4 Forces

Main concept(s)

1. Meaning of force
2. Elastic force and Hooke's law
3. Meaning and examples of contact force and non-contact force
4. Friction and viscous forces
5. Turning effects of a Force, moments and torque of a couple
6. Conditions for Equilibrium
7. Fluid (or hydrostatic) pressure
8. Upthrust, Archimedes' Principle and Principle of Floatation

Learning Outcome(s)

Candidates should be able to:

(a) recall and apply Hooke's law ($F = kx$, where k is the force constant) to new situations or to solve related problems
(b) describe the forces on a mass, charge and current-carrying conductor in gravitational, electric and magnetic fields, as appropriate
(c) show a qualitative understanding of normal contact forces, frictional forces and viscous forces including air resistance (no treatment of the coefficients of friction and viscosity is required)
(d) show an understanding that the weight of a body may be taken as acting at a single point known as its centre of gravity
(e) define and apply the moment of a force and the torque of a couple
(f) show an understanding that a couple is a pair of forces which tends to produce rotation only
(g) apply the principle of moments to new situations or to solve related problems
(h) show an understanding that, when there is no resultant force and no resultant torque, a system is in equilibrium
(i) use a vector triangle to represent forces in equilibrium

Introduction

The concept of forces is fundamental in Physics and spans across many different topics. This chapter briefly overviews two broad categories of forces, namely contact and non-contact forces. Following that, a more in-depth study of some contact forces including resistive forces, elastic force, fluid pressure and upthrust is presented. You will also learn the turning effects of a force as well as the conditions for a system to be in equilibrium.

4.1 The Meaning of Force

- A force is a push or pull.
- Exists as a result of **interaction** between 2 bodies.
- Whenever there is an *interaction* between two bodies, there is a force upon each of the bodies.
- Force is a **vector** quantity that has both magnitude and direction.

4.2 Broad categories of Forces

All forces (interactions) between bodies can be placed into two broad categories:

- **Contact forces** are those types of forces that result when the two interacting bodies are physically in contact with each other.
- **Non-contact forces** are forces resulting from action-at-a-distance. These forces between two interacting bodies, which are able to exert a push or pull on each other despite their physical separation.

Examples

Contact forces		Non-contact forces
Friction	Air Resistance	Gravitational Force
Tension	Applied Force	Electrical Force
Normal Contact Force	Spring Force	Magnetic Force

The usual abbreviations for the forces are shown in the next table:

Type of Force	**Examples**	**Denoted By**
Normal Contact	Force between objects when they are in contact. This force acts normal (perpendicular) to the surfaces in contact.	N
Friction	Force exerted by a surface as an object moves or try to move across it. Direction of force is such that it opposes the relative motion of the object.	f
Tension	Force transmitted through a cable when forces acting from opposite ends pull it tight. The force is directed along the length of the wire and pulls equally on the objects on the opposite ends.	T
Air Resistance	Force which acts upon objects as they travel through the air. The force of air resistance opposes the motion of an object.	f_{air}
Spring	Force exerted by a compressed or stretched spring. A force acts in the direction, which restores the spring to its equilibrium position.	F_{spring}

Gravitational	1. Force of attraction between two masses. 2. The "weight" of an object. [Weight is the force experienced by a mass in a gravitational field.] 3. Acts on the centre of gravity of an object	F_g or W
Nuclear	Cause of beta decay.	--
	Force of attraction between protons and neutrons in the nucleus.	--

4.3 Overview of Non-contact Forces & Force Fields

Two bodies may exert a force on each other even without being in contact. Each of these bodies set up a field of force (or force field) around it. The fields of the two bodies interact, producing a force on each body. Only fields of the same nature can interact e.g. one gravitational field can interact with another gravitational field but does not interact with an electric field or a magnetic field.

Gravitational Forces

Gravitational forces exist between any two bodies with mass. These forces only attract.

(To be discussed later in detail under the topic of Gravitation)

Electric Forces

Electric force occurs between electrically charged bodies. Like charges repel and unlike charges attract.

(To be discussed later in detail under the topic of Electric Field)

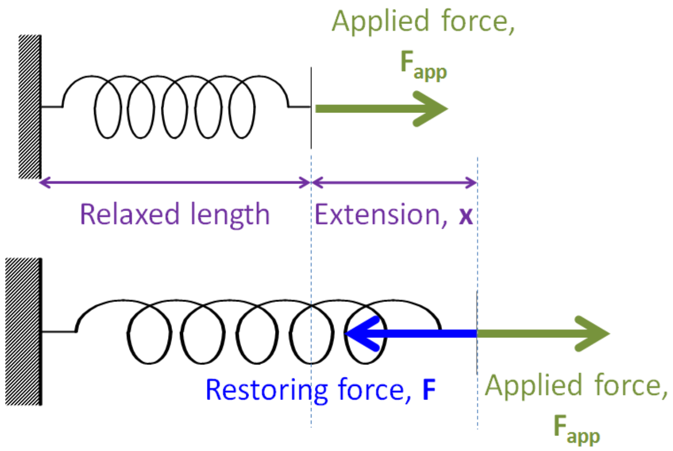
Magnetic Forces

Magnetic forces exist between any two moving electric charges. Moving electric charges and electrical currents create magnetic fields. Consequently, if you have two moving electric charges or current-carrying conductors, these will create magnetic fields. The magnetic force can be attractive or repulsive.

(To be discussed later in detail under the topic of Electromagnetism)

4.4 Elastic (or Restoring) Force & Hooke's Law

4.4.1 Hooke's Law



Consider a stretched spring. In order to return to its original state, a restoring force F is exerted by the spring.

Hooke's Law states that force by an elastic body is **directly** proportional to its extension, provided the limit of proportionality has not been exceeded.

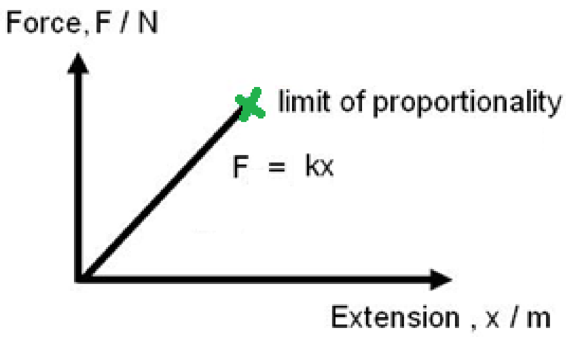
Hooke's Law is given by the equation:

In the form of magnitude only, $F = kx$

In the form of vector, $\boxed{\vec{F} = -k\vec{x}}$

Where :
F is the restoring force by elastic body (N)
k is the force (or spring) constant (N m^{-1}) [i.e. a measure of stiffness]
x is the extension of the body (m)

The negative sign (-) indicates that the restoring force always points in a direction opposite to the extension of the elastic body.



For more information on the graph of Force vs Extension, please refer to Appendix A.

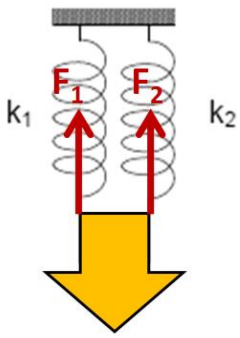
Quiz

The natural length of a spring is L_o. Sketch the graph of elastic force (F) against TOTAL length of the spring (L).

Is F proportional to L?

Solution:

4.4.2 Springs in Parallel



Extension is same for both springs.

So $x_1 = x_2 = x$

However, the restoring forces are not equal if the spring constants differ.

Applied Force is balanced by the Total Restoring Forces, i.e.

Applied Force = $F_1 + F_2$ [Assume springs have negligible mass.]

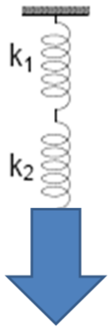
$k_{eff}\, x = k_1 x_1 + k_2 x_2$

Since $x_1 = x_2 = x$,

$\mathbf{k_{eff} = k_1 + k_2}$

where k_{eff} is the effective spring constant.

4.4.3 Springs in Series



Assume springs have negligible mass. Then, restoring force is same for both springs.
So $F_1 = F_2$ = Applied force F

However, the extensions are not equal if the spring constants differ.
Total extension, $x = x_1 + x_2$

$$\frac{F}{k_{eff}} = \frac{F_1}{k_1} + \frac{F_2}{k_2}$$

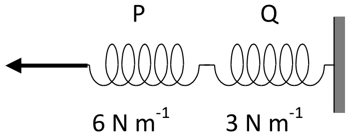
Since $F_1 = F_2 = F$,

$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2}$$

where k_{eff} is the effective spring constant.

Example 1

A spring P of force constant 6 N m^{-1} is connected in series with a spring Q of force constant 3 N m^{-1}as shown in the diagram. One end of the combination is securely anchored and a force of 0.6 N is applied to the other end.



a) What is the extension for each spring?

Solution:

When the 0.6 N force is exerted, an elastic force is generated in the spring, which tends to restore the springs to their original lengths. Since the springs are at rest, by Newton's first law of motion, the forces on each spring must balance out. Hence the restoring force in each spring must equal 0.6 N. Assuming the proportional limit of the spring is not exceeded, Hooke's Law is applied. According to Hooke's Law, the restoring force F in each spring is directly proportional to that particular spring's extension x:

$\text{For spring P, } F = k_P x_P$

$0.6 = 6x_P \quad \therefore x_P = 0.1\,\text{m}$

$\text{For spring Q, } F = k_Q x_Q$

$0.6 = 3x_Q \quad \therefore x_Q = 0.2\,\text{m}$

b) What is the effective force constant, k_{eff}, of the spring combination?

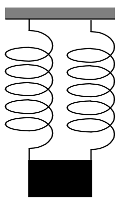
<u>Solution:</u>

$$F = k_{eff} x_T$$
$$0.6 = k_{eff}(0.1 + 0.2)$$
$$k_{eff} = 2\,\text{N m}^{-1}$$

$\text{Alternatively, for springs in series, } \frac{1}{k_{eff}} = \frac{1}{6} + \frac{1}{3}. \qquad \therefore k_{eff} = 2\,\text{N m}^{-1}$

Example 2

When an object of mass m is suspended from a spring, its extension is *e*. Two such springs, of negligible mass, are then connected in parallel to form a combination. If another object of mass 2m is then suspended on the spring combination, what would its extension be?



2 m

<u>Solution:</u>

Let the spring constant of each spring be k.
Each spring has negligible mass, hence when an object of mass m is suspended from it, the total downward force equals mg (i.e. neglecting the weight of the spring) and the elastic force is uniform throughout and equals mg.

mg = ke, $\therefore k = \frac{mg}{e}$

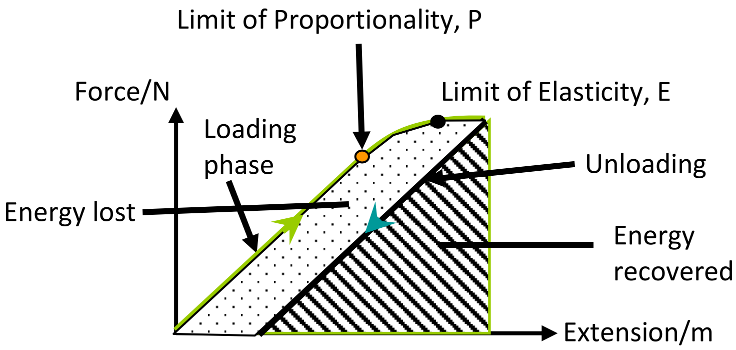
For the parallel spring combination, $k_{eff} = k + k = 2k$.

Hence 2 mg = 2 kx, $2mg = 2\left(\frac{mg}{e}\right)x$, hence $x = e$.

4.4.4 Plastic deformation

Once the limit of elasticity is exceeded in an elastic body, it is said to under plastic deformation.

The graph for the loading and unloading of an elastic body, which has undergone plastic deformation, is as shown below.



Graph for unloading of an elastic body, which has undergone plastic deformation

During unloading, not all the energy is fully recoverable when the elastic body's limit of elasticity, E has been exceeded. The energy is not fully recovered because the spring no longer returns to its original length when the load is removed; the work done on the spring in stretching it from its initial position is not the same as the work done by the spring in returning to the initial position.

The energy lost is given by the dotted area, which is the difference in the area under F-x graph during loading and that of unloading and is usually dissipated as thermal energy and other forms of energy.

MINI-TEST 1

A force of 16 N is required to stretch a spring a distance of 40 cm from its rest position. A second identical spring is fixed parallel and adjacent to this spring. How much force is required to stretch this pair of springs by 20 cm?

4.5 Friction

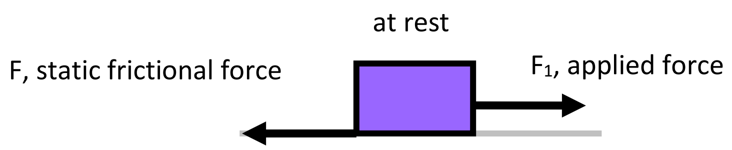
Friction is a contact force that opposes the *relative* motion between two bodies in contact, or, *tends to* oppose one body from moving *relative* to the other.

Magnitude of the frictional force depends on:

1. the nature of the surface, and
2. the magnitude of the normal contact force.

The magnitude is independent of the contact area and speed of motion. Friction occurs for both a static body and a moving body.

Consider a force F_1 applied to a block, which is at rest.



As F_1 is applied and increased slowly, the frictional force F acts in the opposite direction and increases slowly. Initially, F always equals to F_1, causing the block to remain at rest. Frictional force F is known as static friction.

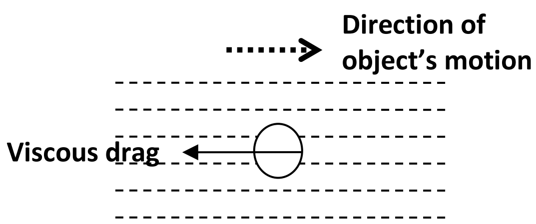
However, once F increases to a maximum value, known as the limiting frictional force, if the applied force F_1 exceeds the limiting frictional force, the block will start to move. Kinetic friction comes into action.

4.6 Viscous Drag

When an object moves through a fluid, that is, either a liquid or a gas, it experiences a force opposing the motion called viscous drag. Air resistance and water resistance are examples of viscous drag. Unlike frictional forces, drag forces equal zero when the body's velocity relative to the fluid is zero, and is dependent on the speed of motion.

The magnitude of viscous drag depends on

- the properties of fluid (e.g. density and viscosity)
- the shape and size of the object, and
- its velocity *relative to* the fluid.



The drag forces act over the whole surface of the body but can be represented by a single force as shown above.

At low speeds, the drag forces are proportional to the speed of the object relative to the fluid, i.e. $F \propto v$. At high speeds, the flow of the fluid becomes turbulent and drag force is proportional to the square or cube of the speed, i.e. $F \propto v^2$ or $F \propto v^3$

4.7 Turning Effect of Forces

4.7.1 Moment of a force (Torque)

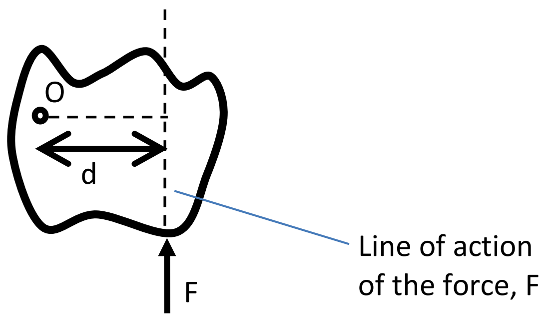
Definition:

Moment of a force about any point is the turning effect of a force which is equal to the product of the force and the perpendicular distance from that point to the *line of action* of the force.

$$M = F \times d$$

where d is the perpendicular distance from the point to the line of action of the force, F.

The moment of a force is a vector.
SI unit: newton metre, N m
Consider the following body pivoted at O and a force F is applied on the body:

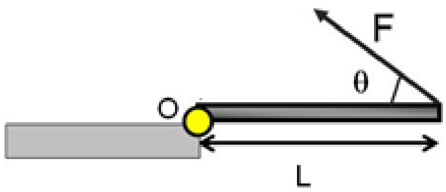


The perpendicular distance may not be immediately apparent. In such cases, draw the line of action first, and then draw the perpendicular distance *d*.
The force causes the body to rotate about O and it produces a moment which causes the body to have a turning effect.

- If the body is initially at rest, the moment will cause it to rotate.
- If the body is initially in rotational motion, it will cause it to rotate with increasing/decreasing rotational speed. The body will have a rotational acceleration.

Quiz

A drawbridge of length L being is raised by a chain attached as shown.
What is the perpendicular distance from O? What is the moment or torque about O?

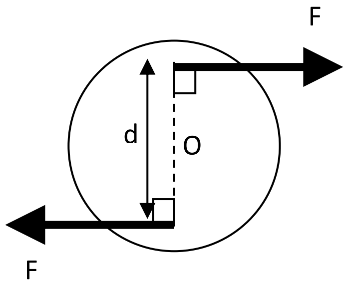


Perpendicular distance between O & the line of action of the force =
Moment or torque about O, τ =

4.7.2 Couple

Definition:

A **couple** consists of <u>two forces</u> of <u>equal magnitude</u> but acting in <u>opposite directions</u> whose <u>lines of action are parallel but separate</u>.



Moment (or torque) of a couple is given by

$$\boxed{\tau = F.d}$$

where: τ is the moment (or torque) of couple
F is a force of the couple
d is the perpendicular distance between the two forces

The net force is <u>zero</u>, as the 2 forces are equal in magnitude and opposite in direction.

- no <u>linear acceleration</u>
- a couple <u>tends to produce rotation only</u>

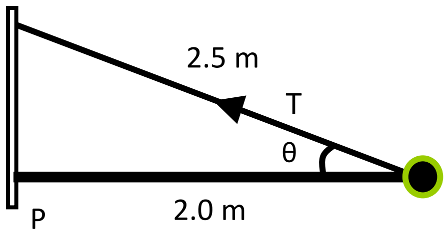
The body will start to <u>rotate</u> about O (the midpoint between the two forces) if it is initially at rest or rotate with an <u>increasing/decreasing rotational speed</u> if it is initially in rotational motion.

4.7.3 Principle of Moments

> The Principle of Moments states that for a body to be in equilibrium, the **sum of** clockwise moments about any pivot must be **equal** to the **sum of** anticlockwise moments about that same pivot.

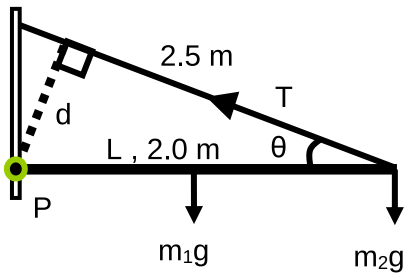
Example 3

A uniform beam of length 2.0 m and mass 1.0 kg, carrying at its end, a toy of mass 0.50 kg, is supported on one end by a pivot, P on the wall and the other by a wire 2.5 m long, as shown in the diagram. Calculate the tension in the wire.



Solution:

Draw the free-body diagram of the beam:



$$\theta = \cos^{-1}\left(\frac{2.0}{2.5}\right) = 36.9^o$$

$$d = 2.0 \sin\theta$$
$$= 1.2 \text{m}$$

Taking moments about P,

Sum of clockwise moments = Sum of anti-clockwise moments

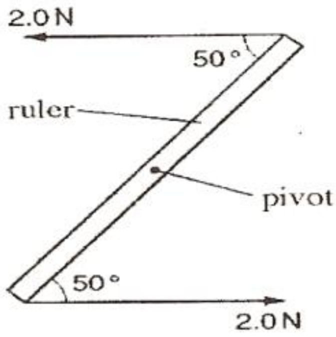
$m_1g \times 0.5\ L + m_2g \times L = T \times d$

(1.0) (9.81) (1.0) + (0.50) (9.81) (2.0) = T x (1.2)

T = 16 N

Example 4

A ruler of length 0.30 m is pivoted at its centre. Equal and opposite forces of magnitude 2.0 N are applied to the ends of the ruler as shown. What is the magnitude of the torque of the couple on the ruler when it is in the position shown?



<u>Solution:</u>
Torque of the couple,
$\tau = F \cdot d$,
d: perpendicular distance between the forces

$d = 0.30 \sin 50^{O} = 0.23$ m,
therefore torque $\tau = 2.0 \times 0.23 = 0.46$ N m

4.8 Equilibrium of Forces

4.8.1 Equilibrium

Definition of Equilibrium:

MEANING of equilibrium	CONDITIONS to achieve equilibrium
An object which, under the action of a number of forces, remains at rest or moving with a constant velocity and if in rotation is rotating at constant angular velocity is said to be in equilibrium. In other words, it has: (1) **zero** linear acceleration, → AND (2) **zero** angular acceleration →	(1) **Net** force acting on the body is **zero** ($\sum F = 0$) (2) **Net** moment of force **about any point** acting on the body is **zero** ($\sum M = 0$)

Note that a *moving* body can also satisfy the two conditions for equilibrium. This state is referred to as dynamic equilibrium.

Linear acceleration means that its linear velocity changes with time. A ZERO linear acceleration means that either the body is not translating, or, the body's translational velocity remains constant with time.

Angular acceleration means that its angular velocity (or rate of rotation) changes with time. ZERO angular acceleration means that either the body is not rotating, or, the body's rotational velocity remains constant with time.

4.8.2 Problem-solving for rigid bodies in Equilibrium (2-D problems)

- Draw the free body diagram(s).

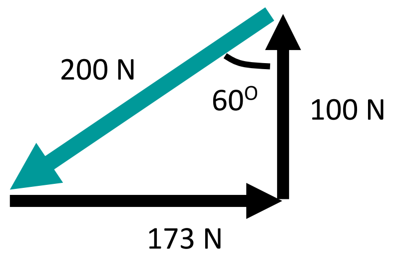
- Based on the free-body diagram(s),

  EITHER:
  Write down equations that fulfil
  (i) condition (1) (∑F =0): The sum of the resolved components of the forces in two mutually perpendicular directions (e.g. x and y directions) are both equal to zero.
  (ii) condition (2) (∑M = 0): Choose a point as the pivot and equate the resultant moment about that pivot to be zero (i.e. apply the Principle of Moments).

  OR:
  Draw a vector diagram: If the forces are coplanar and are in equilibrium, the force vectors when joined head-to-tail should form a closed triangle or a closed polygon.

Example 5
Three men are pushing on a wardrobe. One pushes northward with a force of 100 N and the second eastwards with 173 N. In what direction and with what magnitude is the third pushing for the wardrobe to be stationary?

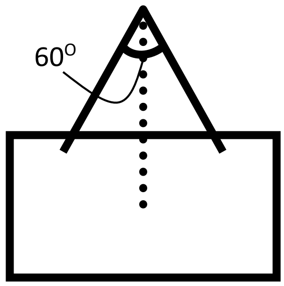
Solution:
Stationary → wardrobe is in equilibrium.
→ Since the 3 forces are coplanar, the 3 forces should form a closed vector triangle when joined head-to-tail.



Magnitude: $\sqrt{100^2 + 173^2} = 200\ N$
Direction: $tan^{-1}\left(\frac{173}{100}\right) = 60°$
The force is 200 N, 60° clockwise from South.

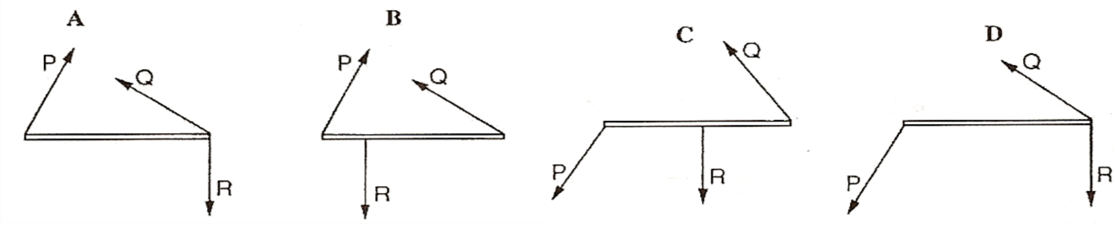
Example 6

A picture weighing 40 N is hung by a wire which passes over a small hook.
The two parts of the wire are inclined at 60^{O} to each other.
Find the tensile force in the wire.



Example 7 (N01/1/5)

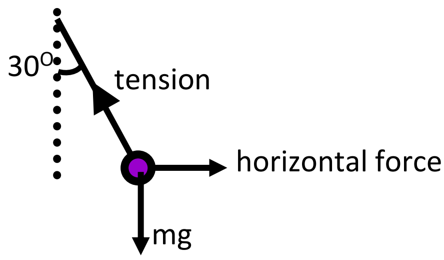
A light rod is acted upon by three forces P, Q and R. Which diagram could show the position and direction of each of the forces when the rod is in equilibrium?



If 3 coplanar and non-parallel forces act on a body which is in equilibrium, the lines of action of all 3 forces must pass through a common point.

Example 8

A body of mass 40 kg is suspended by a rope and is pulled sideways with a horizontal force. Determine the tension and the horizontal force.

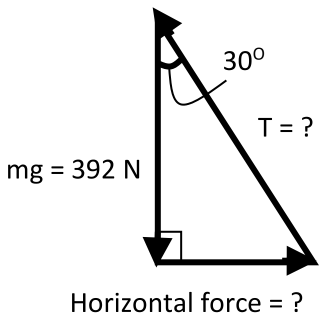


Solution:

Method 1: Constructing a vector diagram

If a body is in equilibrium under the action of three coplanar forces, these forces will form a closed triangle when joined head-to-tail.

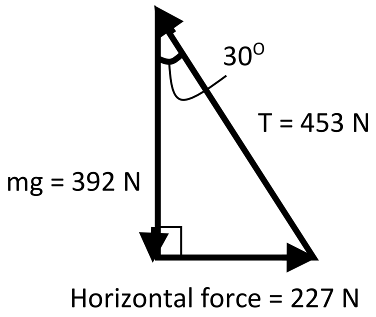
Step 1: Draw the closed vector triangle using ruler, protractor and a chosen scale. Determine and label all known quantities.



Scale: 1 cm: 10 N

mg = 40 (9.81) = 392.4 N

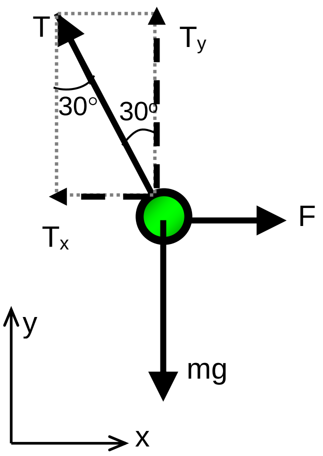
Step 2: Measure the length of the unknown vectors to be determined. Use the scale to calculate their magnitude.



Scale: 1 cm: 10 N

Method 2: Method of resolution of forces

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$



Resolving forces along the x direction (taking → as positive),

$F - T_x = 0$

$F = T \sin 30^{\circ}$

Resolving forces along the y direction (taking ↑ as positive),

$T_y - mg = 0$

$T \cos 30^{\circ} = mg$

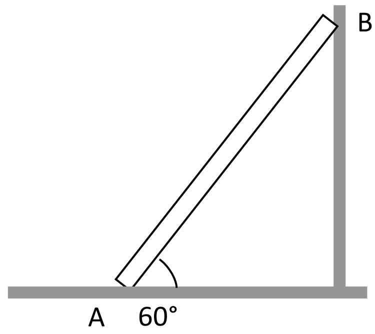
$T = 453$ N

$F = 227$ N

Example 9

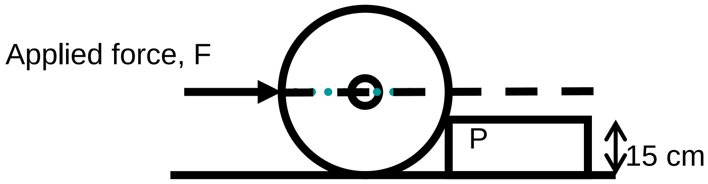
A uniform ladder of length 10 m and weight 200 N leans against a smooth wall so that it is inclined 60^o to the horizontal ground. A boy of weight 500 N stands on the ladder ¼ of the way from its lower end. Calculate the normal reaction at the smooth wall and the magnitude and direction of the force acting on the lower end of the ladder.

<u>Solution:</u>



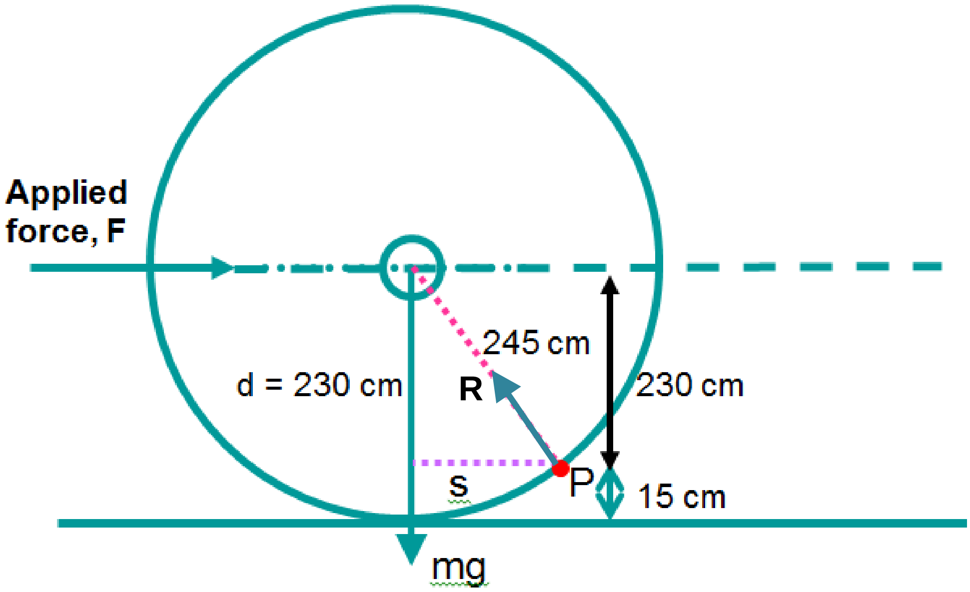
Example 10

What force applied horizontally at the axle of the wheel is required to raise the wheel of diameter 490 cm and weight 100 N over a 15 cm obstacle?



Solution:

Draw the free-body diagram of the wheel:



mg: weight of wheel
R : reaction force exerted by obstacle on wheel

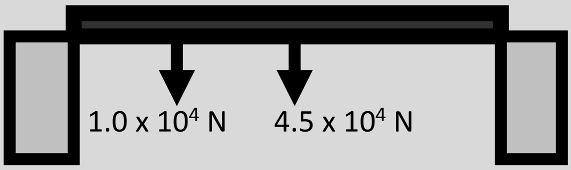
Since 3 non-parallel coplanar forces act on the wheel, and the wheel is in equilibrium, the **lines of action of these 3 forces must pass through one point**. In this case, they all pass through the centre of the wheel.

Taking moments about the pivot P,
Sum of clockwise moments = Sum of anticlockwise moments

F x d = mg x s (R produce zero moment about pivot P)
F x 230 = 100 x 84.4
F = 36.7 N

MINI-TEST 2

A uniform bridge, resting horizontally on supports at each end, weighs 4.5×10^4 N and is 8.0 m long. Find the force acting downward on each support when the bridge carries a load of 1.0×10^4 N placed 2.0 m from one end.



My solution:

4.9 Fluid Pressure and Upthrust (Not in H1 Syllabus but good to know)

4.9.1 Pressure in a fluid

In general, pressure is defined as the perpendicular force acting on per unit area of a surface.

$$P = \frac{F}{A}$$

Where: P is pressure (Pa)
F is force (N)
A is area (m^2)

In a fluid, it can be derived from the above equation that the pressure varies with depth as follows:

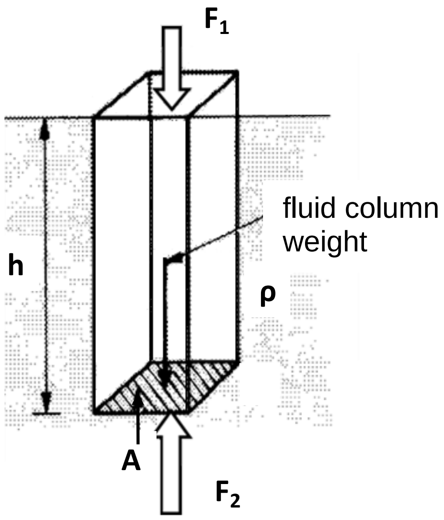
$$\boxed{P = h\rho g}$$

Where: P is pressure at depth h in a fluid (Pa)
h is the depth (m)
ρ is the density of the fluid (kg m^{-3})
g is the acceleration due to gravity (m s^{-2})

Note that all points at the same depth in the same fluid are at the same pressure.

Derivation (required in H2 Syllabus)

Consider a column of fluid as shown below.



fluid Column

h: height of the fluid column.
A: base area of the fluid column.
F_1 , F_2 : perpendicular forces on the upper and lower surfaces of the column respectively.
P_1 , P_2 : pressure on the upper[1] and lower surfaces of the column respectively.
ρ : density of fluid

---

[1] Pressure on the upper surface could be atmospheric pressure if the top of the object is above or at the surface of the liquid.

V : volume of fluid column

Mass of fluid column = Density of fluid x Volume of fluid column
$= \rho V$
$= \rho A h$

Weight of fluid column = mg = $(\rho V)g = (\rho A h)g$

Fluid is static → Upward force = Downward force

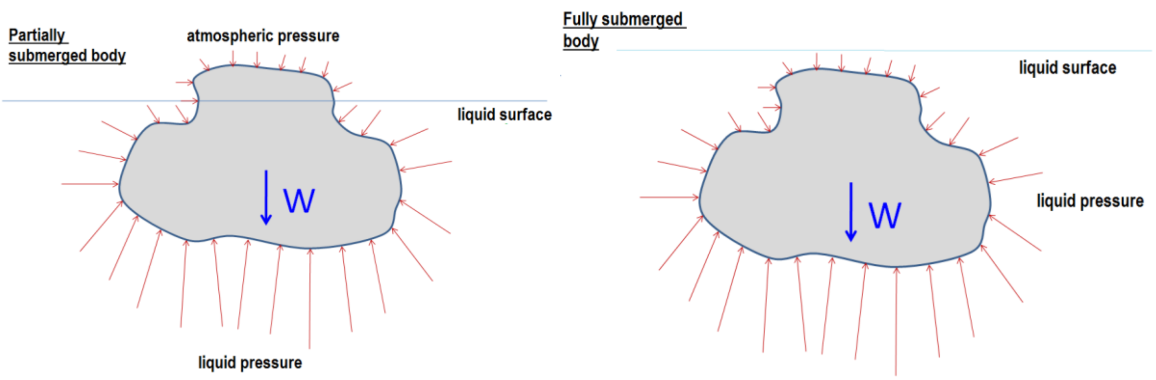
$$F_2 = \text{Weight of fluid column} + F_1$$
$$P_2 A = \rho A h g + P_1 A$$
$$P_2 = \rho h g + P_1$$

Pressure difference (increase) down the fluid column, $\Delta P = P_2 - P_1 = \rho h g$

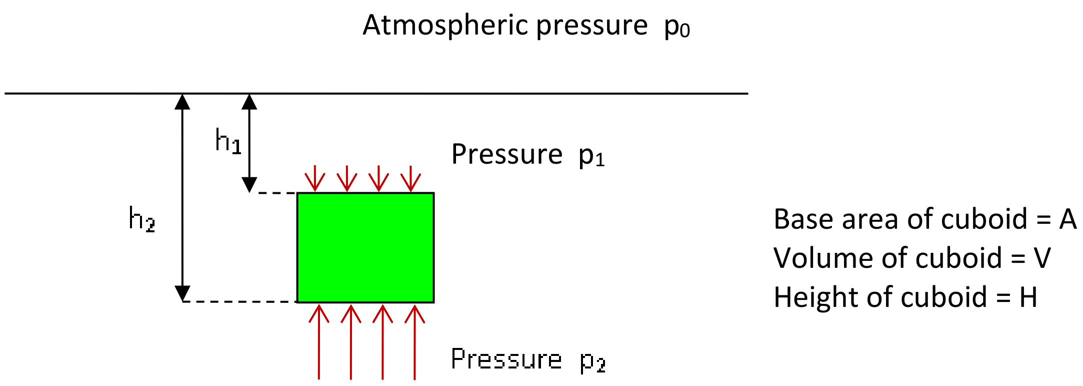
4.9.2 <u>Origin of Upthrust</u>

> Upthrust is the net upward force exerted by a fluid on a body because of the body being immersed (either partially or totally) in the fluid.

Upthrust arises because fluid pressure increases directly proportionally with depth in the fluid. Thus, the pressure at the lower part of a body is higher than the pressure at the upper part of the body. This pressure **differences** results in the sum of the upward force components due to the fluid pressure to be greater in magnitude than the sum of the downward force components due to the fluid pressure. Hence a **net** upward force is exerted by the fluid on the body, which is known as upthrust.



Consider a cuboid <u>fully</u> submerged in a liquid:



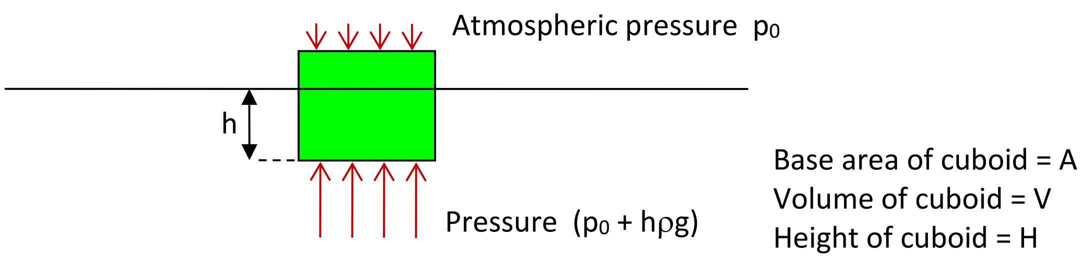
$p_1 = p_0 + h_1\rho g \rightarrow \quad F_1 = p_1A$ (Downward force exerted by fluid on cuboid)

$p_2 = p_0 + h_2\rho g \rightarrow \quad F_2 = p_2A$ (Upward force exerted by fluid on cuboid)

$$\begin{aligned}\text{Upthrust} = F_2 - F_1 &= \rho g(h_2 - h_1)A \\ &= \rho gHA \\ &= \rho gV \qquad \text{------------------------------------(1)}\end{aligned}$$

where V is the volume of the cuboid and also equal to the volume of liquid it displaces.

Consider a cuboid <u>partially</u> submerged in a liquid:



$F_1 = p_0A$ (Downward force exerted by fluid on cuboid)

$F_2 = (p_0 + h\rho g)A$ (Upward force exerted by fluid on cuboid)

$$\begin{aligned}\text{Upthrust} = F_2 - F_1 &= \rho ghA \\ &= \rho gV_{\text{displaced fluid}} \qquad \text{---------------------------------- (2)}\end{aligned}$$

Note that the volume of fluid displaced by the cuboid here is less than its total volume.

Thus, from our finding [expressions (1) and (2)], we may conclude that upthrust is provided by the <u>volume of fluid **displaced**</u> by a submerged or floating object.

4.9.3 Archimedes' Principle

Archimedes' Principle informs us on how to determine upthrust:

Archimedes' Principle states that when an object is totally or partially immersed in a fluid, it experiences a net upward force (upthrust) equal in magnitude and opposite in direction to the **weight** of **fluid displaced**.

i.e. Upthrust = Weight of fluid displaced

$$U = m_{fl.\ displ.}g = \rho_f V_{fl.\ displ.}\ g$$

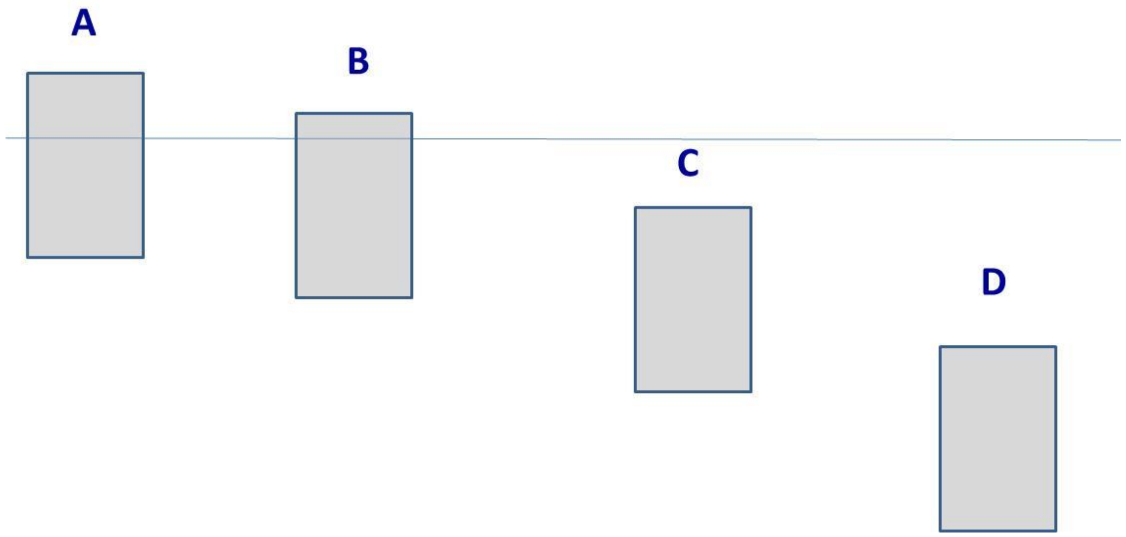
Where: U is upthrust acting on the object (N)
ρ_f is density of fluid displaced (kg m^{-3})
g is gravitational acceleration (m s^{-2})
$V_{fl.\ displ.}$ is volume of fluid displaced (m^3)

Upthrust exists for both an object that is floating and object that is sinking

– When it is **sinking**, means that:
Weight **>** Upthrust
– When it is **floating**, means that:
Weight **=** Upthrust

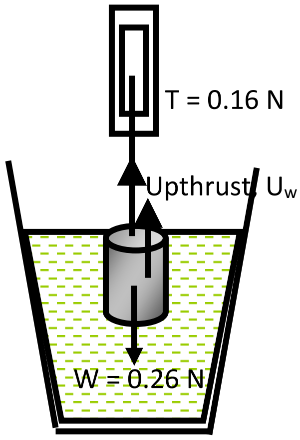
Quiz

Which object experiences the greatest value of upthrust?



Example 11

An object made of glass weighs 0.26 N when suspended in air by a thread attached to a spring balance. When the object is fully immersed in water, the spring balance reading changes to 0.16 N. Given the density of water to be 1000 kg m^{-3}, calculate a) the volume of the object; and b) its density.



Solution:

a) When the object is immersed in water, we expect the spring balance reading to decrease because the pressure differences in the water will generate a net upward force exerted by the water on the object (called upthrust). Upthrust balances the weight of the object partially, hence reducing the force in the spring. The amount of force in the spring gives the spring balance reading.

Since object is in equilibrium,
Upward forces = Downward force

$$T + U_w = W$$

$$\begin{aligned} U_w &= W - T \\ &= 0.26 - 0.16 \\ &= 0.10\ \text{N} \end{aligned}$$

U_w = Weight of fluid displaced = $V_w \rho_w g$

$V_w \rho_w g = 0.10$

$$\Rightarrow V_w = \frac{0.10}{1000(9.81)} = 1.02 \times 10^{-5}\ \text{m}^3$$

b) $\rho = \dfrac{M}{V} = \dfrac{\frac{W}{g}}{V} = \dfrac{\frac{0.26}{9.81}}{1.02 \times 10^{-5}} = 2600\ \text{kg m}^{-3}$

4.9.4 Principle of Flotation

Principle of Flotation states that an object floats in a fluid when it displaces a weight of fluid equal to its own weight.

In other words, a body floating in a fluid must be displacing an amount of fluid whose weight equals its own weight.

Principle of Flotation is derived from (1) balance of vertical forces, and, (2) Archimedes' Principle:
In order for a body to float, the upward force exerted on the body must be equal to the downward force on the body (i.e. it is vertically in equilibrium). In this case, the upward force exerted on the body is the upthrust, whereas the downward force on the body is the gravitational force on the body (i.e. weight of the body).

Weight of floating body, W_o = Upthrust acting on the body, U

According to Archimedes' Principle, the upthrust on the body is equal to the weight of the fluid displaced by the body.

Thus,

Weight of floating body, W_o = Weight of fluid displaced, $W_{fl.displ.}$

The above is the Principle of Flotation.

$m_o g = m_{fl.displ} g$

$V_o \rho_o g = V_{fl.displ} \rho_f g$

$V_o > V_{fl.displ}$ (otherwise it won't be floating!)

Hence, $\rho_o < \rho_f$ in order for an object to float

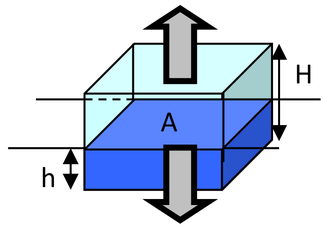
Example 12
A block of ice of weight 1500 N is floating in a fresh-water lake.
a) What is the upthrust on the ice?
b) What is the weight of water displaced?

Solution:

a) 1500 N (since vertical forces are balanced)
b) 1500 N (Archimedes' Principle)

Example 13

A wooden object of density 710 kg m^{-3} floats in water of density 1000 kg m^{-3}. What fraction of the object will be below the surface of the water?



Solution:

The object is floating in equilibrium.
Upthrust = Weight of object
Weight of fluid displaced = Weight of object

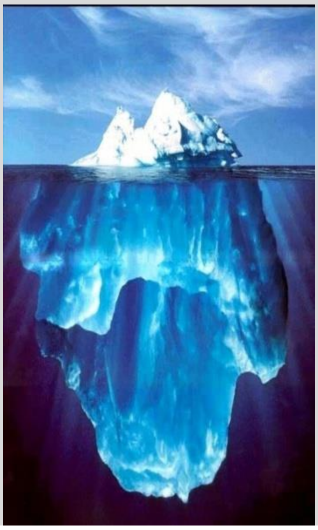
$$m_f g = m_o g \Rightarrow \rho_f V_f = \rho_o V_o$$

$$\frac{V_f}{V_o} = \frac{\rho_o}{\rho_f} \Rightarrow \frac{hA}{HA} = \frac{710}{1000}$$

$$\frac{h}{H} = 0.71$$

MINI-TEST 3

What proportion of an iceberg (floating in water) is submerged?
[density of ice = 0.92 g / cm^3; density of water = 1 g / cm^3]



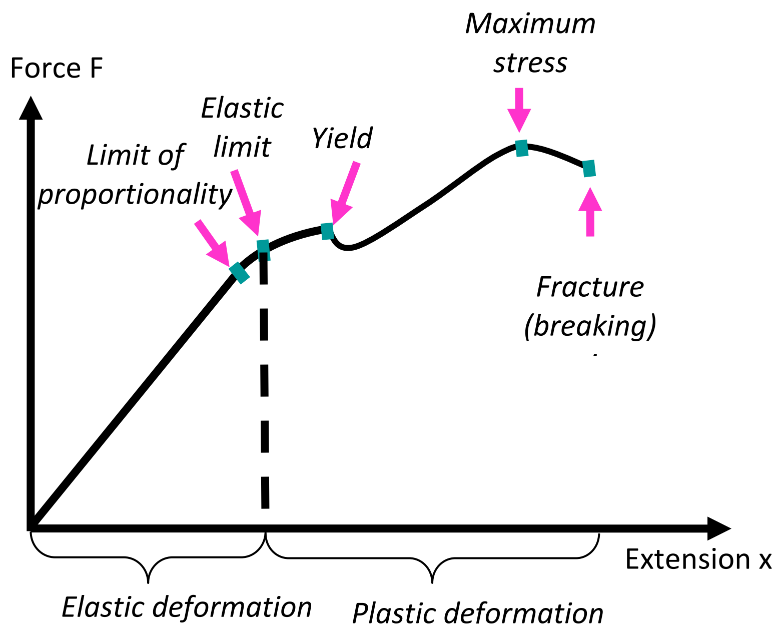
My solution

CHAPTER SUMMARY / SELF-CHECK

1. What the meaning of a force?
2. Can a force exist singly?
3. What is the difference between contact forces and non-contact forces? What are examples of each?
4. What is a force field?
5. What does Hooke's Law state?
6. What is the origin of frictional forces and what does its magnitude depend on?
7. How do viscous drag forces differ from frictional forces?
8. What is the meaning of 'equilibrium'?
9. What are the conditions for an object to be in equilibrium?
10. Can a moving object be in a state of equilibrium?
11. What are the effects on motion that a force acting on a body can produce?
12. How does pressure vary in a fluid? How is the relationship derived?
13. What is upthrust? How does it arise?
14. Does upthrust occur in liquids only, gas only, or both?
15. What is upthrust numerically equal to?
16. Does upthrust exist for both floating and sinking objects in a fluid?
17. What is the condition for an object to float in a fluid?
18. How is the Principle of Flotation derived from Archimedes' Principle and Newton's Laws of Motion?

APPENDIX A

Force-Extension Graph



Graph of Force vs Extension showing an elastic body undergoing elastic and plastic deformation

- ***Limit of proportionality:*** Hooke's law is obeyed, i.e. $F = k x$

- ***Elastic limit:*** The spring will return to its original shape if the force is removed.

- ***Elastic deformation:*** Material will return to its original shape when force is removed.

- ***Plastic deformation:*** Permanent deformation, i.e. material will not return to its original shape even upon removal of applied force.

- ***Yield Point:*** If the force is increased beyond the elastic limit, a point is reached at which there is a marked increase in extension. This is the Yield Point.

Forces Tutorial

Viscous force and Friction

Q1 H1 N13/I/9

In which process do viscous forces create a significant resistance to motion?

A the application of paint to the surface of a wall

B the compress of air while pumping up a car tyre

C the fall of water drops from a faulty tap

D the spreading of petrol on a wet road surface

Q2 H1 N11/II/5a,c

(a) Explain what is meant by friction when used in the context of force. [1]

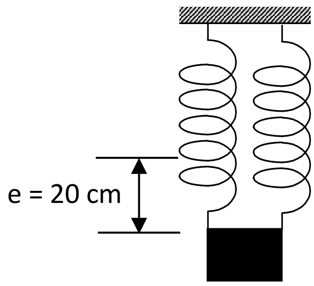
(b) Friction is often regarded as a nuisance.

(i) State two different situations where friction is of critical importance. [2]

(ii) For one of your examples explain why friction is so important. [1]

Hooke's Law

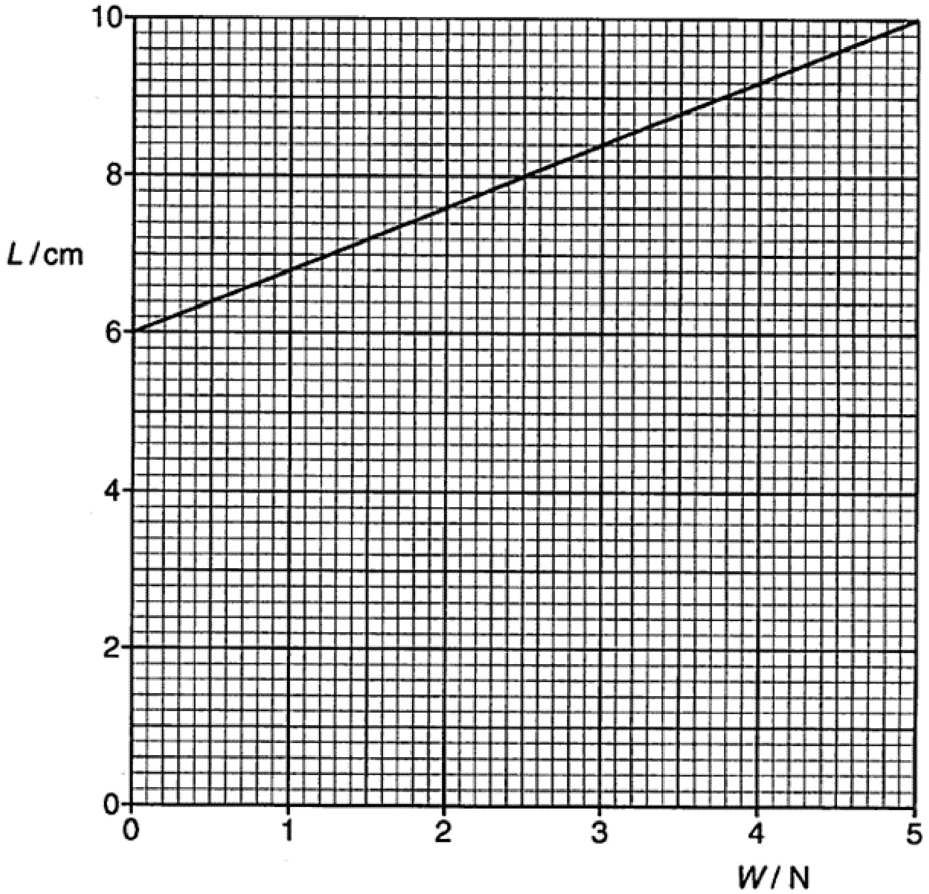
Q3 2 identical springs are connected in parallel to one another as shown in the figure below. This spring combination is subject to a weight of 10 N, resulting in an extension of 20 cm. Determine the spring constant, k, for each spring.



A 5 N m^{-1} **B** 25 N m^{-1} **C** 50 N m^{-1} **D** 100 N m^{-1}

Q4 H2 N10/III/6b (Modified)

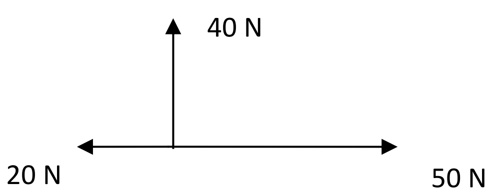
A light helical spring is suspended vertically from a fixed point. Different mases are suspended from the spring. The weight *W* of the mass and the length *L* of the spring are noted. The variation with Weight *W* of the length *L* is shown in the graph below.



Using the graph, determine the spring constant of the spring. [125 N m^{-1}] [2]

Equilibrium and the Turning Effect of Forces

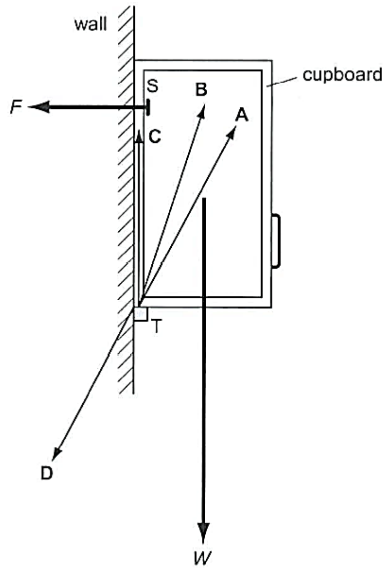
Q5 Three coplanar forces of magnitude 20 N, 40 N and 50 N, act on a body at P in the directions shown in the figure. What is the additional force required to maintain equilibrium?



[50 N, 217°] [3]

Q6 2012/1/8

A cupboard is secured to a wall by a screw at S. It rests on a support at T. Arrows show the directions of the weight W of the cupboard and the horizontal force F of the screw at S on the cupboard. Which arrow best represents the direction of the force on the cupboard from the support T?



Q7 2013/1/8

In order to pull a protruding tooth back into line, a metal band is attached to the tooth, as shown in diagram 1.

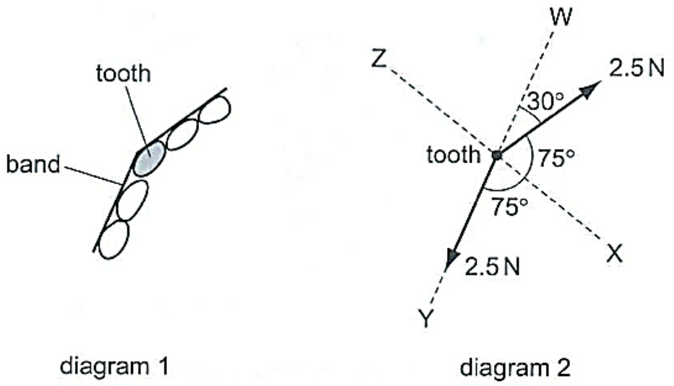


Diagram 2 shows the two forces of 2.5 N which the band exerts on the tooth. The angle between the two parts of the band is 150°.

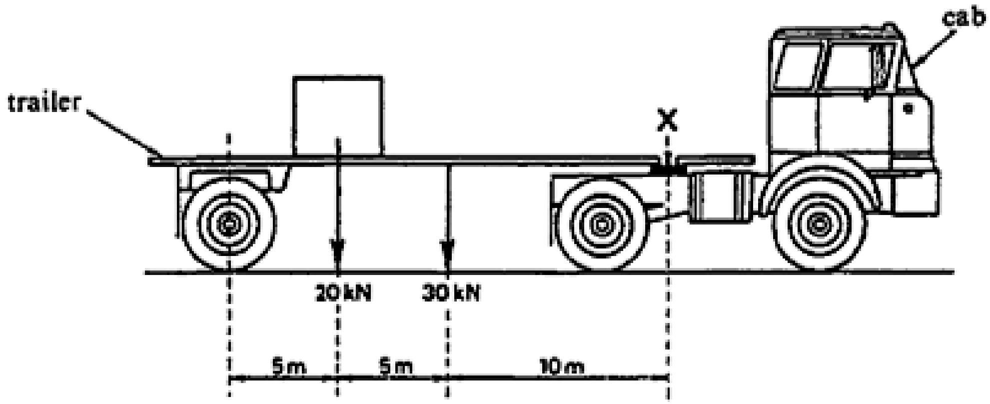
This treatment makes the tooth move so slowly in the gum that it is effectively in equilibrium. For equilibrium to exist, another force must act.

What is the magnitude and direction of this force?

	magnitude	direction
A	0.33 N	W
B	0.33 N	Y
C	1.3 N	X
D	1.3 N	Z

Q8 1986/I/2

A trailer of weight 30 kN is hitched to a cab at the point X as shown in the figure below.



If the trailer carries a weight of 20 kN at the position shown in the diagram, calculate the upward force exerted by the cab on the trailer at the point X? [2]

[20 kN]

Q9 2002/III/1a,b,c (modified)

(a) Define the terms *moment of a force* and *torque of a couple*. For each of the terms draw a sketch to illustrate its meaning. [4]

(b) State the two conditions necessary for a body to be in equilibrium. [2]

(c) During the construction of many modern bridges, sections are added from both banks until the two halves meet at the centre. The figure shows a new section S, of weight 3.0×10^5 N, after it has been attached to an existing part B of a bridge.

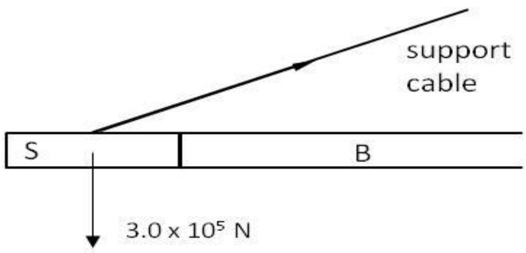


Fig. 1.1

The support cable which keeps section S in equilibrium is at an angle of 25° to the horizontal. The existing part B of the bridge provides a horizontal force on S.

(i) Draw a labelled vector diagram showing the three forces on S. [2]

(ii) Use your diagram to determine the tension in the cable and the horizontal force which B exerts on S. [4]

[7.1×10^5 N, 6.4×10^5 N]

(iii) Since the section S is in equilibrium under the action of three coplanar forces, what can be said about the lines of action of the forces? [1]

Q10 A uniform ladder AB of length 5.0 m and mass 20 kg is propped against a smooth, vertical wall with end B uppermost, and end A on rough, horizontal ground.
End B is 4.0 m above the ground and end A is 3.0 m out from the foot of the wall.
If a man of mass 70 kg stands on the ladder at a point 3 m vertically above the ground, calculate

(a) the reaction force at the wall, [2]

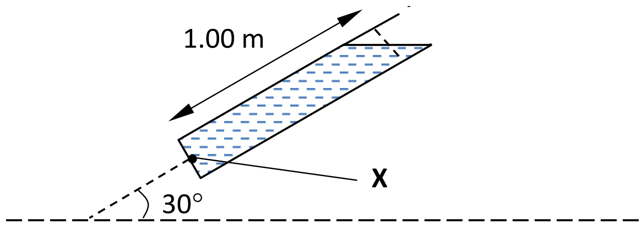
(b) the magnitude of the reaction force at the ground and the angle between this reaction force and the horizontal. [3]

[460 N, 996 N, 62.5^0]

Pressure in a fluid

Q11 N91/I/21

A long narrow tube is filled with water of density 1020 kg m^{-3} to a depth of 1.00 m. The tube is then inclined at 30° to the horizontal as shown.



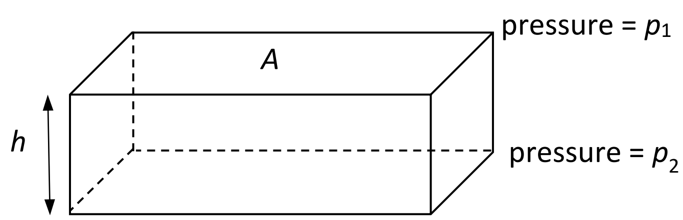
If the atmospheric pressure is 100 kPa, what is the pressure at point **X**, inside the tube?

A 5.00 kPa **B** 10.0 kPa **C** 95.0 kPa **D** 105 kPa

Origin of Upthrust

Q12 N03/I/6

A solid block of material of density ρ, height h and horizontal surface area A is immersed in a liquid. The pressures of the liquid at the upper and lower surfaces are p_1 and p_2 respectively.



Which of the following is an expression for the upthrust on the block? (Given g is the acceleration of free fall)

A $Ah\rho g$ **B** $Ah\rho g + p_1$ **C** p_2A **D** $p_2A - p_1A$

Concept of weight as a force experienced by a mass in a gravitational field

Q13 (9702) N10/P22/Q3

A uniform rectangular sheet of card of weight *W* is suspended from a wooden rod. The card is held to one side, as shown in Fig. 7.

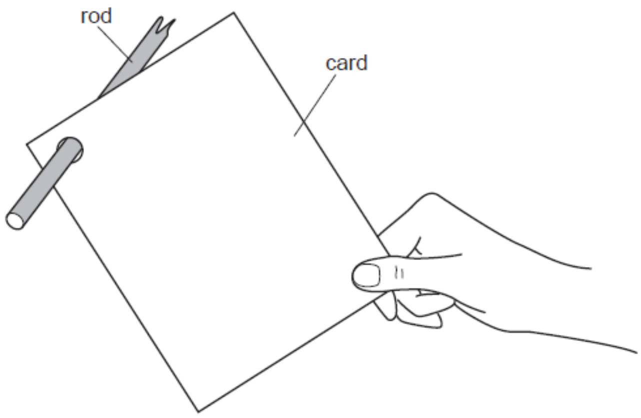


Fig. 7

On Fig. 7,

(a) (i) mark, and label with the letter C, the position of the centre of gravity of the card,

(ii) mark with an arrow labelled *W* the weight of the card.

(b) The card in (a) is released. The card swings on the rod and eventually comes to rest.

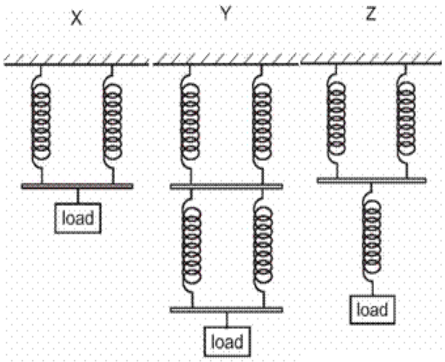
(i) List the two forces, other than its weight and air resistance, that act on the card during the time that it is swinging. State where the forces act.

(ii) By reference to the completed diagram of Fig. 7, state the position in which the card comes to rest. Explain why the card comes to rest in this position.

Assignment Question

A1 **J05/P1/Q20**

A number of similar springs, each having the same spring constant, are joined in three arrangements X, Y and Z. The same load is applied to each arrangement.

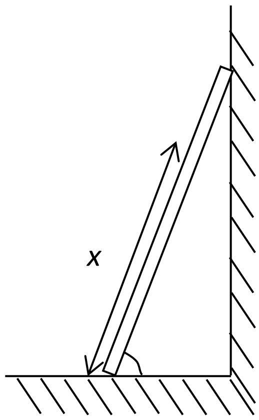


What is the order of increasing extension for these arrangements?

	smallest → largest		
A	X	Y	Z
B	Z	X	Y
C	Z	Y	X
D	Y	X	Z

(You are required to provide a working or an explanation to your answer)

A2 A uniform ladder which is of 3 m long and a mass of 20 kg, leans with its upper end against a smooth vertical wall and its lower end on a rough ground. The angle of elevation of the ladder is 60˚ above the horizontal as shown in the figure.



(a) A technician of mass 60 kg tries to climb up the ladder. Given that the maximum friction between the rough ground and ladder is 300 N, calculate the maximum distance, ***x,*** where the technician could climb before the ladder slips. [3]

(b) Suggest with a reason whether the technician can climb higher if the vertical wall is rough. [2]

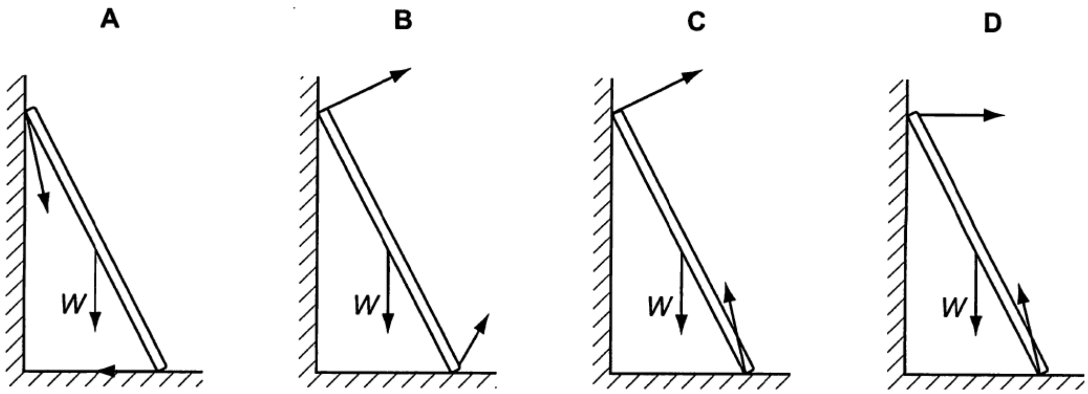
Supplementary Questions

S1 2008/1/8

A ladder of weight W rests against a vertical wall.

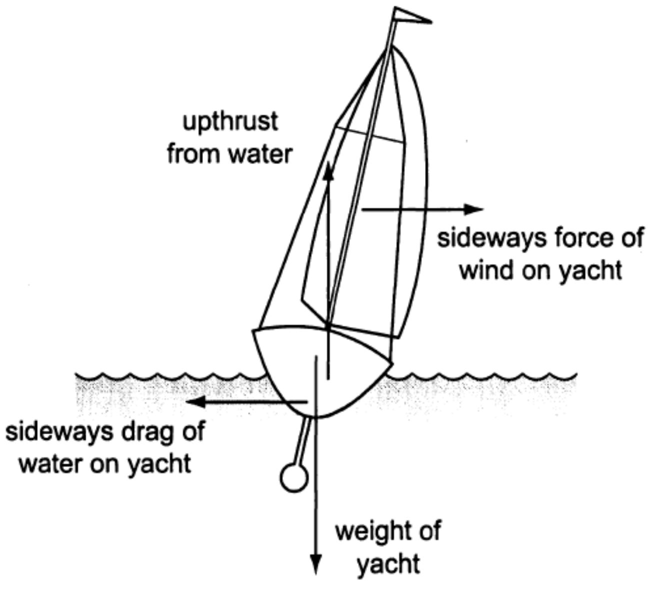
Friction between the ladder and the ground and also between the ladder and the wall prevents the ladder from slipping.

Which diagram shows the directions of the forces on the ladder?



S2 2009/1/8

A yacht that is in equilibrium has two vertical and two horizontal forces acting on it.



Which statement about the forces is **not** correct?

A The sideways drag from the water on the yacht is equal and opposite to the sideways force of the wind on the yacht.

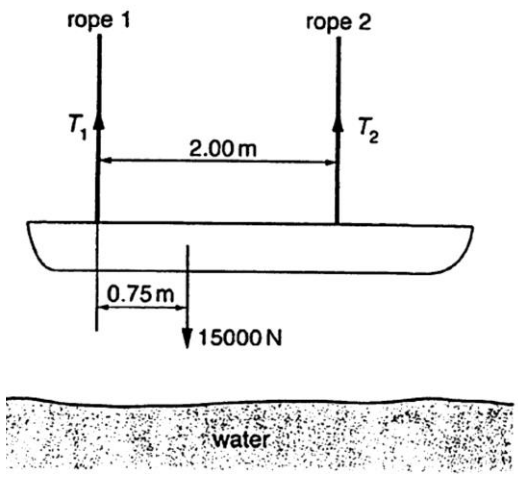
B The resultant of all four forces is zero.

C The torque provided by the vertical forces is the same as the torque provided by the horizontal forces.

D The upthrust from the water is equal and opposite to the weight of the yacht.

S3 2009/II/3

The figure below shows a force diagram that represents a boat that is being lifted by two ropes so that the boat remains horizontal and travels vertically upwards at a constant speed after leaving the water.



The weight of the boat is 15000 N and the tensions in the ropes 1 and 2 are T_1 and T_2 respectively.

(a) The position of the centre of gravity of the boat is not at its midpoint. Suggest what this implies about the distribution of mass in the boat. [1]

(b) Explain two conditions required for the boat to be in a state of equilibrium while it is moving upwards. [2]

(c) Use the principle of moments to determine the tensions in the two ropes. [4]

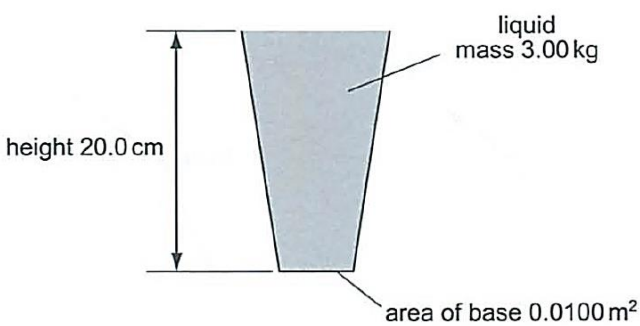
[9400 N, 5600 N]

S4 N04/III/10aii

Derive the expression p = ρgh from the definitions of pressure and density. Explain each stage in your derivation. [3]

S5 N13/1/7

The diagram shows a jar, with the dimensions shown, containing a liquid of mass 3.00 kg and density 600 kg m^{-3}. The liquid exerts a pressure on the base of the jar.

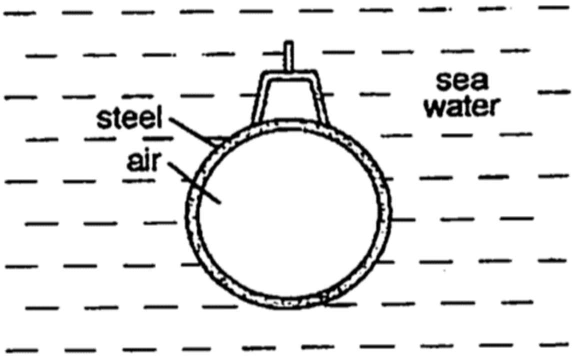


What is the pressure exerted by the liquid on the base of the jar?

A 1.18×10^3 Pa **B** 2.94×10^3 Pa **C** 1.18×10^5 Pa **D** 2.94×10^6 Pa

S6 N04/I/7

A submarine is in equilibrium in a fully submerged position.



What causes the upthrust on the submarine?

A The air exerts a greater upward force on the submarine than the weight of the steel.

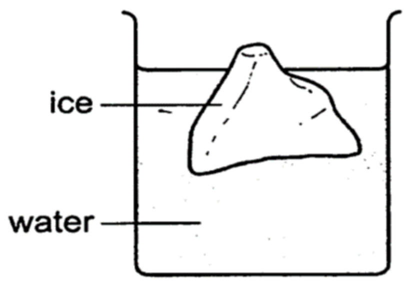
B The air in the submarine is less dense than sea water.

C There is a difference in water pressure acting on the top and bottom of the submarine.

D The submarine displaces its own volume of sea water.

S7 N07/I/8

A lump of ice floats in water as shown.



Which statement is correct?

A The lump of ice floats because the area of its lower surface is larger than the area of its upper surface.

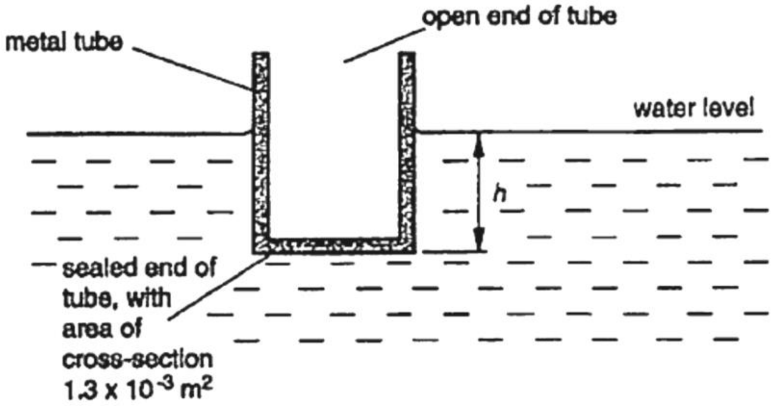
B The pressure difference between the lower and the upper surfaces of the lump of ice give rise to an upthrust equal to its weight.

C The ice has a greater density than the water.

D The mass of water displayed by the ice is equal to the upthrust.

S8 N04/II/1bi

A metal tube of uniform cross-sectional area 1.3 x 10-3 m^2 is sealed at one end. It floats upright in water as shown below.



When the tube is stationary, the base is 5.5 cm below the surface of the water. The pressure p due to water of depth h is given by the expression

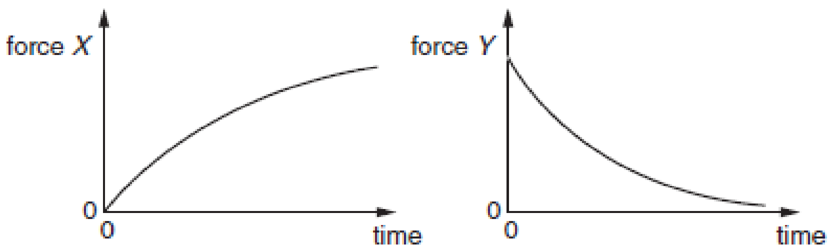
$$p = (9.8 \times 10^3)\ h,$$

where p is in pascals and h is in metres.

(a) Calculate the force acting on the base of the tube [2]
(b) State the direction of the force calculated in (a). [1]
(c) State the weight of the tube. [0.701N, 0.701 N] [1]

S9 An object suspended from a spring balance reads 24 N. When the object is fully submerged in water of density 1000 kg m^{-3}, the weight registered by the spring balance is only 18 N. When it is fully submerged in some other liquid, the spring balance reads 14 N. Find the average density of the object and that of the other liquid. [4000 kg m^{-3}, 1.67×10^3 kg m^{-3}] [4]

S10 A ball falls from rest through air and eventually reaches a constant velocity. For this fall, forces X and Y vary with time as shown.



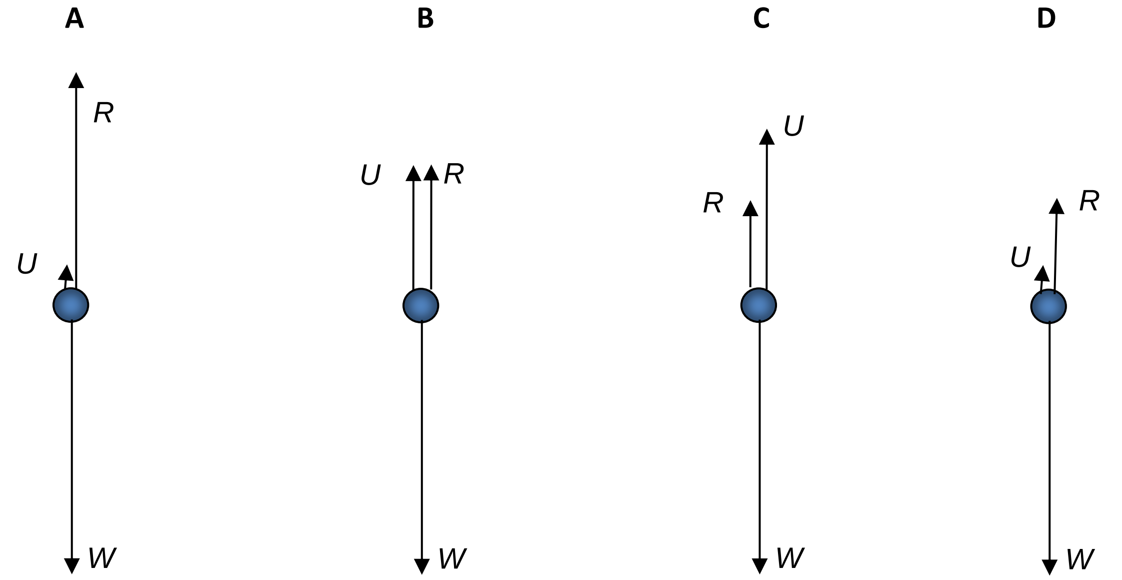
What are forces X and Y?

	force X	force Y
A	air resistance	resultant force
B	air resistance	weight
C	upthrust	resultant force
D	upthrust	weight

S11 N10/P1/Q4

A water droplet in a cloud is falling through air and is in equilibrium. Three forces act on it, its weight W, upthrust U and air resistance R.

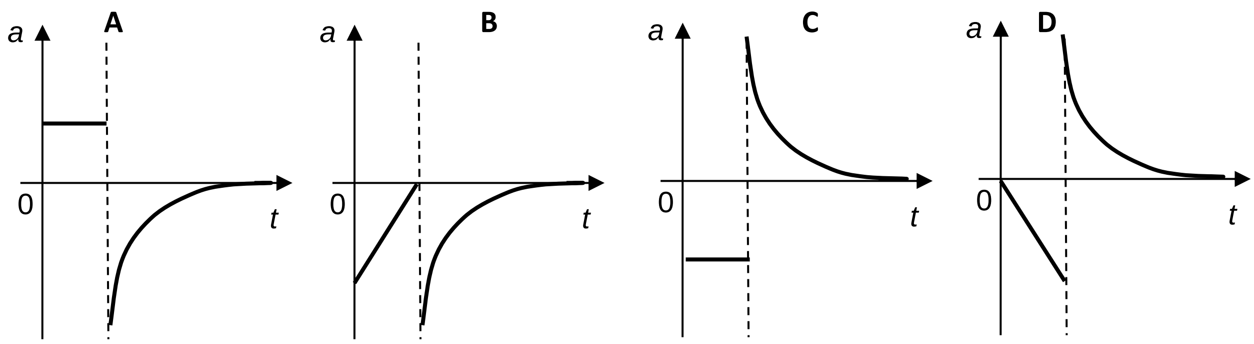
Which diagram, showing these three forces to scale is correct?



S12 **Adapted from J83/P2/Q5 & N83/P2/Q4**

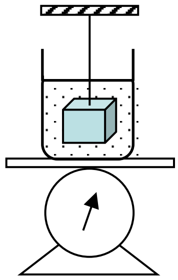
(a) A small metal sphere is released from rest at a height of a few centimeters above the surface of a viscous liquid in a tall wide jar. On entering the liquid, the sphere experiences a viscous drag proportional to its velocity.

Taking upwards as positive, which one of the following graphs best represents the variation of acceleration *a* with time *t* of the sphere?



S13 N84/II/8

The reading of a balance is X when a beaker filled with water is placed on it. A solid of weight Y in air displaced water of weight Z when immersed in the water.



What is the balance reading when the solid is hung in the beaker containing water as shown in the figure?

[2]

[x + Z]