O	JURONG SECONDARY SCHOOL 2023 GRADUATION EXAMINATION SECONDARY 4 EXPRESS/ SECONDARY 5 NORMAL (ACADEMIC)

CANDIDATE NAME	

CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS **4049/01**

PAPER 1
Candidates answer on the Question Paper.
Additional Materials: Writing Paper (1 sheet)

22 August 2023
2 hours 15 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

This document consists of **19** printed pages including this page.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer, and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 A curve has an equation $y = 2x^2 - x + 5$.

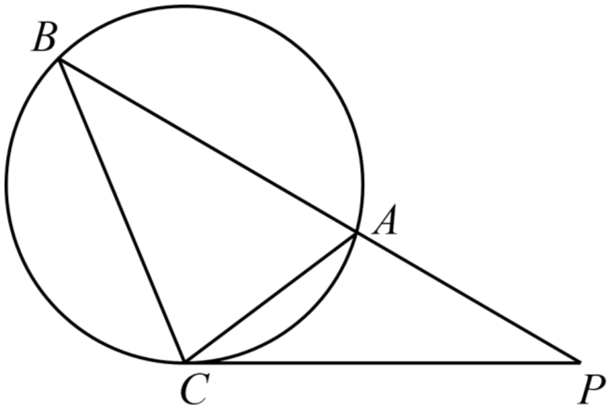
(a) Express $y = 2x^2 - x + 5$ in the form of $a(x-b)^2 + c$. Hence state the coordinates of the turning point. [3]

(b) The line $y = 2x + 7$ intersects the curve at points *A* and *B*. Find the distance *AB*. [3]

2 Express $\dfrac{3x^3 + 10x^2 + x + 1}{x^3 + 3x^2}$ in partial fractions. [6]

3 A curve has equation $y = \frac{1}{3}x^3 + x^2 + kx$, where k is a constant and $k > 1$. Explain why the curve does not have a stationary point. [4]

4 The diagram shows a circle. The line PC is the tangent to the circle at P. A and B are points on the circle such that PAB is a straight line.



Prove that

(a) triangle BPC is similar to triangle CPA, [3]

(b) $PA \times PB = PC^2$. [2]

5 The equation of a curve is $y = -\frac{4}{3}x^3 - (k+1)x^2 - k^2 x$, where k is a constant.

(a) Find the range of values of k for which y is always decreasing. [4]

(b) Given that y has three distinct roots, find the range of values of k. [2]

6 A curve is such that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 3\sin x - 4\cos 2x$. The curve passes through $A(0,1)$ and $B(\pi, 3)$. Find the equation of the curve. [7]

7 For the curve $y = 2x^2$, the tangent at point P where $x = a$, intersect the y-axis at A. The normal to the curve at point P intersects the y-axis at B.

Given that $a > 0$, show that the area of triangle ABP is $\dfrac{a\left(16a^2 + 1\right)}{8}$. [7]

8 **(a)** The equation of a curve is $y = a\sin bx + c$, $a > 0$. The curve attains maximum and minimum values of 4 and 2 respectively, and the period is π radians. Show that $a = 1$, $b = 2$ and show that $c = 3$. [3]

(b) **(i)** Sketch, on the same diagram, the curves $y = \sin 2x + 3$ and $y = 3\cos x$ for $0 \le x \le 2\pi$ radians. [4]

(ii) Find the number of solutions to the equation $\sin 2x + 3 - 3\cos x = 0$ for $0 \le x \le 2\pi$ radians. [1]

9 A container of liquid was heated to a temperature of $90°C$. It was then left to cool in a chiller such that its temperature, $T°C$, t minutes after the heat was removed, is given by $T = Ae^{-qt}$, where A and q are constants.
Measured values of t and T are given in the following table.

t (minutes)	2	4	6	8
T°C	66.674	49.393	36.591	27.107

(a) Explain why $A = 90$. [1]

(b) Plot $\ln T$ against t and draw a straight line to illustrate the information. [3]

(c) Use the graph to estimate the value of q. [3]

(d) Use your graph to estimate the temperature of the liquid 5 minutes after it was left to cool. [2]

10 The length, breadth and height of a cuboid is $3p$ cm, p cm and $(1-p)$ cm respectively. The volume of the cuboid is $\frac{4}{9}$ cm^3.

(a) Show that $27p^3 - 27p^2 + 4 = 0$. [2]

(b) Show that $3p - 2$ is a factor to $27p^3 - 27p^2 + 4$. [2]

(c) Hence, find p and compute the surface area of the cuboid. [5]

11 **(a)** **(i)** Find the first 4 terms, in ascending powers of x, of the expansion of $\left(2-kx\right)^6$ where k is a non-zero constant. [2]

(ii) Given that the coefficient of x^3 is 30 times the coefficient of x, find the possible value(s) of k. [2]

(iii) Hence, show that there is no term in x^2 in the expansion of $\left(1-135x^2\right)\left(2-kx\right)^6$. [2]

(b) Explain why there is no odd powers of x in the expansion of $\left(x+\frac{1}{x}\right)^{2n}$ for $n \in \mathbb{N}$. [4]

12 A particle travels in a straight line so that, at time t seconds after leaving a fixed point O, its displacement from O is s metres and its velocity is v ms^{-1} , where $v = 3e^{t} - 60e^{-3t}$.

Find

(a) the initial velocity of the particle, [1]

(b) the value of t when the particle is instantaneously at rest, [3]

(c) the acceleration of the particle when $t = \ln 8$, [2]

(d) an expression for s in terms of t, [4]

(e) the total distance travelled in the first 5 seconds. [3]

THE END