

Anglo - Chinese School

(Independent)



FINAL EXAMINATION 2021

YEAR THREE EXPRESS

ADDITIONAL MATHEMATICS

PAPER 1

4049/01

Tuesday

5 October 2021

1 hour 30 minutes

Candidates answer on the Question Paper.

No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your index number in the space at the top of this page. Write in dark blue or black pen. You may use a soft pencil for any diagrams or graphs. Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.

Write your answers in the spaces provided under each question. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. The use of a scientific calculator is expected, where appropriate. You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question. The total number of marks for this paper is 60.





1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^{n} = a^{n} + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^{2} + \dots + \binom{n}{r}a^{n-r}b^{r} + \dots + b^{n},$$

where *n* is a positive integer and
$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\cos ec^2 A = 1 + \cot^2 A$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^{2} = b^{2} + c^{2} - 2bc \cos A$$
$$\Delta = \frac{1}{2}ab \sin C$$

Answer all the questions.

1 Solve the following pair of simultaneous equations

$$-3xy + 2y^2 = -1$$
$$2x - y = 1$$

2 Given that $\sin \theta = \frac{8}{17}$ and that θ is obtuse, find the exact value of (i) $\tan \theta$,

(ii) $\cos(-\theta)$,

[1]

[2]

(iii) $\sec(180^\circ - \theta)$.

3 The figure shows part of a straight line graph obtained by plotting $\frac{1}{y}$ against $\frac{1}{x^2}$. Express y in terms of x. [4]



4 Express the following in partial fractions.

(a)
$$\frac{3x^2-2x+2}{2x(x^2+2)}$$
,

[5]

(b)
$$\frac{x^2 + 4x + 5}{x^2 + 4x + 3}$$
.

[4]

- 5 The expression $f(x) = ax^3 + x^2 + bx + 15$ leaves a remainder of 11 and 14 when divided by (x 2) and (x + 1) respectively.
 - (i) Determine the value of *a* and of *b*.

[5]

(ii) Prove that (5 - 2x) is a factor of f(x).

[1]

(iii) Explain why f(x) = 0 has only one real root.

[3]

(b) The diagram below shows part of the graph of the function $y = a \sin bx + c$.



Find the value of *a*, of *b* and of *c*.

[3]

- 7 Solve the following equations.
 - (i) $2\log_4 3x = \log_3 27 + \log_2 (x-5)$, [4]

(ii) $\ln(4^x + 2) = x \ln 2 + \ln 3$,

[5]



- 8 The points P(-3, 0) and Q(1, 2) lie on the circle C_1 such that PQ is the diameter of the circle.
 - (i) Find the centre and radius of circle C_1 .

(ii) Given that the equation of the tangent to C_1 at point *R* is y = 2x + 8, find the coordinates of *R*. [4]

[2]

(iii) A circle C_2 is obtained by reflecting C_1 in the tangent line y = 2x + 8. Find the equation of C_2 . [3]

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Answers

1.
$$x=1, y=1$$
 or $x=\frac{3}{2}, y=2$
2. (i) $\frac{8}{15}$ (ii) $-\frac{15}{17}$ (iii) $\frac{17}{15}$
3. $y=\frac{x^2}{2-5x^2}$
4. (a) $\frac{1}{2x}+\frac{x-1}{x^2+2}$ (b) $1+\frac{1}{x+1}-\frac{1}{x+3}$
5. (i) $a=-2, b=4 = 2$ (iii) $x=\frac{5}{2}$
6. (a) $x=22.5^\circ, 67.5^\circ, 112.5^\circ, 157.5^\circ$ (b) $a=6, b=\frac{1}{4}, c=-5$
7. (i) $x=8$ (ii) $x=1$ or $x=0$ (iii) $x=119$
8. (i) $C(-1,1)$, radius = $\sqrt{5}$ units (ii) $R(-3,2)$ (iii) $(x+5)^2+(y-3)^2=5$