REGENT SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2021
SECONDARY FOUR EXPRESS

NAME: ______________________________ INDEX NUMBER: ________

CLASS: ________ SETTER: MDM TONG

ADDITIONAL MATHEMATICS 4049/01

Paper 1

13 September 2021

2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

	TARGET
PARENT'S SIGNATURE	

/ **90**

This document consists of **18** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 The function $f(x) = x^3 - a^2x + b^2$, where *a* and *b* are positive constants, leaves a remainder of 1 when divided by $x+3$. The function $g(x) = 3ax^3 - 2x^2 + bx - 6$ is exactly divisible by $x-1$.

(a) Find the value of *a* and of *b*. [5]

(b) Using the values of *a* and *b* found in part **(a)**, show that g (x) has only one real root. [2]

2 The equation of a curve is $y = a\cos bx - c$ for $0 \le x \le 2\pi$, where *a*, *b* and *c* are constants. The amplitude of the curve is 3 and $\left(\frac{\pi}{2}, -5\right)$ is a minimum point of the curve.

(a) Find the values of *a*, *b* and *c*, [3]

(b) Using the values of of *a*, *b* and *c* found in part **(a),** sketch the graph of $y = a\cos bx - c$ for $0 \le x \le 2\pi$. [3]

3 **(a)** Write down the first three terms in the expansion, in ascending powers of x, of $(2+ax)^8$, where a is a constant. Give the terms in their simplest form. [2]

(b) In the expansion of $(1+2x-4x^2)(2+ax)^8$ the coefficient of x^2 is 22 times the coefficient of x. Given that a is positive, calculate the value of a. [4]

(c) Explain why there is no even power of x in the expansion of $\left(\frac{x^3}{2}+\frac{2}{3x}\right)^{15}$. [3]

4 The equation of a curve $y = \dfrac{3x+8}{x+2}$, $x \neq -2$.

(a) Explain, with working, whether *y* is an increasing or decreasing curve. [4]

(b) Find the equation of the normal at the point P where the curve cuts the y-axis. [3]

(c) Given that y is decreasing at a constant rate of 0.1 units per second, find the rate of change of x at point P. [2]

5 A glass of hot water is left to cool in a refrigerator. Its temperature T °C, after t minutes, is given by $T = T_0 e^{-at}$, where T_0 and a are constants. The initial temperature of the liquid substance is 85°C and the temperature falls to 70°C ten minutes later.

(a) Find the estimated temperature of the glass of water after one hour, giving your answer correct to one decimal place. [4]

(b) Estimate at least after how many minutes, the temperature of the glass of water fall below 40°C. [2]

(c) Is it possible for the temperature of the liquid substance to fall to 0°C? Justify your answer by showing relevant mathematical workings. [2]

6 For a particular curve $\frac{dy}{dx} = \cos 2x - 3\sin x + 1$, $0 \le x \le 2\pi$. The curve has a stationary point $P(k, \frac{7\sqrt{3}}{4})$ where *k* is an acute angle in radian.

(a) Show that $k = \frac{\pi}{6}$. [4]

(b) Find the equation of the curve. [6]

(c) Determine the nature of the stationary point *P*. [3]

7 **Solutions to this question by accurate drawing will not be accepted.**
Given that the four points $A(-1,7)$, $B(k,0)$, $C(11,-1)$ and $D(7,6)$ are the vertices of the rhombus *ABCD*, find

(a) the length of AC, [1]

(b) the equation of the perpendicular bisector of *AC*, [3]

(c) the value of k, [1]

(d) the area of rhombus $ABCD$. [2]

8 **(a)** Prove that $\dfrac{\cot^2 x - 1}{\cot^2 x + 1} = \cos 2x$. [3]

(b) Solve the equation $\dfrac{\cot^2 x - 1}{\cot^2 x + 1} = 0.5$, for $0^\circ \le x \le 360^\circ$. [3]

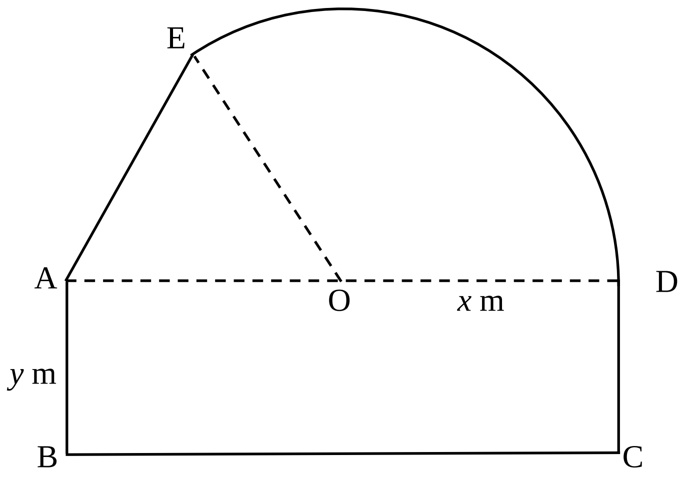
9 A particle Q travels in a straight line so that its velocity, v m/s, at time t seconds is given by $v = t^2 - 10t + 24$. The particle first crosses the fixed point O at t = 1 s.

(a) Find the time interval during which the particle's velocity is decreasing. [2]

(b) Find the values of t when the particle is instantaneously at rest. [2]

(c) Find the total distance travelled by the particle in the first 5 seconds. [4]

10 A sheep enclosure *ABCDEA* consists of a rectangle *ABCD*, an equilateral triangle *EOA* and *DOE* is a sector of a circle, centre *O* and radius *x* m. The points *A*, *O* and *D* lie on a straight line with *OD* = *x* m.



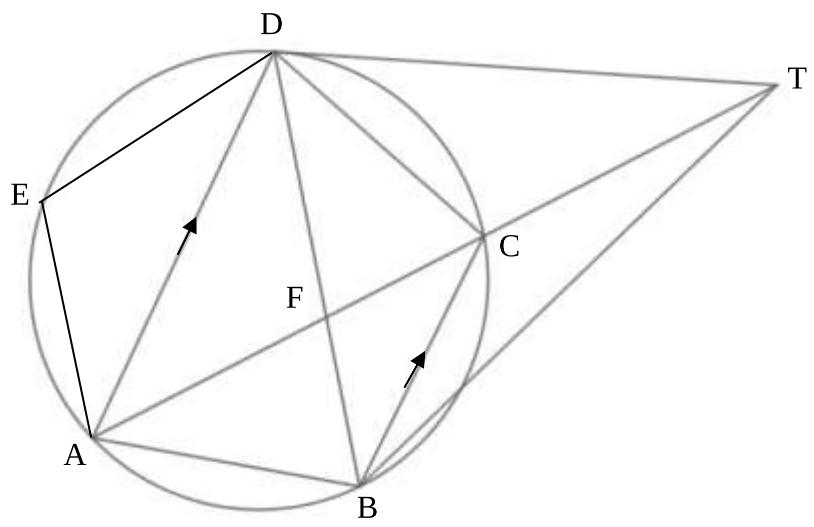
(a) Given that *AB* = *y* m and the area of the enclosure is 1000 m^2, show that $y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)$. [3]

(b) Given that the perimeter of the enclosure is P m, show that [2]

$$P = \frac{1000}{x} + \frac{x}{12}\left(4 \quad 36 \quad 3 \quad 3\sqrt{}\right).$$

(c) Find the value of x that make P a minimum. [5]

11



The diagram shows a point *D* on a circle, and *TD* is a tangent to the circle. Points *A*, *B*, *C*, *D* and *E* lie on the circle such that *DC* bisects angle *TDB* and *TCA* is a straight line. The lines *TA* and *DB* intersects at *F*. AD is parallel to BC.
Given that angle *BDC* = θ.

(a) Show that *CB* = *CD*. [3]

(ii) Show that angle *AED* = 3θ.

[4]

END OF PAPER