with a SI unit of kilogram (kg) is a measure of its inertia; the larger the mass, the larger the inertia of the body.

The ***weight*** of a body is defined as the force acting on it due to gravitational attraction by another body. Mathematically, the weight of a body at a point in space is given by $W = m\,g$, where m is the mass of the body and g the gravitational field strength at that point in space. Hence, the mass of a body will remain constant throughout the universe but its weight is dependent on the value of g at the point at which it is placed.

3.2.1 Linear Momentum

Linear momentum of a body is defined as the product of its mass and its velocity.

$$\boxed{\vec{P} = m\vec{v}}$$

Linear momentum is a vector quantity and it takes the same direction as the velocity of the body.

Momentum is the property of a body by virtue of its mass and velocity. For a body to gain a larger momentum, it will require more effort to accelerate it. Conversely, the more momentum a body possesses, the harder it will be to stop it.

3.3 Newton's 2nd Law of Motion

Newton's Second Law of Motion states that [1][the rate of change of momentum of a body is directly proportional to the net external force acting on it] and [2][takes place in the direction of the net external force].

$$\boxed{\frac{d\vec{P}}{dt} \propto \vec{F}_{net}}$$

* Since momentum (and hence change in momentum) is a vector quantity, Newton's 2nd Law as stated above can be interpreted in 2 parts as marked with the first part of the statement giving the magnitude of rate of change of momentum (how fast the change in momentum is taking place) while the second part provides the direction in which the change in momentum takes place.

Mathematically,

$$\frac{d\vec{P}}{dt} \propto \vec{F}_{net}$$

$$\frac{d\vec{P}}{dt} = k\vec{F}_{net}, \text{where } k \text{ is the proportionality constant}$$

In the case where SI units are used, k = 1. (since 1 N is defined as the force which produces an acceleration of 1 m s^{-2} when it is applied to a mass of 1 kg.) Hence,

$$\frac{d\vec{P}}{dt} = \vec{F}_{net}$$

Thus ***force*** may be defined as the rate of change of momentum.

$$
\begin{aligned}
\vec{F}_{net} &= \frac{d\vec{P}}{dt} \\
&= \frac{d(m\vec{v})}{dt} \\
&= m\frac{d\vec{v}}{dt}, for\ analysis\ where\ the\ mass\ of\ the\ body\ remains\ constant \\
\therefore \vec{F}_{net} &= m\vec{a}
\end{aligned}
$$

In analyzing situations in which the mass of the body remains constant, the recommended procedure is as follows:

1. Draw a free body diagram, identify and label all forces acting on the body.
2. Determine the net force acting on the body in the direction of acceleration.
3. Equate the net force to mass × acceleration to determine the unknowns. This mass is the mass of the free body considered.

* If a system is moving with velocity *v* and acceleration *a*, all bodies in that system will also move with velocity *v* and acceleration *a*.

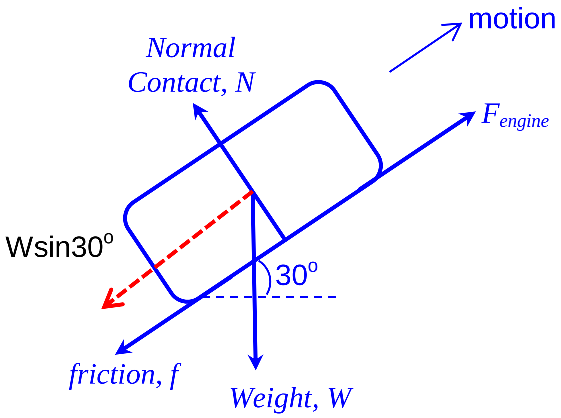
Example 1a

A car of mass 800 kg is moving up a hill inclined at 30° to the horizontal. The total frictional force on the car is 1000 N. Calculate the force *F* due to the engine on the car when the car is

(a) accelerating up the plane at 2.0 m s^{-2}.

(b) moving with a steady velocity of 15 m s^{-1}.

Considering a free body diagram of the car:



(a) Considering forces acting along the slope, taking the direction up the slope to be positive:

Using Newton's 2nd Law: $F_{net} = ma$

$$F_{engine} - f - W\sin 30^o = ma$$

$$F_{engine} = ma + f + W\sin 30^o$$
$$= (800)(2.0) + 1000 + (800)(9.81)\sin 30^o$$
$$= 6520 \text{ N}$$

(b) Considering forces acting along the slope, taking the direction up the slope to be positive:

Using Newton's 2nd Law: $F_{net} = ma$

$$F_{engine} - f - W\sin 30^o = 0$$
$$F_{engine} = f + W\sin 30^o$$
$$= 1000 + (800)(9.81)\sin 30^o$$
$$= 4920 \text{ N}$$

Example 1B

A 1000 kg block hangs on a rope. Find the tension in the rope if

(a) the block is stationary;

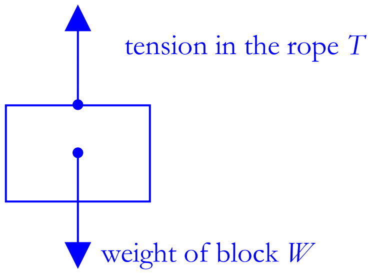
(b) the block is moving upward at a **constant speed** of 5.0 ms^{-1};

(c) the block is accelerating upward at 5.0 ms^{-2}.

You should identify the forces acting on the block, draw a free-body diagram then explicitly make use of Newton's 2nd Law.

(a) 9810 N

(b) 9810 N

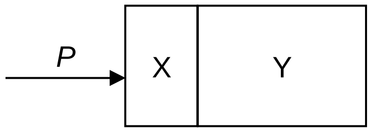


(c) Taking upwards as positive and using Newton's 2nd law:

$F_{net} = ma$
$T - W = ma$
$T = ma + W$
$= (1000)(5.0) + (1000)(9.81)$
$= 14810$ N

Example 2

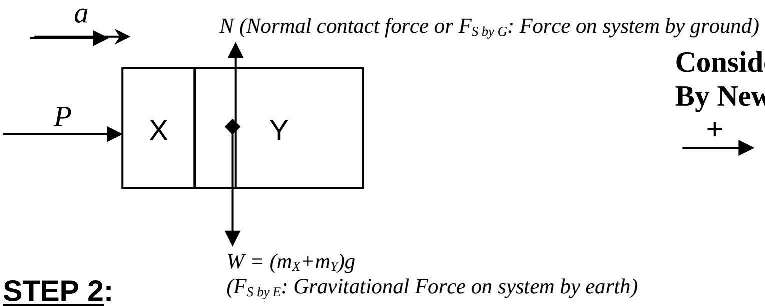
Two blocks X and Y, of masses m and 3m respectively, are accelerated along a smooth horizontal surface by a force P applied to block X as shown. What is the magnitude of the force exerted by block X on block Y during this acceleration?



Note: To solve this problem, you have to consider the free body diagram of X or Y depending on which one will give you the answer in shortest number of steps.

<u>STEP 1</u>:

- **Considering the free body of the 2 blocks as 1 system:**



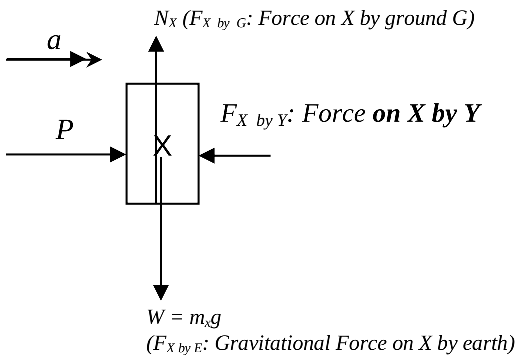
Considering the horizontal direction,
By Newton's 2nd Law: F_{net} = ma

$\xrightarrow{+}$: $P = (m+3m)a$

$$a = \frac{P}{4m}$$

<u>STEP 2</u>:

- **Considering a free body of block X**



Considering the horizontal direction,
By Newton's 2nd Law

$\xrightarrow{+}$ **: F_{net} = ma**

$$P - F_{XbyY} = m_x a$$

$$F_{XbyY} = P - m_x a$$

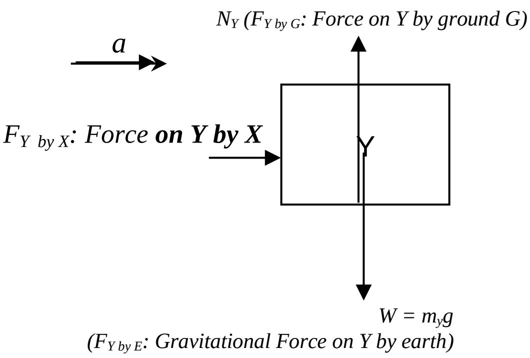
$$= P - m\left(\frac{P}{4m}\right)$$

$$= \frac{3P}{4}$$

By Newton's 3rd Law

$$\left|F_{XbyY}\right| = \left|F_{YbyX}\right| = \frac{3P}{4}$$

- **Considering a free body of block Y**



Considering the horizontal direction,
By Newton's 2nd Law

$\xrightarrow{+}$ **: F_{net} =ma**

$$F_{YbyX} = m_y a$$

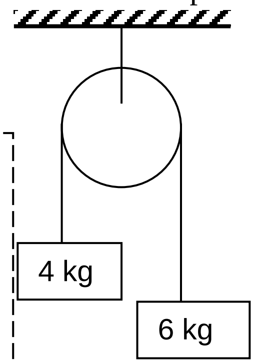
$$= 3m\left(\frac{P}{4m}\right)$$

$$= \frac{3P}{4}$$

Example 3

In the figure shown, masses 4 kg and 6 kg are connected by an inextensible thread looped over a smooth pulley.

Calculate the acceleration of the masses and the tension in the thread.



Apply "General Approach to Solving Problem"

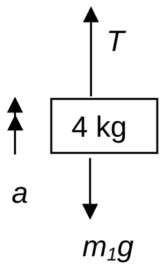
Key question to solving question:
What does "connected by an inextensible thread" and "smooth" imply?
It means that m_1 and m_2 share the same acceleration. The tensions acting on m_1 and m_2 are the same

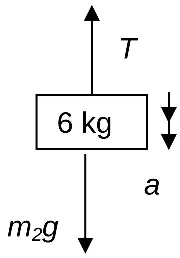
Is the acceleration of the masses g? Why?
No. Since m_1 and m_2 are not free-falling masses (i.e. acted only by their own weights), they do not fall at free-falling acceleration g.

The directions of acceleration should also be "continuous", i.e. if m_1 accelerates upwards, m_2 accelerates downwards, and vice versa.

Free Body Diagrams

In order to move upwards, the tension must be larger than the 4 kg weight. The net force acting on the 4 kg mass is upwards. Hence acceleration of the 4 kg mass is upwards.





In order to move downwards, the 6 kg weight must be larger than the tension. The net force acting on the 6 kg mass is downwards. Hence acceleration on the 6 kg mass is downwards.

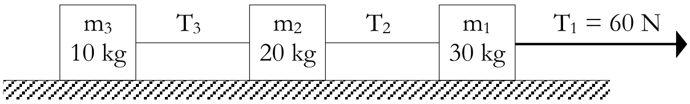
By Newton's Second Law,
4 kg mass : (taking upwards positive):
$F_{net} = ma$
$T - (4)(9.81) = 4a$ … (1)

6 kg mass : (taking downwards positive):
$F_{net} = ma$
$(6)(9.81) - T = 6a$ … (2)

Solving (1) and (2)
$a = 1.96$ m s^{-2}
$T = 47.1$ N

Example 4

Three blocks are connected as shown in the figure below on a horizontal frictionless table and pulled to the right with a force T_1 = 60 N. If m_1 = 30 kg, m_2 = 20 kg and m_3 = 10 kg, find the tensions T_2 and T_3.



Clue: All the blocks must be accelerating at the same rate.

Consider a free-body consisting of m_1, m_2 and m_3:
Apply Newton's 2nd Law → $F_{net} = T_1 = (60)\ a \rightarrow a = 1.0\ m\ s^{-2}$

Consider a free-body consisting of m_2 and m_3:
Apply Newton's 2nd Law → $F_{net} = T_2 = (30)(1.0) = 30\ N$

Consider a free-body consisting of m_3:
Apply Newton's 2nd Law → $F_{net} = T_3 = (10)(1.0) = 10\ N$

3.4 What is "weightlessness"?

We do not experience our weight or the gravitational force by the Earth directly. Instead, it is through the contact force that the **ground exerts on us** that we feel our weight. Normally, when we are on stationary ground, we feel our weight as constant.

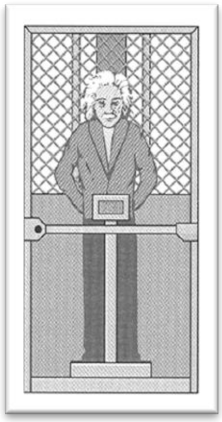
However, if we are in an accelerating lift, we begin to feel funny. This is due to the acceleration of the lift. The contact force that the floor exerts on us changes and we feel our 'weight' has changed.

If the lift cable was cut and the lift falls freely, the person then feels the sensation of **'weightlessness'**. When a person experiences **"weightlessness"**, it does not mean that the person has no weight. **'Weightlessness'** refers to the state where the body **does not experience** the effects of **contact forces**. In this case, the lift floor exerts **zero** contact force on the person.

If that person in the falling lift was to stand on a weighing scale, the reading on the scale would read zero. This is because weighing scales **do not measure our weight directly**. Instead, they measure the contact force or pressure our bodies exert on the weighing scale.

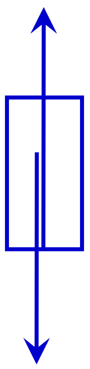
Remember your **actual weight** does not change. However, the contact force exerted on the weighing scale can change and hence the reading on the weighing scale changes giving you an **apparent weight**. (i.e. for calculation purposes the **apparent weight** is equal to the contact force on the *weighing scale)*

Example 5



A man of mass 80 kg stands on a platform scale in a lift.

(a) Draw a free-body diagram to show the forces acting on the man.



$N_{Man\ by\ Scale}$: Normal contact force on man by scale

* $|N_{Man\ by\ scale}| = |-F_{scale\ by\ Man}|$

* $F_{scale\ by\ Man}$ gives the reading on the scale

$W_{Man\ by\ earth}$, Gravitational force on man by earth= (80)(9.81)=785 N

(b) Calculate the reading on the scale during the motion of the lift:

i. Lift is stationary

<u>Lift is going up to a higher level:</u>

ii. accelerating at 2.0 m s^{-2}.

iii. moving at a constant speed of 5.0 m s^{-1}.

iv. decelerating at 2.0 m s^{-2}.

<u>Lift is going down to a lower level:</u>

v. accelerating downwards at 2.0 m s^{-2}.

vi. moving at constant speed of 5.0 m s^{-1}.

vii. decelerating at 2.0 ms^{-2}.

viii. Lift is faulty and free-falling:

Taking vectors in the upward direction to be positive:

i.

$$\boldsymbol{N - W = 0}$$
$$\boldsymbol{N = W = mg = 785\ \mathrm{N}}$$

ii.

$$\boldsymbol{N - W = ma}$$
$$\boldsymbol{N = (80)(2.0) + W}$$
$$\boldsymbol{= 945\ \mathrm{N}}$$

iii.

$$\boldsymbol{N - W = 0}$$
$$\boldsymbol{N = W = 785\ \mathrm{N}}$$

iv.

$$\boldsymbol{N - W = ma}$$
$$\boldsymbol{N = (80)(-2.0) + W}$$
$$\boldsymbol{= 624\ \mathrm{N}}$$

OR:

Taking vectors in the downward direction to be positive:

iv.

$$\boldsymbol{W - N = ma}$$
$$\boldsymbol{N = W - (80)(-2.0)}$$
$$\boldsymbol{= 624\ \mathrm{N}}$$

v. W- N = ma

N = W-ma

vi. W – N = 0

N = W

vii. N- W = ma

N = ma +W

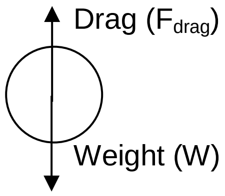
viii. N – W = ma

Free fall => a =g

N – mg = mg

N = 0

3.5 Air resistance and Terminal velocity



If there was no air resistance or drag, all objects close to the earth should fall with the same acceleration g.

With air resistance, the **resultant** force experienced by the object is reduced. In this case, it's **W - F_{drag}**. Since, the mass of the object does not change, from Newton's 2nd law, a reduction in the resultant external force will cause the acceleration to be lower. This is why with air resistance an object takes a longer time to fall.

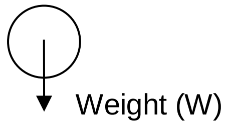
<u>With air resistance:</u>
Using Newton's 2nd law,
Fnet = ma'
W- F_{drag} = ma'
a'= (mg - F_{drag})/m
= g - F_{drag}/m

<u>Without air resistance:</u>
Using Newton's 2nd law,
Fnet = ma

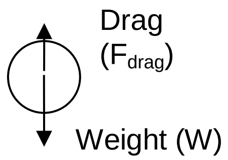
mg = ma
a = g

Therefore a'< a or a'<g.
Now, air resistance **increases** with speed. The faster the object moves, the greater the amount of air resistance it experiences. Initially, when an object falls from rest, it has zero speed and it experiences an acceleration of g. However, as it **accelerates**, it increases in speed and the air resistance **increases**.

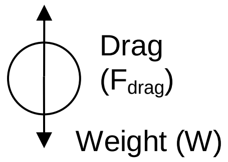
This in turn reduces the resultant force experienced by the object until the air resistance is equal to the weight of the object. At this point, the object has **zero** resultant force and **zero** acceleration. The velocity of the object stops increasing and achieves a final value which we call the terminal velocity.



Object at rest – Resultant force is its weight, acceleration is g.



Object accelerates or increases in velocity –
Resultant force is $\mathbf{W - F_{drag}}$,
Acceleration is ($\mathbf{W - F_{drag}}$)/m. Acceleration is reduced!



Object stops accelerating, reaches terminal velocity –
Resultant force is $\mathbf{W - F_{drag}}$, $\mathbf{W = F_{drag}}$ so Resultant force = 0 N
Acceleration is 0 m s^{-2}.

Examples of "weightlessness"

- A free-falling parachutist before parachute is deployed (ignoring air resistance)
- A free-falling bungee jumper before the cord experiences tension
- An astronaut in a spacecraft which is in orbit around the Earth

Note: In all the examples above, the person is accelerating with g. Their weight is constant. The gravitational force exerted on them is constant.

Thinking Question:
Are these examples illustrate "weightlessness" as well?

a) A parachutist falling to earth with his parachute opened. The **harness** of the parachute **pulling** on him.
b) A scuba diver floating underwater. No. He feels the **upthrust** of water.
c) *A bungee jumper when the cord is under tension. No. He feels the* ***tension*** *of the cord.*

3.6 Impulse

Problems in Newtonian mechanics that have been analyzed thus far using concepts of kinematics and forces are generally limited to situations involving a constant net force (and thus a constant acceleration). In the real world, this is seldom the case. Understanding the concept of impulse will allow a better analysis of real world phenomena which usually involves forces that vary with time.

When we exert a force F on a body for a time period t, it is can be said that an impulse is exerted by the force on the body. The ***impulse*** of a force is defined as the integral of a force over the time interval during which the force acts.

Mathematically, the impulse of the force F acting on the object between times t_1 and t_2 is given by

$$\textbf{impulse} = \int_{t_1}^{t_2} \boldsymbol{F}\,\mathrm{dt}$$

Graphically, impulse is given by the area under the F-t graph.

3.6.1 Impulse-Momentum Theorem

Considering Newton's Second Law of Motion,

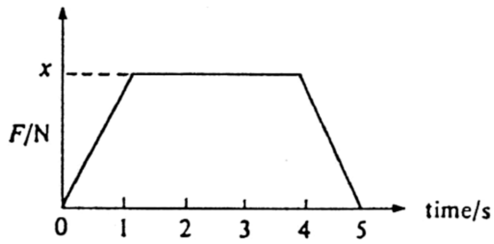
$$\frac{d\vec{P}}{dt} = \vec{F}_{net}$$

$$\Rightarrow \int_{t_1}^{t_2} \vec{F}_{net}\, dt = \Delta\vec{P}\Big|_{from\ t_1\ to\ t_2}$$

The above relationship gives rise to the *impulse-momentum theorem* which states that the impulse of the net force acting on a body between times t_1 and t_2 is equal to the body's change in momentum within this time interval.

Example 6

When a force F, varying as shown, is applied to a mass of 10 kg, the gain in momentum in 5 s is 40 kg m s^{-1}.



Determine the value x.

Change in Momentum = Area under $F - t$ graph

$$\mathbf{40 = \tfrac{1}{2}(5+3)\,x}$$

$$\mathbf{x = 10\ N}$$

3.6.2 Average Force

In situations where forces act only briefly on a body (e.g. the force of the ground on a bouncing ball, impact of a bullet on its target), it is difficult and sometimes impractical to ascertain the variation of the forces with time.

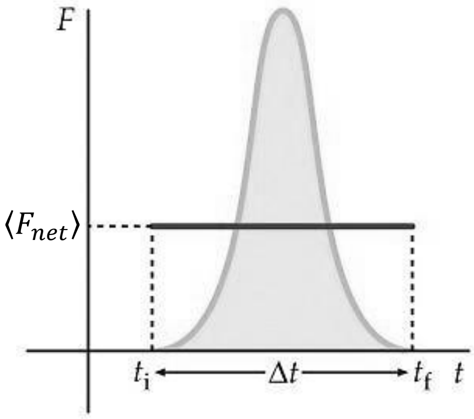
For ease of analysis in such situations, the average force that causes the same change in momentum is usually considered instead.

$$\vec{F}_{net} = \frac{d\vec{P}}{dt}$$

$$\langle \vec{F}_{net} \rangle = \frac{\Delta \vec{P}}{\Delta t}$$

$$\therefore \langle \vec{F}_{net} \rangle \Delta t = \Delta \vec{P}\Big|_{for\ the\ time\ interval\ \Delta t}$$

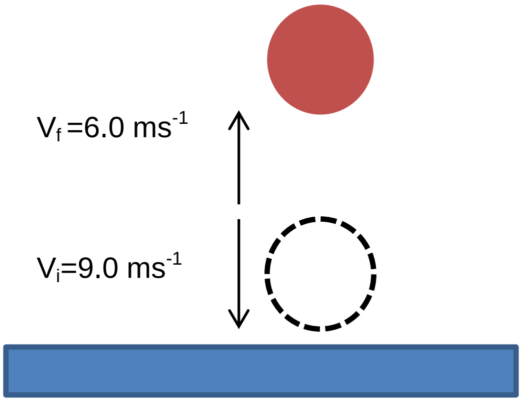
Graphically, this method of analysis ensures that the area under the F-t graph is equal to the area under the $\langle F_{net} \rangle$-t graph for the time interval Δt in which F acts.



Hence it should be noted that the $\langle F_{net} \rangle$ determined will, in general, be smaller than the peak force experienced by the body.

Example 7

A basketball of mass 0.62 kg is bounced horizontally off a flat vertical surface. The basketball has an initial speed of 9.0 m s^{-1} and rebounds with a speed of 6.0 m s^{-1}. If the basketball is in contact with the surface for 15 ms, determine the magnitude of the average force the surface exerts on the basketball.



Taking upwards as positive.

$$\textit{Resultant force on ball}, F_{net} = \frac{\Delta P_{ball}}{\Delta t}$$

$$= \frac{m_{ball} \Delta V_{ball}}{\Delta t}$$

$$= \frac{(0.62)[6.0 - (-9.0)]}{15 \times 10^{-3}} = 620\text{ N}$$

F_{avg} – mg = Fnet = 620
Favg = 620 + (0.62)(9.81) = 626 N

Example 8

A conveyor belt is used to transfer luggage at an airport. It consists of a horizontal endless belt running over driving rollers, moving at a constant speed of 1.5 m s^{-1}. The rate at which baggage is placed on the belt is 20 kg s^{-1}. What is its value of the driving force F needed?

In 1 s, the change in horizontal momentum experienced by the baggage

$= m\Delta v$

= 20 (1.5 – 0)

= 30 kg m s^{-1}

Hence the average force needed (to change the horizontal momentum of the baggage)

$$= \frac{\Delta P_{baggage}}{t}$$

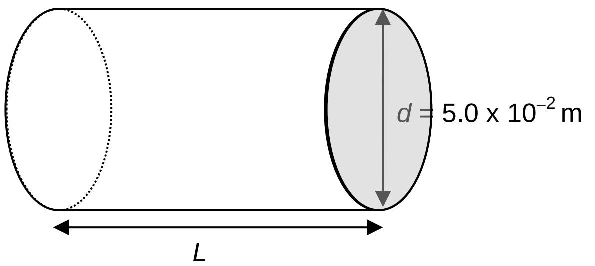
$$= \frac{30}{1} = 30 \text{ N}$$

Example 9

A jet of water leaves the nozzle of a hose of diameter 5.0 × 10^{-2} m with a speed of 0.400 m s^{-1}. The water is directed perpendicularly to the wall and it can be assumed that the water does not rebound. The density of water is 1000 kg m^{-3}.

Calculate the force exerted on the wall by the water. [0.314 N]

Given:
Diameter of hose, d =5.0 x 10^{-2} m
Speed of water, v = 0.400 m s^{-1}
Density of water, ρ = 1000 kg m^{-3}



Taking:
Horizontal distance travelled by water, L
Time taken, t
Volume of water, V

Using Newton's 2$^{\text{nd}}$ Law,

Force by wall on water= $\langle F_{net} \rangle$ = *mass per unit time* $\times \Delta v$

$$= \frac{m}{\Delta t}\Delta v = \frac{\rho(Vol)}{\Delta t}\Delta v \qquad \text{since } m = \rho(Vol)$$

$$= \frac{\rho \times \pi(\frac{d}{2})^2 L}{t}\Delta v$$

$$= \rho \times \pi(\frac{d}{2})^2 v\Delta v \qquad \text{(Since v=L/t)}$$

$$= \rho\pi(\frac{d}{2})^2 v(0 - v)$$

$$= -\rho\pi(\frac{d}{2})^2 v^2$$

$$= -\frac{(1000)\pi(5.0 \times 10^{-2})^2(0.400)^2}{4}$$

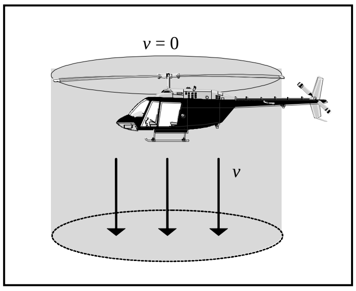
$$= -0.314 \text{ N}$$

By Newton's 3rd Law, Force on wall by water = Force by water by wall = 0.314 N

General Equation: Force on wall by water = $\rho A v^2$ [water does not rebound]

Example of moving air mass:

Helicopter hovering at constant height:
Taking downwards as positive,



Rate of gain in momentum of air =d*p*/d*t* , downwards
(Force on air by helicopter)

By Newton's 3rd law,
Force imparted on helicopter by air, = *dp/*d*t* , upwards
= Mg, weight of helicopter
when the helicopter is hovering.

3.7 Principle of Conservation of Linear Momentum

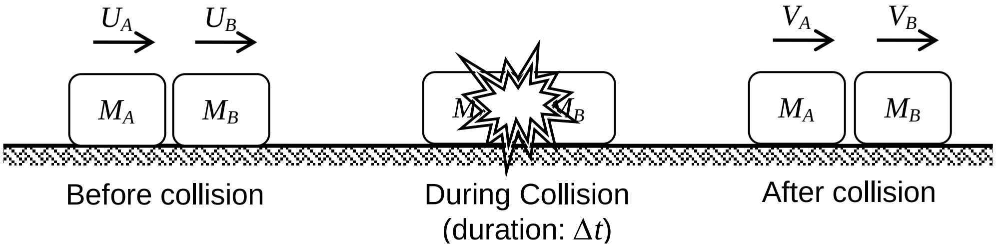
Considering Newton's Second Law of Motion,

$$\frac{d\vec{P}}{dt} = \vec{F}_{net}$$

and the situation in which no net force acts on a body, then it must follow that that the body will experience no change of linear momentum. This is consistent with Newton's First Law of Motion (which is usually seen as a special case of Newton's Second Law of Motion) and this idea gives rise to the Principle of Conservation of Linear Momentum.

The ***Principle of Conservation of Linear Momentum*** states that the total linear momentum of a system of interacting particles remains unchanged provided no net external force acts on the system.

Considering body A with mass M_A and velocity U_A colliding head-on with body B with mass M_B and velocity U_B along a frictionless surface. The duration of impact is Δt. After the collision, body A and B move off with velocities V_A and V_B respectively.



By Newton's Third Law of Motion:

$$\vec{F}_{A\ on\ B} = -\vec{F}_{B\ on\ A}$$

In the absence of other forces, $\vec{F}_{A\ on\ B} = \frac{d\vec{P}_{of\ B}}{dt}$ and $\vec{F}_{B\ on\ A} = \frac{d\vec{P}_{of\ A}}{dt}$

$$\therefore \frac{d\vec{P}_{of\ B}}{dt} = -\frac{d\vec{P}_{of\ A}}{dt}$$

$$\frac{\Delta\vec{P}_{of\ B}}{\Delta t} = -\frac{\Delta\vec{P}_{of\ A}}{\Delta t}$$

$$m_B V_B - m_B U_B = -(m_A V_A - m_A U_A)$$

$$m_A U_A + m_B U_B = m_A V_A + m_B V_B$$

The final relationship shows that the total initial momentum before collision is equal to the total final momentum before collision, implying that momentum is conserved!

Do note that although the total momentum of the system (consisting of bodies A and B) remains unchanged, the momenta of body A and body B individually has changed due to the net force that they exert on one another during the collision.

Example 10

The diagram shows two trolleys, X and Y, about to collide and gives the momentum of each trolley before the collision.

After the collision, the directions of motion of both trolleys are reversed and the magnitude of the momentum of X is then 2 N s. What is the magnitude of the corresponding momentum of Y?

By Conservation of Momentum:

$$\sum \vec{P}_i = \sum \vec{P}_f$$

$\rightarrow +ve$: $\mathbf{20 + (-12) = (-2) + P_{Yf}}$

$$\mathbf{P_{Yf} = 10\ Ns}$$

to the right

Even as the Principle of Conservation of Linear Momentum is used to analyze collisions, different collisions have their specific characteristics which will in turn yield other relationships that can aid in their analysis.

3.7.1 Elastic Collisions

A collision in which no mechanical energy is lost is called an ***elastic collision***. As such, kinetic energies of the colliding bodies are conserved.

- Considering an elastic head on collision between body A and body B along a frictionless surface:
  - Note: In **head-on** collisions, the velocity/velocities of the colliding objects are collinear (directed along the same straight line) both before and after the collision.

- Since no external force acts on the 2 body system,
- by Conservation of Linear Momentum,

$$m_A\vec{U}_A + m_B\vec{U}_B = m_A\vec{V}_A + m_B\vec{V}_B$$
$$m_A(\vec{U}_A - \vec{V}_A) = m_B(\vec{V}_B - \vec{U}_B) \ldots\ldots(1)$$

> Since the collision is elastic,
> by Conservation of Kinetic Energy

$$\frac{1}{2}m_A\vec{U}_A^{\ 2} + \frac{1}{2}m_B\vec{U}_B^{\ 2} = \frac{1}{2}m_A\vec{V}_A^{\ 2} + \frac{1}{2}m_B\vec{V}_B^{\ 2}$$
$$m_A\left(\vec{U}_A^{\ 2} - \vec{V}_A^{\ 2}\right) = m_B\left(\vec{V}_B^{\ 2} - \vec{U}_B^{\ 2}\right)$$
$$m_A(\vec{U}_A - \vec{V}_A)(\vec{U}_A + \vec{V}_A) = m_B(\vec{V}_B - \vec{U}_B)(\vec{V}_B + \vec{U}_B) \ldots\ldots(2)$$

$$\frac{(2)}{(1)}: \qquad (\vec{U}_A + \vec{V}_A) = (\vec{V}_B + \vec{U}_B)$$
$$\vec{U}_A - \vec{U}_B = \vec{V}_B - \vec{V}_A$$

This equation is known as the relative speed relation between two bodies undergoing elastic collision. Hence, for an elastic collision, the relative speed of approach is always equal to the relative speed of separation, regardless of the masses of the bodies.

Elastic collisions are ideal and the only real example is the collision of molecules. In reality, collisions between everyday objects will normally result in a loss of mechanical energy typically through thermal energy, sound and plastic deformation of the bodies.

3.7.2 Inelastic Collisions

In real life, everyday situations, collisions are usually inelastic. An ***inelastic collision*** is a collision where kinetic energy is not conserved (more specifically kinetic energy is lost). In a ***perfectly inelastic collision***, the colliding bodies will coalesce with one another and move off with the same velocity.

General Approach to Solving Problems involving Collisions of Two Bodies or Separation of Two Bodies from a Single Body.

1. Identify whether the question is a separation, elastic collision or inelastic collision.

2. Draw diagrams of what happens before and after the collision / separate (velocity and mass of object to be included).

3. In the absence of external forces acting on the system of colliding bodies, use the Principle of Conservation of Linear Momentum to form an equation. Take note that the direction of motion is important.

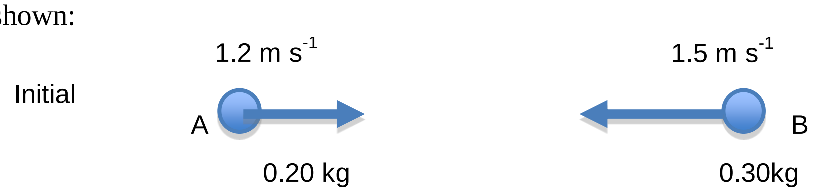
4. a. If collision is known to be elastic, use the fact that kinetic energy is conserved in the collision or that relative speed of separation = relative speed of approach to form another equation.

   b. If collision is perfectly inelastic collision, the two bodies will stick/coalesce together move with a common velocity. Account for the loss in KE.

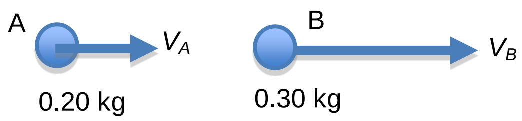
Note: The solving of simultaneous equations involving Conservation of KE may lead to complicated quadratic equations. This can be avoided by using the method of Relative Speeds of Approach and Separation shown below.

Example 11

Two balls A and B collide with each other head-on and elastically. Their masses and initial velocities are as shown:



Determine their velocities after collision. [V_A = 2.04 m s^{-1} left; V_B = 0.660 m s^{-1} right]



Take note that direction of final velocities are randomly assumed to be to the right.

Using principle of conservation of momentum (taking right direction to be positive)
Sum of initial momentum = Sum of final momentum

$M_AU_A + M_B U_B = M_A V_A + M_BV_B$

The positive signs in the generic equation should not be changed. But, V_A and V_B adopted different signs based on their directions when substituted into the equation.

$(0.20)(1.2) + (0.30)(-1.5) = (0.20)(V_A) + (0.30)V_B$

$-0.21 = 0.20V_A + 0.30V_B$ … (1)

Method 1

As Collision is Elastic, Kinetic energy is conserved

Sum of initial kinetic energy = Sum of final kinetic energy

$\frac{1}{2} M_A U_A^2 + \frac{1}{2} M_B U_B^2 = \frac{1}{2} M V_A^2 + \frac{1}{2} M_B V_B^2$

$\frac{1}{2}(0.20)(1.2)^2 + \frac{1}{2}(0.30)(-1.5)^2 = \frac{1}{2}(0.20)V_A^2 + \frac{1}{2}(0.30)V_B^2$

$0.4815 = 0.10V_A^2 + 0.15V_B^2$ … (2)

Solving (1) and (2) Simultaneously

V_A = -2.04 m s^{-1} (leftwards)
V_B = 0.660 m s^{-1} (rightwards)

Take note that if the assumed directions of velocities were wrong, the final values calculated will be negative. There is no need to re-calculate, just take note when describing the direction, if necessary.

Method 2:

As Collision is Elastic,

Relative speed of separation = Relative speed of approach

$V_B - V_A = 1.2 + 1.5$
$= 2.7$ … (2)

Solving Eqn (1) and (2) Simultaneously

V_A = -2.04 m s^{-1} (leftwards)
V_B = 0.660 m s^{-1} (rightwards)

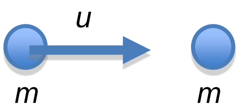
Take note that if the assumed directions of velocities were wrong, the final values calculated will be negative. There is no need to re-calculate, just take note when describing the direction, if necessary.

Example 12

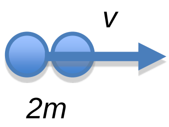
A particle of mass *m* moving with speed *u* makes a head-on collision with an identical particle which is initially at rest. The particles coalesce and move off with a common velocity.

(a) Determine the common speed of the particles after the collision.
(b) Determine the ratio of the kinetic energy of the system after the collision to that before.
(c) Explain what happens to the kinetic energy that is 'lost'.

Initial



Final



(a) Using the principle of conservation of momentum,
Sum of initial momentum = Sum of final momentum

$$mu = 2mv$$

$$v = \frac{1}{2}u$$

(b) Initial Kinetic Energy of system = $\frac{1}{2}mu^2$

Final Kinetic Energy of system $= \frac{1}{2}(2m)v^2$

$$= mv^2$$

$$= m\left(\frac{1}{2}u\right)^2 \qquad \left(\text{As } v = \frac{1}{2}u\right)$$

$$= \frac{1}{4}mu^2$$

Ratio of Final KE to Initial KE $= \dfrac{\frac{1}{4}mu^2}{\frac{1}{2}mu^2} = \dfrac{1}{2}$

(c) This kinetic energy of the system has become an increase in internal energy in the particles as well as sound.

3.7.3 Special Case: Separation of Objects

- Problems involving separation of objects involve a single mass which separates, such as 'explodes' or splits up.

- By using the principle of conservation of momentum, the directions and the velocities of the different pieces may be analyzed.

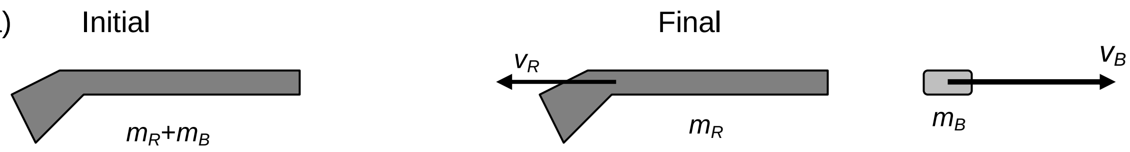
Example 13 (Separation of Objects)

a) Calculate the recoil velocity of a 5.0 kg rifle that shoots a 0.020 kg bullet out with a velocity of 620 m s^{-1}. [2.5 m s^{-1}]

b) Explain whether the rifle or the bullet has a higher kinetic energy.

---

a)



where m_R is the mass of the rifle and m_B the mass of the bullet.

By principle of conservation of momentum,
(Taking right direction to be positive),

Sum of initial momentum = Sum of final momentum

$$(m_R + m_B) \times 0 = m_R\,(-v_R) + m_B\,v_B$$
$$0 = -m_R\,v_R + m_B\,v_B$$
$$m_R\,v_R = m_B\,v_B$$
$$(5.0)(v_R) = (0.020)(620)$$
$$v_R = 2.5 \text{ m s}^{-1}$$

(Notice how small the velocity of the rifle really is)

Note that another useful method of determining KE is by the following formula:

$$KE = \frac{p^2}{2m}$$

b) Since $KE = ½\, mv^2$ and $p = mv$, $KE = p^2 / 2m$. Since the magnitude of the momentum of the rifle and the bullet is the same, and the mass of the bullet is smaller than that of the rifle, the bullet has a larger kinetic energy.

LOSS OF ENERGY DURING COLLISIONS

In most collisions, provided the time of interaction is rather short, momentum is usually conserved. However **some loss in kinetic energy usually takes place**. The loss of kinetic energy is usually converted to sound, heat or other forms of electromagnetic energy.

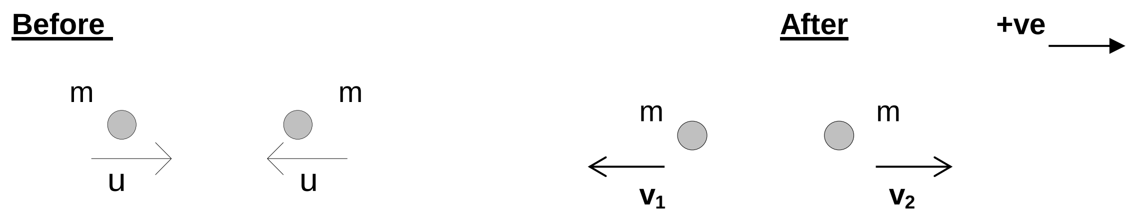
Summary

	Elastic Collisions		***Perfectly Inelastic Collisions***	
	Before & After Collision	*During Collision*	*Before & After Collision*	*During Collision*
Total Linear Momentum*	*Conserved*		*Conserved*	
Total Energy of System	*Conserved*		*Conserved*	
Total k.e. of the System	*Conserved*	*Not conserved, converted to pe*	*Not conserved*	*Not conserved*
Relative Speed Relation	*Applicable*	*Not applicable*	*Not applicable*	*Not applicable*

Appendix

(I) Special Examples of Totally Elastic Collision

1) Masses and speed of the particles are the same



By Principle of conservation of momentum,

$$mu + m(-u) = m(-v_1) + mv_2$$

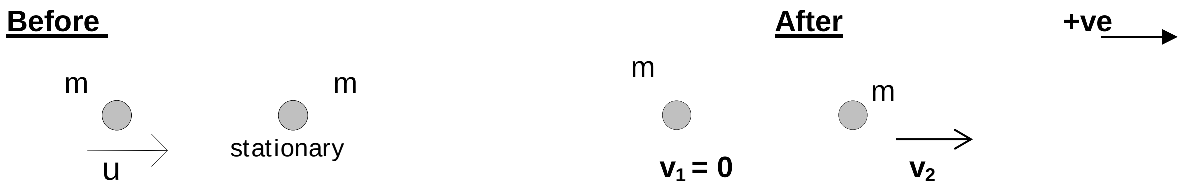
$$v_1 = v_2$$

By Principle of conservation of energy,

$$\tfrac{1}{2} mu^2 + \tfrac{1}{2} mu^2 = \tfrac{1}{2} mv_1^2 + \tfrac{1}{2} mv_2^2$$
$$u^2 = v_1^2$$
$$u = v_1$$

Conclusion:
Under totally elastic condition, the two objects of the after collision will move off in the opposite direction to one another, each with the same speed as before.

2) Masses of particles are the same with one particle at rest



By Principle of conservation of momentum,

$$mu + m(0) = mv_1 + mv_2$$
$$u = v_1 + v_2 \quad \text{--- [1]}$$

By Principle of conservation of energy,

$$\tfrac{1}{2}mu^2 + \tfrac{1}{2}m(0)^2 = \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}mv_2^2$$
$$u^2 = v_1^2 + v_2^2 \quad \text{--- [2]}$$

Subst eqn[1] in eqn [2]: $(v_1 + v_2)^2 = v_1^2 + v_2^2$

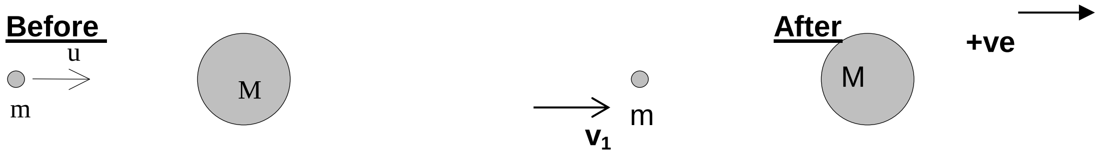
$2v_1v_2 = 0$,

therefore $v_1 = 0$

From eqn [1]: $v_2 = u$

Conclusion:
Under totally elastic condition, after collision the stationary particle will move off with a speed of the initial particle while the initially moving particle will come to rest.

3) Mass of the incoming particle being much smaller than the stationary particle (M >> m)



By Principle of conservation of momentum,

$$mu + M(0) = mv_1 + Mv_2$$
$$m(u - v_1) = Mv_2 \quad \text{---[1]}$$

By Principle of conservation of energy,

$$\tfrac{1}{2}mu^2 + \tfrac{1}{2}M(0)^2 = \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}Mv_2^2$$
$$m(u^2 - v_1^2) = Mv_2^2 \quad \text{--- [2]}$$

Eqn[2]/Eqn[1] : $u + v_1 = v_2$ --- [3]

From Eqn[1]: $u - v_1 = v_2 M/m$ --- [4]

Eqn[3]+ Eqn[4] : $2u = v_2[1+(M/m)]$

$v_2 = 2u\,[m/(M+m)]$

M>>m, $v_2 \approx 0$

Therefore, from Eqn [3], $u \approx -v_1$

Conclusion:
Under totally elastic condition, after collision the stationary particle of much bigger mass will remain stationary while the incoming particle will move off in the opposite direction of it's initial motion with it's speed approximately unchanged.