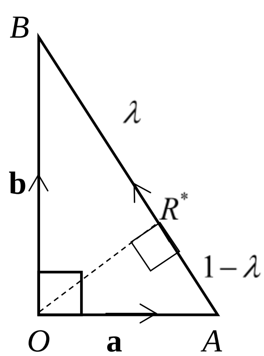
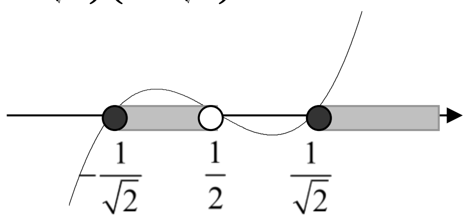
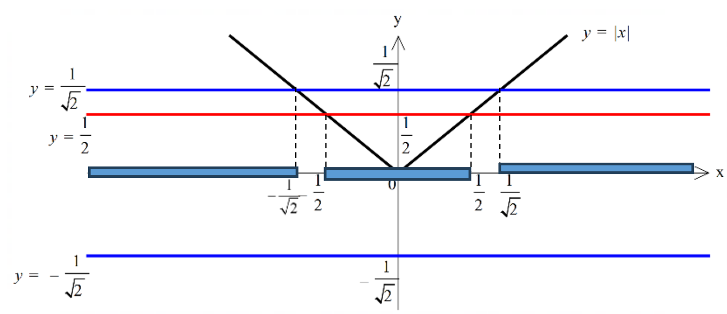
Q1 **[4]**	**Method 1** $\frac{\mathrm{d}y}{\mathrm{d}x}=a+\frac{b}{x}-\frac{c}{x^2}$ At $x=\frac{3}{2}$, $\frac{\mathrm{d}y}{\mathrm{d}x}=0 \quad \Rightarrow a+\frac{2}{3}b-\frac{4}{9}c=0 \quad ----(1)$ The gradient of D at x = 1 is equal to the gradient of the line $y=x-5$. At $x=1$, $\frac{\mathrm{d}y}{\mathrm{d}x}=1 \quad \Rightarrow a+b-c=1 \qquad ----(2)$ y-coordinate of D at $x=1$ is $1-5=-4$. Substituting $x=1$ and $y=-4$ into equation of D, we get $a+c=-4 \quad ----(3)$ From GC, $a=2, b=-7, c=-6$. **Method 2** $\frac{\mathrm{d}y}{\mathrm{d}x}=a+\frac{b}{x}-\frac{c}{x^2}$ At $x=\frac{3}{2}$, $\frac{\mathrm{d}y}{\mathrm{d}x}=0 \quad \Rightarrow a+\frac{2}{3}b-\frac{4}{9}c=0 \quad ----(1)$ Gradient of D at $x=1$ is $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=1}=a+b-c$. y-coordinate of D at $x=1$ is $a+c$. So, the equation of tangent to D at $x=1$ is $y-(a+c)=(a+b-c)(x-1) \Rightarrow y=(a+b-c)x-b+2c$ Comparing this line with $y=x-5$, we get $a+b-c=1 \qquad ----(2)$ $-b+2c=-5 \qquad ----(3)$ From GC, $a=2, b=-7, c=-6$.

Q2	Solution
(a) **[1]**	The set of all possible positions of R is the **line** that passes through points A and B, or The set of all possible positions of R is the **line** that passes through point B (or A) and parallel to the vector $\overrightarrow{AB}$.
(b) **[1]**	$\mathbf{a}\cdot\mathbf{b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos 90°=0$ since $\cos 90°=0$ **Additional Note:** That the scalar product is 0 because 2 vectors are perpendicular (or the angle between them is $90°$) is a consequence of this definition.
(c) **[4]**	$\overrightarrow{OR^*}=\lambda\mathbf{a}+(1-\lambda)\mathbf{b}$ for some $\lambda\in\mathbb{R}$. $\overrightarrow{OR^*}\cdot(\mathbf{a}-\mathbf{b})=0$ $\left[\lambda\mathbf{a}+(1-\lambda)\mathbf{b}\right]\cdot(\mathbf{a}-\mathbf{b})=0$ $\lambda\lvert\mathbf{a}\rvert^2-(1-\lambda)\lvert\mathbf{b}\rvert^2+(1-2\lambda)\mathbf{a}\cdot\mathbf{b}=0$ Since **a** and **b** are perpendicular, $\mathbf{a}\cdot\mathbf{b}=0$ $\therefore\lambda=\dfrac{\lvert\mathbf{b}\rvert^2}{\lvert\mathbf{a}\rvert^2+\lvert\mathbf{b}\rvert^2}$ $\overrightarrow{OR^*}=\dfrac{\lvert\mathbf{b}\rvert^2}{\lvert\mathbf{a}\rvert^2+\lvert\mathbf{b}\rvert^2}\mathbf{a}+\dfrac{\lvert\mathbf{a}\rvert^2}{\lvert\mathbf{a}\rvert^2+\lvert\mathbf{b}\rvert^2}\mathbf{b}$ $AR^*:BR^*=1-\lambda:\lambda$ $=\dfrac{\lvert\mathbf{a}\rvert^2}{\lvert\mathbf{a}\rvert^2+\lvert\mathbf{b}\rvert^2}:\dfrac{\lvert\mathbf{b}\rvert^2}{\lvert\mathbf{a}\rvert^2+\lvert\mathbf{b}\rvert^2}$ $=\lvert\mathbf{a}\rvert^2:\lvert\mathbf{b}\rvert^2$ 

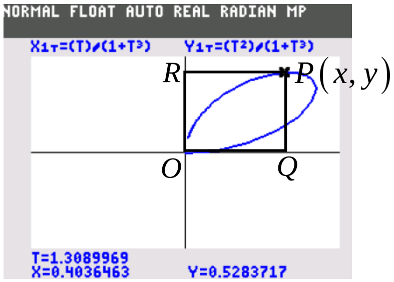
Q3	Solution
[4]	$\dfrac{6x^2+2x-3-2(x+1)(2x-1)}{2x-1} \geq 0, \qquad x \neq \dfrac{1}{2}$ $\dfrac{2x^2-1}{2x-1} \geq 0$ $2\left(x-\dfrac{1}{\sqrt{2}}\right)\left(x+\dfrac{1}{\sqrt{2}}\right)(2x-1) \geq 0$  $-\dfrac{1}{\sqrt{2}} \leq x < \dfrac{1}{2}$ or $x \geq \dfrac{1}{\sqrt{2}}$ **Additional Notes:** You are strongly advised to use ( ) if you are making algebraic errors in arriving at $\dfrac{2x^2-1}{2x-1} \geq 0$. For students who multiply by $(2x-1)^2$ in the first step, you should always factorize first before any expansion, as shown below: $(6x^2+2x-3)(2x-1)-2(x+1)(2x-1)^2 \geq 0$ $\Leftrightarrow (2x-1)\left[(6x^2+2x-3)-2(x+1)(2x-1)\right] \geq 0$ This will avoid unnecessary algebraic manipulation.
[3]	 By replacing x with $\|x\|$,

	$-\frac{1}{\sqrt{2}} \leq \lvert x \rvert < \frac{1}{2}$ or $\lvert x \rvert \geq \frac{1}{\sqrt{2}}$ $x \leq -\frac{1}{\sqrt{2}}$ or $-\frac{1}{2} < x < \frac{1}{2}$ or $x \geq \frac{1}{\sqrt{2}}$ **Additional Note:** Perhaps the simplest way to think of solving modulus inequalities like $\lvert x \rvert < \frac{1}{2}$ would be to tell yourself if the **magnitude*** of *x* is smaller than $\frac{1}{2}$, then *x* itself should not be too far from the origin, that is $-\frac{1}{2} < x < \frac{1}{2}$. Similarly, if $\lvert x \rvert \geq \frac{1}{\sqrt{2}}$, then *x* has to be at least $\frac{1}{\sqrt{2}}$ from the origin, and thus $x \leq -\frac{1}{\sqrt{2}}$ or $x \geq \frac{1}{\sqrt{2}}$.

Q4	Solution

(a) **[5]**

Area of rectangle $OQPR$, $A = xy = \dfrac{\lambda^3}{\left(1+\lambda^3\right)^2}$



$$\frac{\mathrm{d}A}{\mathrm{d}\lambda} = \frac{\left(1+\lambda^3\right)^2\left(3\lambda^2\right) - \lambda^3(2)\left(1+\lambda^3\right)\left(3\lambda^2\right)}{\left(1+\lambda^3\right)^4}$$

$$= \frac{\left(1+\lambda^3\right)^2\left(3\lambda^2\right) - \lambda^3(2)\left(1+\lambda^3\right)\left(3\lambda^2\right)}{\left(1+\lambda^3\right)^4}$$

$$= \frac{3\lambda^2\left(1-\lambda^3\right)}{\left(1+\lambda^3\right)^3}$$

or stationary A, $\dfrac{\mathrm{d}A}{\mathrm{d}\lambda} = 0$, giving $\lambda = 1$ (since $\lambda > 0$)

MTD 1 : First Derivative Test

$$\frac{\mathrm{d}A}{\mathrm{d}\lambda} = \frac{3\lambda^2\left(1-\lambda^3\right)}{\left(1+\lambda^3\right)^3}$$

Since $\dfrac{3\lambda^2}{\left(1+\lambda^3\right)^3} > 0$ for all $\lambda > 0$,

λ	1^-	1	1^+
$\left(1-\lambda^3\right)$	+ve	0	− ve
$\dfrac{\mathrm{d}A}{\mathrm{d}\lambda}$	+ve	0	− ve

Maximum A when $\lambda = 1$.

MTD 2 : Second Derivative Test

	$\frac{\mathrm{d}A}{\mathrm{d}\lambda}=\frac{3\left(\lambda^2-\lambda^5\right)}{\left(1+\lambda^3\right)^3}$ $\frac{\mathrm{d}^2A}{\mathrm{d}\lambda^2}=3\left[\frac{\left(1+\lambda^3\right)^3\left(2\lambda-5\lambda^4\right)-\left(\lambda^2-\lambda^5\right)(3)\left(1+\lambda^3\right)^2\left(3\lambda^2\right)}{\left(1+\lambda^3\right)^6}\right]$ $=3\left[\frac{4\lambda^7-12\lambda^4+2\lambda}{\left(1+\lambda^3\right)^4}\right]$ When $\lambda=1$, $\frac{\mathrm{d}^2A}{\mathrm{d}\lambda^2}=-\frac{9}{8}<0$ Maximum A when $\lambda=1$.
(b) **[4]**	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}\lambda}.\frac{\mathrm{d}\lambda}{\mathrm{d}x}=\frac{\left(1+\lambda^3\right)(2\lambda)-\lambda^2\left(3\lambda^2\right)}{\left(1+\lambda^3\right)^2}.\frac{\left(1+\lambda^3\right)^2}{1-2\lambda^3}=\frac{2\lambda-\lambda^4}{1-2\lambda^3}$ $\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}y}{\mathrm{d}x}.\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{2\lambda-\lambda^4}{1-2\lambda^3}.\frac{\mathrm{d}x}{\mathrm{d}t}$ When $\lambda=1$, $\frac{\mathrm{d}y}{\mathrm{d}t}=(-1)(1)=-1$ unit per sec. Rate of change of the y-coordinate of the point P at this instant is -1 unit per second or Rate of decrease of the y-coordinate of the point P at this instant is 1 unit per second.

Q5	**Solution**
(a) **[3]**	$\sum_{r=0}^{n}\left[(n+2)r+n\right]$ $=\frac{n+1}{2}\left[n+n(n+2)+n\right]$ $=\frac{n+1}{2}\left(n^2+4n\right)$ $=\frac{n}{2}(n+1)(n+4)$ **Alternative** $\sum_{r=0}^{n}\left[(n+2)r+n\right]=\sum_{r=0}^{n}(n+2)r+\sum_{r=0}^{n}n$ $=(n+2)\sum_{r=1}^{n}r+(n+1)n$ $=(n+2)\cdot\frac{n}{2}(n+1)+(n+1)n$ $=\frac{n}{2}(n+1)(n+4)$
(b) **[3]**	$\sum_{r=1}^{n}(r+2)^3=3^3+4^3+\ldots+(n+1)^3+(n+2)^3$ $\sum_{r=1}^{n}(r+2)^3=\sum_{r=3}^{n+2}r^3$ $=\sum_{r=1}^{n+2}r^3-1^3-2^3$ $=\frac{1}{4}(n+2)^2(n+3)^2-9$
(c) **[3]**	$1^3-2^3+3^3-4^3+5^3-6^3+\ldots+(2n-1)^3-(2n)^3$ $=\sum_{r=1}^{2n}r^3-2\sum_{r=1}^{n}(2r)^3$ $=\sum_{r=1}^{2n}r^3-16\sum_{r=1}^{n}r^3$ $=\frac{1}{4}(2n)^2(2n+1)^2-16\left[\frac{1}{4}n^2(n+1)^2\right]$

	$= n^2(2n+1)^2 - 4n^2(n+1)^2$ $= n^2\left[(2n+1)^2 - (2n+2)^2\right]$ $= -n^2(4n+3)$

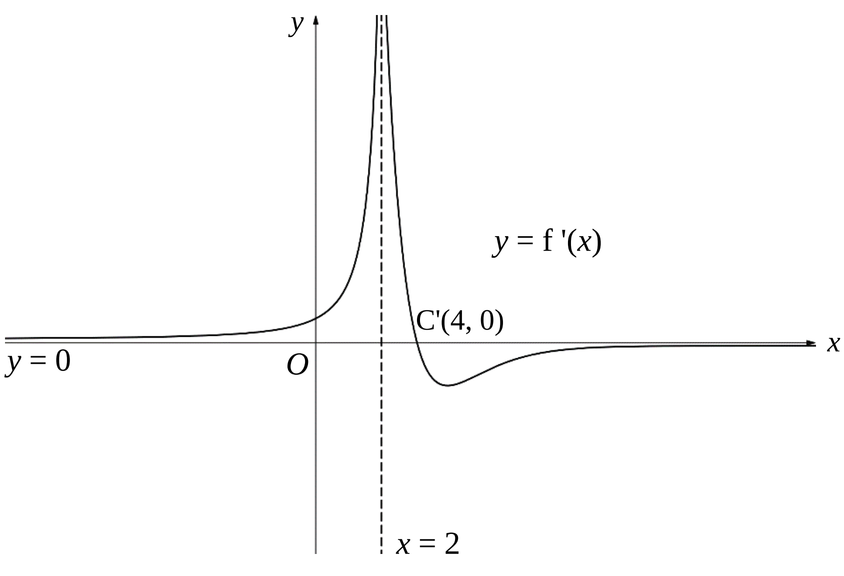
Q6	Solution
(a) **[5]**	$e^y = 1 + \sin 3x$ ------ (1) Differentiating w.r.t. x, $e^y \frac{dy}{dx} = 3\cos 3x$ Differentiating w.r.t. x, $e^y \frac{d^2y}{dx^2} + e^y \frac{dy}{dx}\frac{dy}{dx} = -9\sin 3x = -9\left(e^y - 1\right)$ (from (1)) $e^y \frac{d^2y}{dx^2} + e^y \left(\frac{dy}{dx}\right)^2 + 9e^y = 9$ Dividing throughout by e^y, $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 9 = 9e^{-y}$ (shown) Differentiating w.r.t. x, $\frac{d^3y}{dx^3} + 2\frac{dy}{dx}\frac{d^2y}{dx^2} = 9e^{-y}\left(-\frac{dy}{dx}\right)$ When $x = 0$, $y = 0$; $\frac{dy}{dx} = 3$; $\frac{d^2y}{dx^2} = -9$; $\frac{d^3y}{dx^3} = 27$ $\Rightarrow y = 0 + 3x + \frac{-9}{2!}x^2 + \frac{27}{3!}x^3 + \ldots$ $\therefore y = 3x - \frac{9}{2}x^2 + \frac{9}{2}x^3 + \ldots$

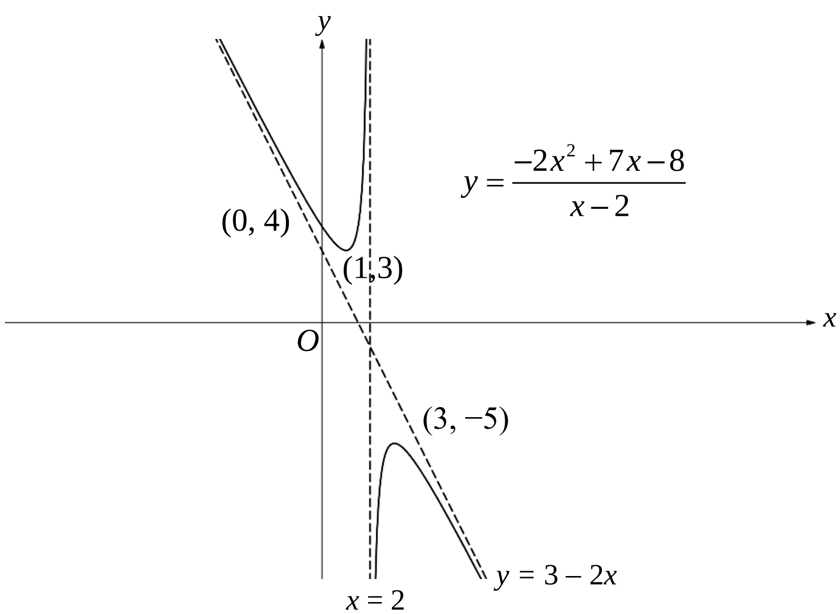
(b) **[3]**	$\mathrm{e}^y = 1+\sin 3x$ $\Rightarrow y = \ln\left(1+\sin 3x\right)$ $= \ln\left(1+\left(3x-\frac{(3x)^3}{3!}\right)+\ldots\right)$ $= \left(3x-\frac{(3x)^3}{3!}\right)-\frac{\left(3x-\frac{(3x)^3}{3!}\right)^2}{2}+\frac{\left(3x-\frac{(3x)^3}{3!}\right)^3}{3}+\ldots$ $= 3x-\frac{27x^3}{6}-\frac{9x^2}{2}+\frac{27x^3}{3}+\ldots$ $= 3x-\frac{9}{2}x^2+\frac{9}{2}x^3+\ldots$ which is same as the expansion for y found in part **(a)**, up to and including the term in x^3.

Q7	Solution
(a) **[5]**	Since $a,\ b \in \mathbf{R}$, then all the coefficients of the polynomial are real, complex roots will occur in conjugate pairs. Given $1+3\text{i}$ is a root, then $1-3\text{i}$ is also a root. $\left[x-(1+3\text{i})\right]\left[x-(1-3\text{i})\right]$ $=\left[(x-1)-3\text{i}\right]\left[(x-1)+3\text{i}\right]$ $=(x-1)^2-(3\text{i})^2$ $=x^2-2x+10$ $x^3+ax^2+18x+b=(x^2-2x+10)(x+p)$ Compare coefficients of $x\ :18=-2p+10 \Rightarrow p=-4$ $x^2: a=p-2 \Rightarrow a=-6$ $x^0: b=-40$ The other roots are 4 and $1-3\text{i}$. **Alternative:** Since $x=1+3\text{i}$ is a root of the given equation, $(1+3i)^3+a(1+3i)^2+18x+b=0$ $(1+3i)(-8+6i)+a(-8+6i)+18(1+3i)+b=0$ $-8-24i+6i-18-8a+6ai+18+24i+b=0$ $(-8-8a+b)+(6a+36)i=0$ Equating real and imaginary parts: $6a+36=0 \Rightarrow a=-6$ $\therefore b=8a+8=-40$ Since the coefficients of the equation $x^3+ax^2+18x+b=0$ are all real numbers, the complex roots occur in conjugate pairs and so $1-3\text{i}$ is also a root. Let the third root be a real number *k*. Note that

	$\left[x-(1+3\text{i})\right]\left[x-(1-3\text{i})\right]$ $=\left[(x-1)-3\text{i}\right]\left[(x-1)+3\text{i}\right]$ $=(x-1)^2-(3\text{i})^2$ $=x^2-2x+10$ Therefore, we have $x^3-6x^2+18x-40=(x^2-2x+10)(x-k)$. Comparing constants, we have $k=4$. So $a=-6,\ b=-40$ and the other roots are $1-3\text{i}$ and 4.
(b) **[5]**	$w^*+z=4-6\text{i}$ ------ (1) $w-2z=1+10\text{i}$ ------ (2) $2(1)+(2)$: $2w^*+w=9-2\text{i}$ Sub $w=c+d\text{i}$: $3c-d\text{i}=9-2\text{i}$ Comparing real part: $3c=9\Rightarrow c=3$ Comparing imaginary part: $d=2$ $\therefore w=3+2\text{i}$ From (1) : $z=4-6\text{i}-(3-2\text{i})=1-4\text{i}$ $\therefore w=3+2\text{i},\ z=1-4\text{i}$ **Alternative 1** Substitute $z=4-6\text{i}-w^*$ into (2) : $w-2(4-6\text{i}-w^*)=1+10\text{i}$ $\Rightarrow 2w^*+w=9-2\text{i}$ Sub $w=c+d\text{i}$: $3c-d\text{i}=9-2\text{i}$ Comparing real part: $3c=9\Rightarrow c=3$ Comparing imaginary part: $d=2$ $\therefore w=3+2\text{i}$ From (1) : $z=4-6\text{i}-(3-2\text{i})=1-4\text{i}$ $\therefore w=3+2\text{i},\ z=1-4\text{i}$ **Alternative 2** Let $w=a+bi$ and $z=g+hi$. Then we have

	$a-bi+g+hi=4-6i$ $\Rightarrow a+g=4$ ---------- (1) $h-b=-6$ ---------- (2) and $a+bi-2(g+hi)=1+10i$ $\Rightarrow a-2g=1$ ---------- (3) $b-2h=10$ ---------- (4) From (1) and (3), $3g=3\Rightarrow g=1$ and $a=3$. From (2) and (4), $-h=4\Rightarrow h=-4$ and $b=2$. $\therefore w=3+2i$ and $z=1-4i$

Q8	Solution
(a) **[3]**	
(b)(i) **[3]**	$a=-2$ By long division, $y=(b-4)-2x+\dfrac{2b-16}{x-2}$. $b-4=3\Rightarrow b=7$ (shown)

	Alternative $y=\dfrac{ax^2+bx-8}{x-2}=3-2x+\dfrac{A}{x-2}$ $=\dfrac{(3-2x)(x-2)+A}{x-2}$ Comparing the numerators, Coefficient of x^2 : $a=-2$ Coefficient of x : $b=3+4=7$ <u>Note</u> Coefficient of x^0 : $-8=-6+A \Rightarrow A=-2$
(b)(ii) **[3]**	

(b)(iii) **[2]**	$y=3-2x-\frac{2}{x-2}$ ↓ Replace x with $x+2$ $y=3-2(x+2)-\frac{2}{(x+2)-2}$ $\Rightarrow y=-1-2x-\frac{2}{x}$ ↓ Replace y with $2y$ $2y=-1-2x-\frac{2}{x}$ $\Rightarrow y=-\frac{1}{2}-x-\frac{1}{x}$

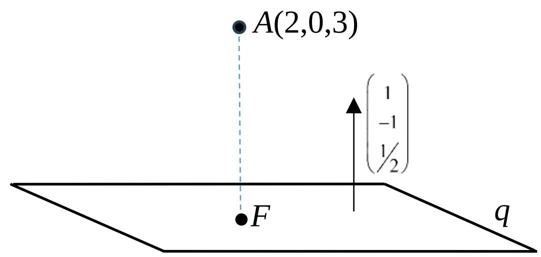
Q9	Solution
(a) **[2]**	Since l lies in plane q, $\begin{pmatrix}5\\4\\0\end{pmatrix}\cdot\begin{pmatrix}a\\-1\\b\end{pmatrix}=1\Rightarrow 5a-4=1 \quad \therefore a=1$ $\begin{pmatrix}-2\\-1\\2\end{pmatrix}\cdot\begin{pmatrix}a\\-1\\b\end{pmatrix}=0\Rightarrow -2a+1+2b=0$ ----- (2) Substitute $a=1$, $-2+1+2b=0\Rightarrow b=\frac{1}{2}$ **Alternative** Equation of l : $\mathbf{r}=\begin{pmatrix}5\\4\\0\end{pmatrix}+\alpha\begin{pmatrix}-2\\-1\\2\end{pmatrix}, \alpha\in\mathbb{R}$. Generate 2 points on l, say $(5,4,0)$ and $(3,3,2)$.

	$\begin{pmatrix}5\\4\\0\end{pmatrix}\cdot\begin{pmatrix}a\\-1\\b\end{pmatrix}=1\Rightarrow 5a-4=1 \quad \therefore a=1$ $\begin{pmatrix}3\\3\\2\end{pmatrix}\cdot\begin{pmatrix}a\\-1\\b\end{pmatrix}=1\Rightarrow 3a-3+2b=1$ Substitute $a=1,\ 2b=1 \Rightarrow b=\frac{1}{2}$
(b) **[3]**	$p:\quad \mathbf{r}=\begin{pmatrix}2\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}1\\-3\\-1\end{pmatrix}+\mu\begin{pmatrix}-2\\1\\2\end{pmatrix},\quad \lambda,\mu\in\mathbb{R}$ A vector perpendicular to $p=\begin{pmatrix}1\\-3\\-1\end{pmatrix}\times\begin{pmatrix}-2\\1\\2\end{pmatrix}=\begin{pmatrix}-5\\0\\-5\end{pmatrix}=-5\begin{pmatrix}1\\0\\1\end{pmatrix}$ Let θ be the acute angle between the planes p and q. $\begin{pmatrix}1\\0\\1\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\\frac{1}{2}\end{pmatrix}=\sqrt{2}\sqrt{\frac{9}{4}}\cos\theta$ $\frac{3}{2}=\frac{3}{2}\sqrt{2}\cos\theta$ $\cos\theta=\frac{1}{\sqrt{2}}$ $\theta=45°$
(c) **[4]**	Distance from A to q = AF $=\left\|\overrightarrow{AQ}\cdot\frac{1}{\sqrt{\frac{9}{4}}}\begin{pmatrix}1\\-1\\\frac{1}{2}\end{pmatrix}\right\|$ $=\frac{2}{3}\left\|\left[\begin{pmatrix}5\\4\\0\end{pmatrix}-\begin{pmatrix}2\\0\\3\end{pmatrix}\right]\cdot\begin{pmatrix}1\\-1\\\frac{1}{2}\end{pmatrix}\right\|$ $=\frac{2}{3}\left\|3-4-\frac{3}{2}\right\|=\frac{5}{3}$

Alternative
Solving simultaneously

$l_{AF}: \quad \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ -1 \\ \frac{1}{2} \end{pmatrix}$ and

$q: \quad \mathbf{r} \cdot \begin{pmatrix} 1 \\ -1 \\ \frac{1}{2} \end{pmatrix} = 1,$

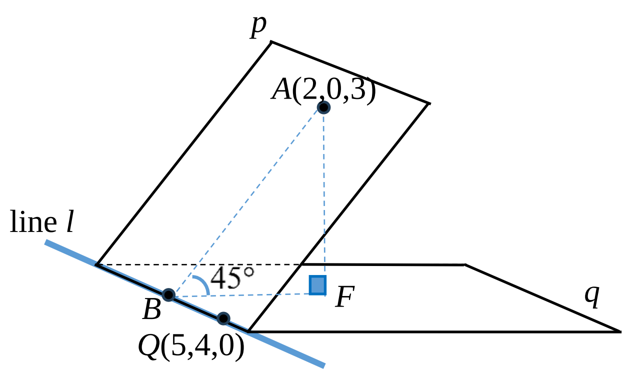


$$\left[\begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ -1 \\ \frac{1}{2} \end{pmatrix} \right] \cdot \begin{pmatrix} 1 \\ -1 \\ \frac{1}{2} \end{pmatrix} = 1$$

$$2 + \frac{3}{2} + \alpha \left(1 + 1 + \frac{1}{4} \right) = 1$$

$$\alpha = -\frac{10}{9}$$

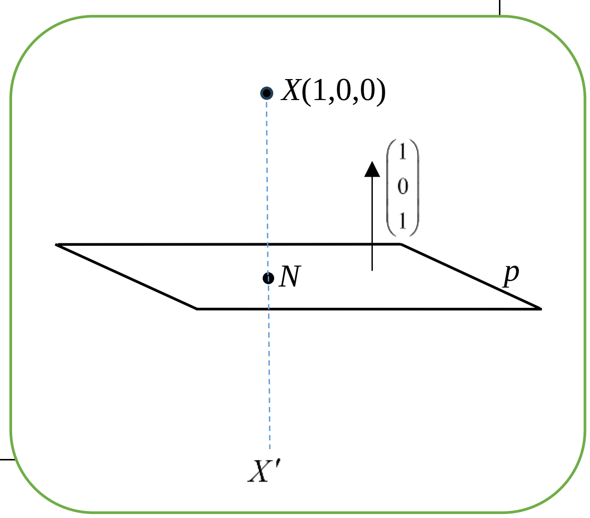
Distance from A to $q = AF = \left| -\frac{10}{9} \begin{pmatrix} 1 \\ -1 \\ \frac{1}{2} \end{pmatrix} \right| = \left(\frac{10}{9} \right) \left(\frac{3}{2} \right) = \frac{5}{3}$



From **part (b)**, the angle between the 2 planes is $45°$.

Distance from A to $l = AB = \dfrac{AF}{\sin 45°} = \dfrac{\frac{5}{3}}{\frac{1}{\sqrt{2}}} = \dfrac{5}{3}\sqrt{2}$

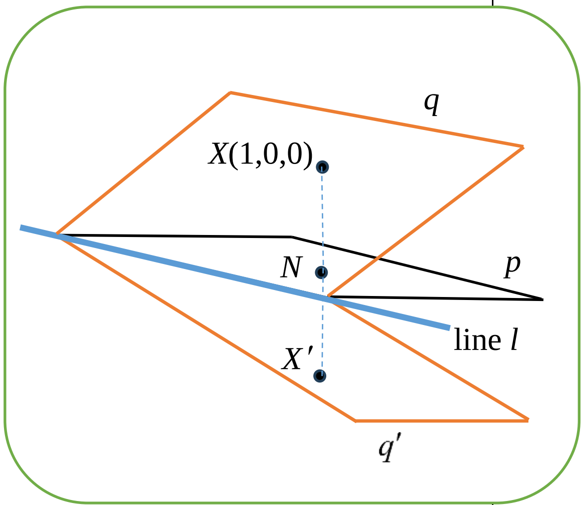
	Alternative Since the angle between the 2 planes is $45°$, Distance from A to q = Distance from F to l Hence distance from A to l $= AB = \sqrt{\left(\frac{5}{3}\right)^2 + \left(\frac{5}{3}\right)^2} = \frac{5}{3}\sqrt{1^2+1^2} = \frac{5}{3}\sqrt{2}$
(d) **[3]**	Since $\theta = 45°$, q and q' are perpendicular. Since l lies on q and q', A normal to q' = $\begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 1 \\ -1 \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{3}{2} \\ 3 \\ 3 \end{pmatrix} = \frac{3}{2}\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ Equation of q': $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = 13$ Cartesian equation of q': $x + 2y + 2z = 13$ **Alternative (If you can't see that $q \perp q'$)** $X(1,0,0)$ is a point on plane q. Let X' be the point of reflection of X about plane p. **<u>To find X'</u>** Solving simultaneously $p: \ \mathbf{r} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 5$ and $L_{XX'}: \ \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\left[\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\right] \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 5 \Rightarrow 1 + 2\beta = 5 \Rightarrow \beta = 2$ $\overrightarrow{ON} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$



$$\overrightarrow{ON} = \frac{\overrightarrow{OX} + \overrightarrow{OX'}}{2} \Rightarrow \overrightarrow{OX'} = \begin{pmatrix} 5 \\ 0 \\ 4 \end{pmatrix}$$

<u>To find equation of q'</u>

A vector parallel to q' $= \begin{pmatrix} 5 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 5 \\ 4 \\ 0 \end{pmatrix} = 4\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$



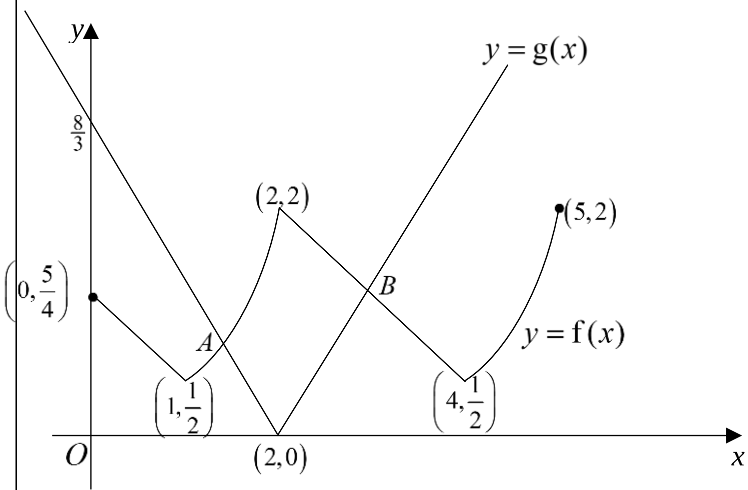
Since line l lies on q',

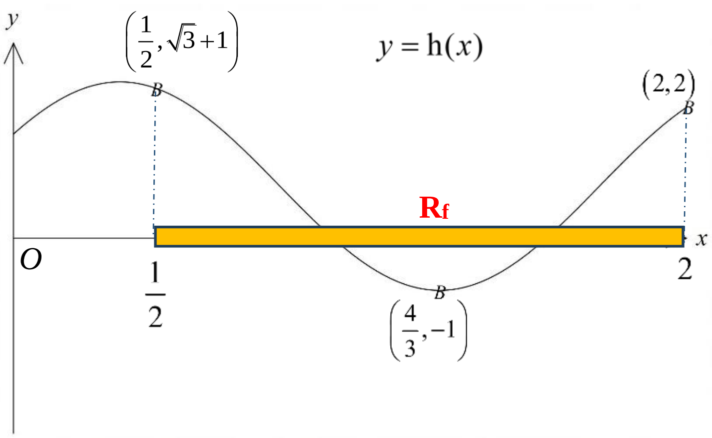
$$q': \quad \mathbf{r} = \begin{pmatrix} 5 \\ 4 \\ 0 \end{pmatrix} + s\begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \quad s, t \in \mathbb{R}$$

A normal to q' $= \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

Equation of q': $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = 13$

Cartesian equation of q': $x + 2y + 2z = 13$

Q10	Solution
(a) **[1]**	By taking $x=0$ in $\mathrm{f}(x)=\mathrm{f}(x+3)$, $\mathrm{f}(0)=\mathrm{f}(3)=\frac{1}{4}\left[-3(3)+14\right]=\frac{5}{4}$.
(b) **[3]**	
(c) **[4]**	Finding the *x*-coordinates of the points of intersection *A* and *B*, Point *A*: $\frac{4}{3}(2-x)=\frac{1}{2}x^2$ $3x^2+8x-16=0$ $(3x-4)(x+4)=0$ $x=\frac{4}{3}$ or $x=-4$ (reject since $1<x<2$ at *A*) Point *B*: $\frac{4}{3}(x-2)=-\frac{3}{4}x+\frac{7}{2}$ $16(x-2)=-9x+42$ $x=\frac{74}{25}$ or 2.96 Hence the solution to the inequality is $\left(\frac{4}{3},\frac{74}{25}\right)$.
(d) **[4]**	From the graph above, $\mathrm{R_f}=\left[\frac{1}{2},2\right]$ and $\mathrm{D_h}=[0,2]$.

	Since $\mathrm{R_f} \subseteq \mathrm{D_h}$, hf exists.  $\mathrm{R_f} = \left[\frac{1}{2}, 2\right] \xrightarrow{\mathrm{h}} \left[-1, \sqrt{3}+1\right]$ $\therefore \mathrm{R_{hf}} = \left[-1, \sqrt{3}+1\right]$

Q11	Solution
(a)(i) [1]	Amount in John's account, including the interest, at the end of the first year is \$1.05*x*.

(ii) [3]

Year	Amt at Beginning	Amt at the End
1	x	$1.05x$
2	$1.05x+x$	$(1.05x+x)(1.05)$ $=1.05x+1.05^2x$
3	$1.05^2x+1.05x+x$	$(1.05^2x+1.05x+x)1.05$ $=1.05x+1.05^2x+1.05^3x$

Total amount in the account at the end of *n* years

$=1.05x+1.05^2x+1.05^3x+\ldots+1.05^nx \quad \cdots(*)$

$=\dfrac{1.05x(1.05^n-1)}{1.05-1}$

$=21x(1.05^n-1)$

(iii) [4]

$21(10000)(1.05^n-1) > 500000$

$\Rightarrow 1.05^n \geq \dfrac{50}{21}+1$

$\Rightarrow n \geq \dfrac{\ln\left(\dfrac{71}{21}\right)}{\ln(1.05)} \approx 24.967$

Alternative : Use GC table

n	$21(10000)(1.05^n-1)$
24	$467271 \not> 500000$
25	$501135 > 500000$

At the start of 25th year, total = \$467271 + \$10000 <\$500000.

Or total = $\dfrac{501135}{1.05} \approx 477271 < \$500\,000$

The total in his account first exceeds \$500 000 at the <u>end</u> of the <u>25th</u> year.

(b)(i)
[2]

Year r	**Amount Sarah invested at the start of year r**	**No. of years interest accumulated**	**Amt accumulated due to this investment at the end of nth year**
1	6000	n	$\$6000(1.05)^n$
2	6000+400(1)	$n-1$	$\$(6000+400)(1.05)^{n-1}$
3	6000+400(2)	$n-2$	$\$[6000+2(400)](1.05)^{n-2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	6000+400(n–1)	1	$\$[6000+400(n-1)](1.05)^{n-(n-1)}$

The $\$[6000+400(r-1)]$ invested at the start of the rth year will become $\$[6000+(400)(r-1)](1.05)^{n-(r-1)}$ after n complete years since it accumulated interest for $n-(r-1)$ years.

Alternative

Yr	**Amt at Beginning**	**Amt at the End**
1	6000	(6000)1.05
2	$6000(1.05)+6400$	$6000(1.05)^2+6400(1.05)$
3	$6000(1.05)^2$ $+6400(1.05)+6800$	$6000(1.05)^3$ $+6400(1.05)^2+6800(1.05)$

The total amount at the end of n years will be $\sum_{r=1}^{n}[6000+400(r-1)](1.05)^{n-(r-1)}$,

where $a=6000, b=400$.

(b)(ii)
[2]

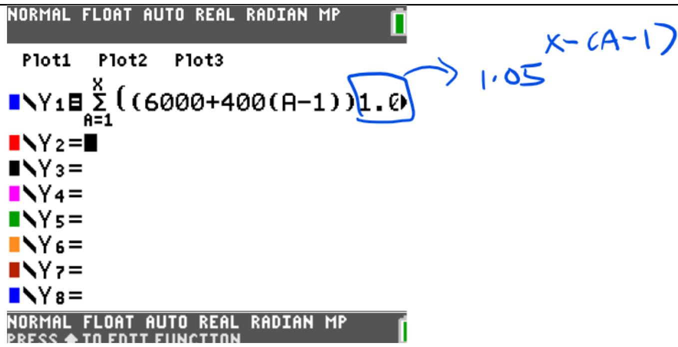
$\sum_{r=1}^{n}[6000+400(r-1)](1.05)^{n-(r-1)}>500000$

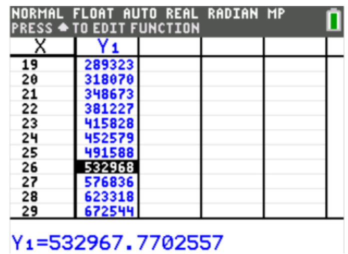
From GC,

n	$\sum_{r=1}^{n}[6000+400(r-1)](1.05)^{n-(r-1)}$
25	491588 $\not>$ 500000
26	532968 > 500000

It takes 26 complete years.

GC Method 1: Using Y=





GC Method 2: Guess and Check (