Candidate Name	Form Class	Index Number

ANG MO KIO SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2024
SECONDARY FOUR EXPRESS / FIVE NORMAL ACADEMIC

ADDITIONAL MATHEMATICS **4049/02**
Paper 2
27 August 2024
2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is **90**.

For Examiner's Use
90

This document consists of **19** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots\dots\dots\dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots\dots\dots\dots\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Show that the solution of the equation $100^x + 10^{x+1} - 24 = 0$ is in the form $x = \lg a$,

where a is an integer. [4]

2 Explain why $x-1$ is a factor of $3x^3-5x+2$. Hence solve the equation $3x^3-5x+2=0$, giving your answers in exact form. [5]

3 **(a)** Given that $y=(x-1)\sqrt{4x+1}$, show that $\frac{\mathrm{d}y}{\mathrm{d}x}$ can be written in the form $\frac{px+q}{\sqrt{4x+1}}$ where p and q are constants. [4]

(b) Hence evaluate $\int_2^6 \frac{x}{\sqrt{4x+1}}\,\mathrm{d}x$. [5]

4 **(a)** Given that the curve $y = -3x^2 + px + q$ is always negative for $x < -2$ or $x > 3$, find the values of p and q. [3]

(b) **(i)** It is now given that $p = 1$. Find the values of q for which the line $y = qx + 1$ is a tangent to the curve $y = -3x^2 + px + q$ at point R. [4]

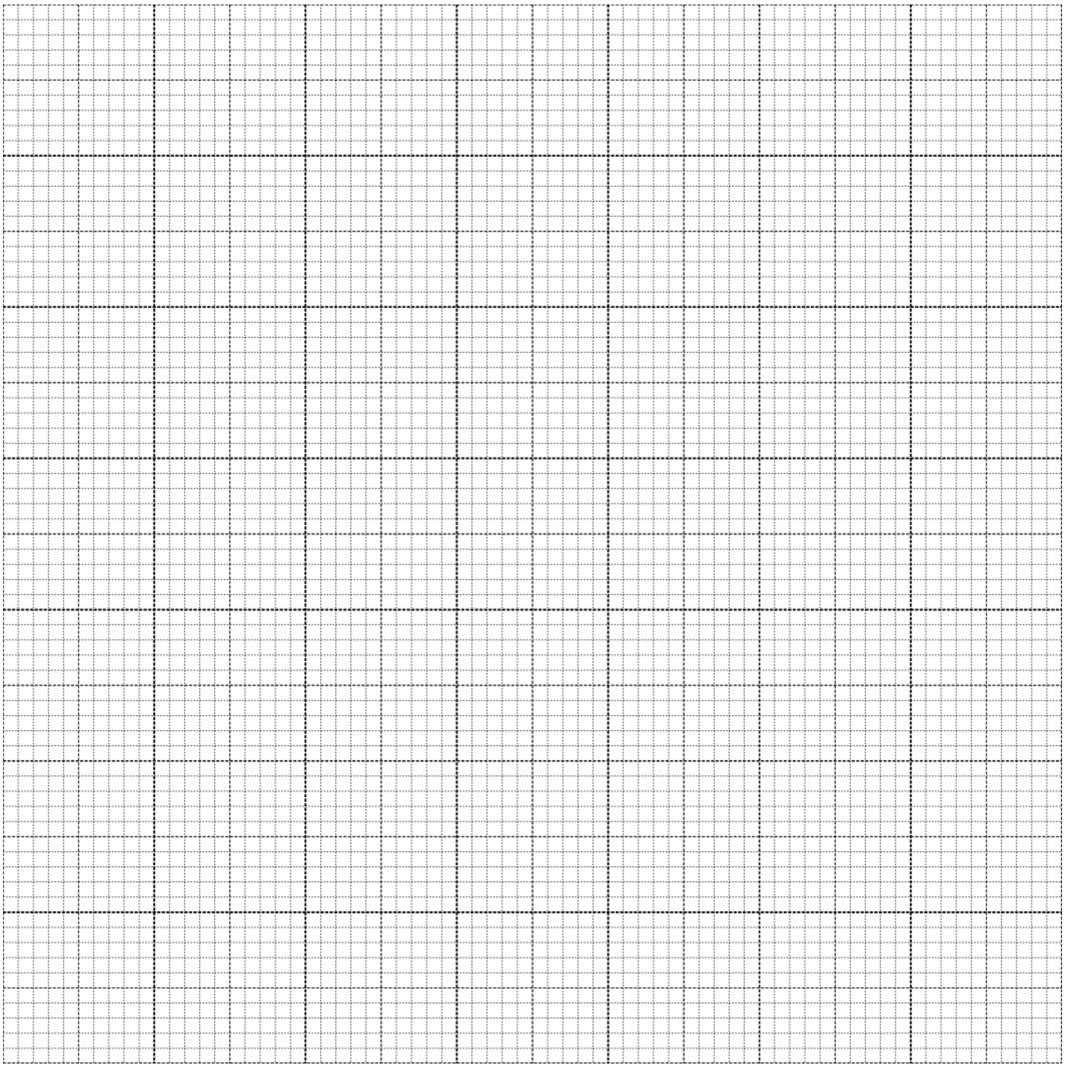
(ii) Using the smaller value of q from **part (i)**, find the coordinates of point R. [3]

5 **(a)** The table shows the time, t hours, before a scuba diver wearing a wet suit develops hypothermia when submerged in water of various temperature T °C.

Water Temperature T	2.2	5.0	7.8	10	12.8
Number of hours t	1.4	1.92	2.63	3.36	4.59

The relationship can be modelled by the formula $t = a(1.064)^{kT}$ where a and k are constants.

(i) Plot the graph of $\lg t$ against T and draw a straight line to illustrate the information. [2]



(ii) Use your graph to

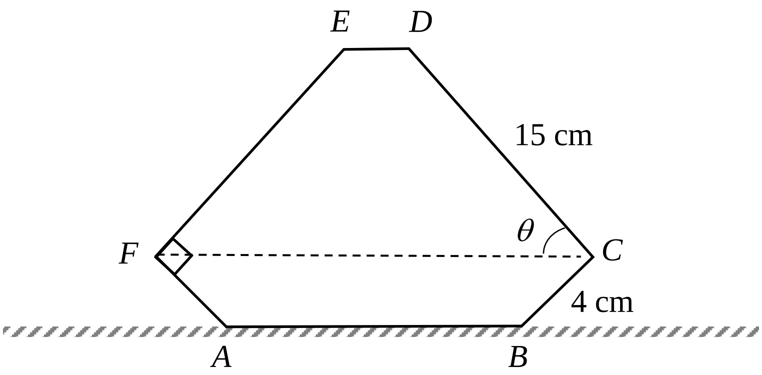
(a) find time taken for the diver to develop hypothermia when the water temperature is 0°C, [2]

(b) find the value of k. [3]

(iii) Explain why this model might not be applicable for another scuba diver. [1]

(b) The variables x and y are related in such a way that when y^2 is plotted against x^2y, a straight line is obtained. This line passes through the points $(-1,\ -1)$ and $(1,\ 3)$. Find an equation connecting x and y. [3]

6



The diagram shows a cross-section of a symmetrical aircraft wheel chock. *AB* is in contact with the ground. *AB*, *FC* and *ED* are horizontal. The lengths of *BC* and *CD* are 4 cm and 15 cm respectively. Angle *AFE* is 90° and angle *DCF* is θ, where $0° < \theta < 90°$.

(a) Show that the vertical height of *D* from the ground is $(4\cos\theta + 15\sin\theta)$ cm. [2]

(b) Express $4\cos\theta + 15\sin\theta$ in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [4]

(c) For the chock to secure the aircraft wheel, the angle θ should be between 40° and 50°.
If D is 14 cm above the ground, determine if the chock can secure the aircraft wheel. [3]

7 Driving along a straight road, a delivery rider passes a point A with a speed of 12 m/s and sees the traffic light ahead at point B turned amber. The traffic light will stay amber for 3 seconds before turning red. He speeds up and passes point B with a speed of 21 m/s.

Between points A and B, the velocity, v m/s, of the rider is given by $v = ke^{0.24t} + 8$, where t is the time in seconds after passing A and k is a constant to be found.

(a) Show that $k = 4$. [1]

(b) Find the time taken for the rider to reach point B and determine if he manages to pass point B before the traffic light turned red. [4]

(c) Calculate the average speed of the rider between A and B. [5]

8 **(a)** Write down the general term of $\left(x+\frac{k}{2x}\right)^n$, where k is a constant and n is a positive integer. Hence explain why x have odd powers in every term if n is an odd integer. [2]

(b) Write down and simplify the coefficient of x^7 in the expansion of $\left(x+\frac{k}{2x}\right)^{11}$, giving your answer in terms of k. [2]

(c) Show that $\left(x^2 - \frac{k}{2} + \frac{k^2}{4x^2}\right)\left(x + \frac{k}{2x}\right)^{12}$ can be expressed as $\left(x^3 + \frac{k^3}{8x^3}\right)\left(x + \frac{k}{2x}\right)^{11}$. [2]

(d) Hence or otherwise, given that the coefficient of x^4 in the expansion of $\left(x^2 - \frac{k}{2} + \frac{k^2}{4x^2}\right)\left(x + \frac{k}{2x}\right)^{12}$ is -577, find the value of *k*. [3]

9 Solutions to this question by accurate drawing will not be accepted.

The circle C_2 is the reflection of circle C_1 about the line $y = -x$ such that the x-coordinates of the points of intersection of C_1 and C_2 are -15 and -5.

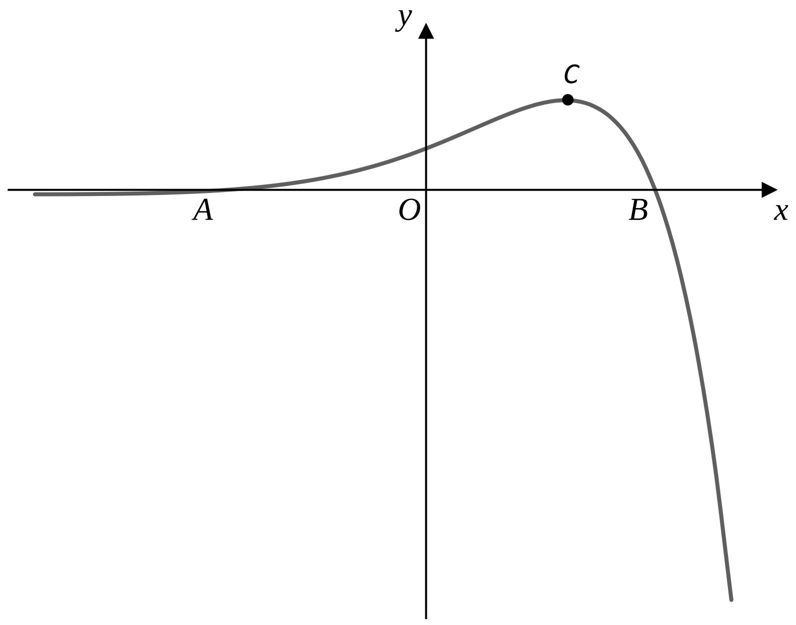
(a) Find the equation of the line passing through the centres of C_1 and C_2. [2]

The radius of circle C_1 is 10 units.

(b) Find the coordinates of the centres of circles C_1 and C_2. [4]

(c) Given that circles C_1 and C_2 are inscribed in a larger circle C_3, find the equation of circle C_3 in the form of $(x-a)^2+(y-b)^2=p+q\sqrt{2}$, where a, b, p and q are integers. [4]

10



The diagram shows the curve $y = e^{\sqrt{3}x}\cos x$ for $-2 \le x \le 2$ radians. The curve intersects the x-axis at points A and B. Point C is the maximum point of the curve.

(a) Find the coordinates of point C, leave your answer in exact form. [8]

(b) Show that the area of triangle *ABC* is $\frac{\pi}{4}e^{\frac{\pi}{\sqrt{3}}}$ units^2. [5]

END OF PAPER

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