Paya Lebar Methodist Girls' School (Secondary)
Preliminary Examination 2017
Secondary 4 Express / 5 Normal Academic

Name: ____________________________ (   )   Class: _______

Centre Number	S				

Index Number				

ADDITIONAL MATHEMATICS **4047/01**

Paper 1 — 17 August 2017

Additional Materials: Answer Paper (8 sheets)
Graph Paper (1 sheet)

2 hours

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number, name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper and Graph Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

My Target is:

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 State the values between which each of the following must lie

(a) the principal value of $\tan^{-1} x$, [1]

(b) the principal value of $\cos^{-1} 2x$. [2]

2 The function f is defined, for all values of x, by $f(x) = (x+3)(1-2x)^2$.
Find the range of values of x for which f is a decreasing function. [3]

3 In the expansion of $(2x-1)^2\left(1+\dfrac{p}{x}\right)^8$, where p is a positive constant, there is no term in $\dfrac{1}{x^3}$. Find the possible values of the constant p. [5]

4 A curve has the equation $y = 4x^2 - 24x + 30$.

(i) Express $4x^2 - 24x + 30$ in the form $[a(x+h)]^2 + k$. [1]

(ii) Show that the minimum point of the curve has coordinates $(3, -6)$. [1]

(iii) Sketch the graph of $y = |4x^2 - 24x + 30|$, indicating clearly the **exact** x-intercept(s) and y-intercept. [3]

A line of gradient m passes through the point $(0, -10)$.

(iv) Given that $0 < m \leq 10$, determine the **exact** value of m, for which the line intersects the graph of $y = |4x^2 - 24x + 30|$ at one real and distinct point. [2]

5 **(i)** Factorise completely $2x^3 + 7x^2 + 4x - 4$. [3]

(ii) Express $\dfrac{12x^2+32x+31}{2x^3+7x^2+4x-4}$ in partial fractions. [5]

[Turn over

6 The equation of a curve is $y = ax^2 - 3x + 4 - a$, where a is a constant.

(i) In the case where $a = -2$, find the set of values of x for which the curve lies completely below the line $y = -3$. [3]

(ii) In the case where $a = 3$, show that the line $y = 3x - 2$ is a tangent to the curve. [2]

(iii) Determine if there is any other value of a for which the line $y = 3x - 2$ intersects the curve at only one point. [3]

7 **(i)** Prove that $\dfrac{\sin\theta}{(\sec\theta - \tan\theta)(\sin\theta + 1)} = \tan\theta.$ [4]

(ii) Find all the values of θ between 0 and π for which

$$\frac{\sin\theta}{(\sec\theta - \tan\theta)(\sin\theta + 1)} = 1 - \sec^2\theta.$$ [4]

8 An auction house claimed that it is worthwhile to invest in their art pieces as the value of one of their art pieces has been increasing exponentially since it was produced.

The value, $\$V$, of this art piece is related to t, the number of years since it was produced at the start of the year 1995.

The variables V and t can be modelled by the equation $V = 10\,000 + ae^{kt}$, where a and k are constants. The table below gives values of V and t at the start of some of the years 2000 to 2015.

Year	2000	2005	2010	2015
t years	5	10	15	20
$\$V$	16 000	20 260	27 545	40 000

(i) Plot a suitable straight line graph to show that the model is valid for the years 2000 to 2015. [3]

(ii) Estimate the value of a and of k. [3]

(iii) A claim was made that in the year 2065, this art piece will increase in value by 500 times from the time it was produced. Do you agree? Justify your answer. [2]

9 The point A lies on the curve $y = x \ln x^2$, $x > 0$. The tangent to the curve at A is parallel to the line $y - 3x = 1$.

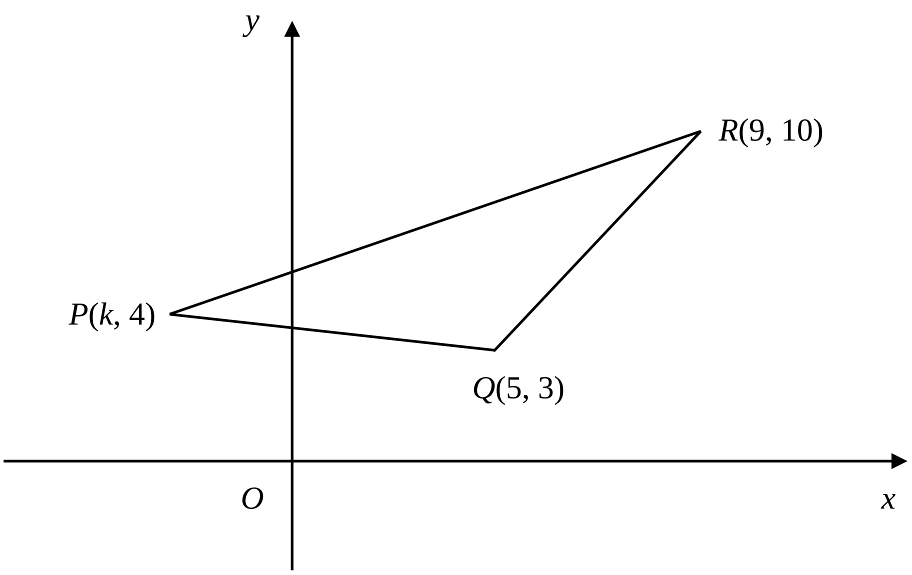
(i) Find the **exact** coordinates of A. [4]

The normal to the curve $y = x \ln x^2$ at A meets the line $y - 3x = 1$ at B.

(ii) Show that the x-coordinate of B can be expressed in the form $\frac{1}{10}(p\sqrt{e} + q)$, where p and q are integers to be found. [4]

10 **Solutions to this question by accurate drawing will not be accepted.**

The diagram shows an isosceles triangle PQR in which $PQ = QR$. The vertices of the triangle are at the points $P(k, 4)$, $Q(5, 3)$ and $R(9, 10)$.



(i) Find the value of k. [4]

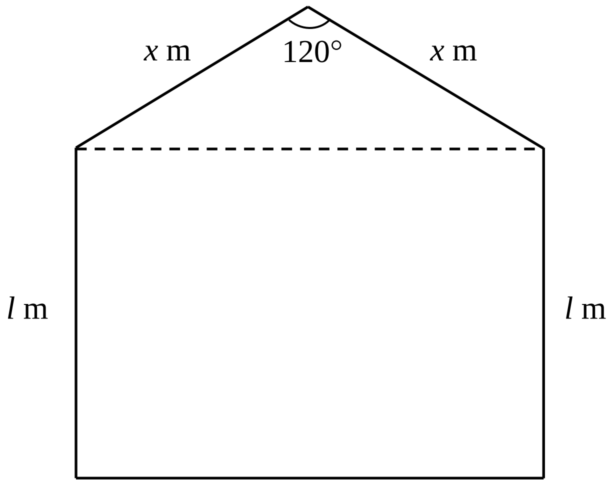
A line is drawn from Q to cut the y-axis at S such that $PS = SR$.

(ii) Find the equation of QS and the coordinates of S. [4]

(iii) Find the ratio of the area of triangle PQR to the area of quadrilateral $PQRS$. [3]

[Turn over

11 A farmer uses 160 m of fencing to enclose a plot of his land in a shape that comprises an isosceles triangle and a rectangle, with the dimensions shown.



(i) Show that the area of the plot is $\dfrac{320\sqrt{3}\,x - (3\sqrt{3} + 6)\,x^2}{4}$ m^2. [4]

(ii) Given that x can vary, find the value of x for which the area of the plot is stationary. [4]

(iii) Explain why this value of x gives the farmer the largest possible area for the plot. Find this area and give your answer correct to the nearest square metre. [3]

End of Paper