JURONG JUNIOR COLLEGE
Preliminary Examinations

MATHEMATICS
Higher 2

9740 / 1
30 August 2016
3 hours

Additional materials: Answer Paper
Cover Page
List of Formulae (MF 15)

READ THESE INSTRUCTIONS FIRST

Write your name and civics class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together, with the cover page in front.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages.

[Turn over

1 In the finals of a General Knowledge Quiz, a team is required to answer 25 questions. Each question that is correctly answered scores 5 points, while a question that is wrongly answered is deducted 3 points. If the answer is partially correct, the team scores 2 points.

After 24 questions, the results are shown in the following table.

Number of Questions			
Correct	**Partially Correct**	**Wrong**	**Points**
a	b	c	79

If the team answers the last question wrongly, then the total number of questions answered correctly and partially correct is four times the number of questions answered wrongly. By forming a system of linear equations, find the values of a, b and c. [4]

2 A sequence $u_1, u_2, u_3, \ldots$ is such that $u_1 = \frac{1}{3}$ and

$$u_{r+1} = u_r - \frac{1}{(2r-1)(2r+3)}, \quad \text{for all } r \geq 1.$$

(i) Use the method of mathematical induction to prove that $u_n = \dfrac{n}{4n^2 - 1}$. [4]

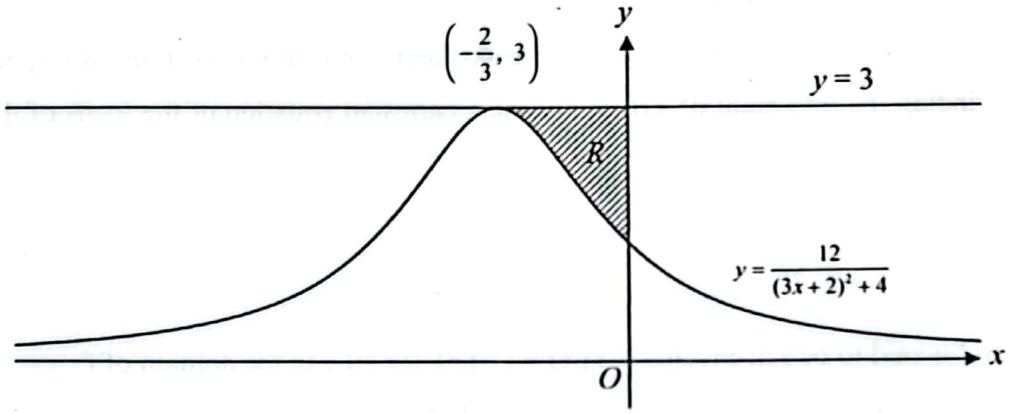
(ii) Hence prove that the sum of the first n terms of the series

$$\frac{1}{5 \times 9} + \frac{1}{7 \times 11} + \frac{1}{9 \times 13} + \cdots$$

is $\dfrac{3}{35} - \dfrac{n+3}{4(n+3)^2 - 1}$. [3]

(iii) Give a reason why the series in part (ii) is convergent and state the sum to infinity. [2]

3 The diagram shows the curve C with equation $y=\dfrac{12}{(3x+2)^2+4}$ which has a turning point at $\left(-\dfrac{2}{3},\ 3\right)$. The region R is bounded by C, the y-axis and the line $y=3$.



(i) Find the exact area of R. [5]

(ii) R is rotated through 2π radians about the y-axis. Find the volume of the solid of revolution formed, giving your answer to 4 decimal places. [3]

4 Let $y=\tan\left(2\tan^{-1}x+\dfrac{\pi}{4}\right)$.

(i) Show that $\left(1+x^2\right)\dfrac{dy}{dx}=2\left(1+y^2\right)$. [2]

(ii) Hence find the Maclaurin series for y, up to and including the term in x^2. [4]

Denote the answer to part **(ii)** of the Maclaurin series by $g(x)$ and $f(x)=\tan\left(2\tan^{-1}x+\dfrac{\pi}{4}\right)$.

(iii) Find, for $-0.4\le x\le 0.4$, the set of values of x for which the value of $g(x)$ is within ± 0.5 of the value of $f(x)$. [2]

[Turn over

5 A curve C has parametric equations

$$x = 2\sin 2t, \qquad y = \cos 2t, \qquad \text{for } 0 \le t < \pi.$$

(i) Show that the equation of the normal to C at the point P with parameter θ is

$$(2\cos 2\theta)x - (\sin 2\theta)y = m\sin 2\theta \cos 2\theta,$$

where m is an integer to be determined. [3]

(ii) The normal to C at the point P cuts the x-axis and y-axis at points A and B respectively. By finding the mid-point of AB, determine a cartesian equation of the locus of the mid-point of AB as θ varies. [5]

6 A function f is said to be self-inverse if $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$ for all x in the domain of f.

The function g is defined by

$$\mathrm{g} : x \mapsto \sqrt{\frac{x^2+2}{x^2-1}}, \quad x > 1.$$

(i) Sketch the curve $y = \mathrm{g}(x)$, stating the equations of the asymptotes clearly. [2]

(ii) Define g^{-1} in a similar form and show that g is self-inverse. [4]

(iii) Show that $\mathrm{g}^2(x) = x$ and that $\mathrm{g}^3(x) = \mathrm{g}(x)$. Hence find the values of x for which

$$4 - \mathrm{g}^{50}(x) = \left[\mathrm{g}^{51}(x)\right]^2.$$

[4]

7 (a) Find $\int \frac{\cos(\ln x)}{x^2}\,\mathrm{d}x$. [4]

(b) Using the substitution $u = \sqrt{x+3}$, find $\int_1^6 \frac{x-2}{x\sqrt{x+3}}\,\mathrm{d}x$, giving your answer in the form

$$a + \frac{b}{\sqrt{3}}\ln\left(\frac{c-\sqrt{d}}{c+\sqrt{d}}\right),$$

where a, b, c and d are constants to be determined. [6]

8 The complex number z satisfies the following inequalities:

$$|z|^2 \leq 4 \quad \text{and} \quad -\frac{\pi}{6} \leq \arg(z+\sqrt{3}-\mathrm{i}) \leq 0.$$

(i) On an Argand diagram, sketch the region R in which the point representing z can lie. [4]

(ii) Find exactly the minimum and maximum possible values of $|z-2\mathrm{i}|$. [3]

(iii) Determine the number of roots of the equation $z^{100} = 2^{100}$ that lie in the region R. [3]

9 It is given that $\mathrm{f}(x) = x + \dfrac{m^2}{x-2}$, where $0 < m < 1$.

(i) Sketch the graph of $y = \mathrm{f}(x)$, showing clearly the coordinates of the turning points and the equation(s) of any asymptote(s). [5]

(ii) By inserting a suitable graph to your sketch in (i), find the set of values of k, in terms of m, for which the equation $x^2 - (2+k)x + (m^2 + 2k) = 0$ has two distinct positive roots. [4]

(iii) The curve $y = \mathrm{f}(x)$ undergoes the transformations A, B and C in succession:

A: A translation of -2 units in the direction of x-axis,

B: A stretch parallel to the x-axis with scale factor of $\dfrac{1}{2}$, and

C: A translation of -2 units in the direction of y-axis.

Given that the resulting curve is $y = 2x + \dfrac{1}{8x}$, find the value of m. [2]

10 The point A has position vector $\begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix}$ and the line l has equation $\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$, where $\lambda \in \mathbb{R}$.

(i) Find the position vector of the foot of the perpendicular from A to l. [4]

(ii) Show that a cartesian equation of the plane π_1 which contains A and l is

$$x + y + 2z = 1.$$ [2]

The equation of the plane π_2 is $x + 7z = c$, where c is a constant.

(iii) Given that π_1 and π_2 intersect in a line L, show that a vector equation of L is

$$\mathbf{r} = \begin{pmatrix} c \\ 1-c \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -7 \\ 5 \\ 1 \end{pmatrix}, \text{ where } \mu \in \mathbb{R}.$$ [2]

Another plane π_3 has equation $2x - y + dz = 5$, where d is a constant.

(iv) Find the values of c and/or d if all three planes π_1, π_2 and π_3

(a) meet in the line L, [3]

(b) have only one point in common. [1]

11 At the beginning of May 2016, Sam borrowed \$50 000 from a bank that charges him a special rate of 0.2% interest at the end of every month. Sam pays back \$1 000 for every instalment at the beginning of every month, starting from June 2016.

(i) Show that the total amount with interest that Sam still owes the bank at the end of the month after the nth instalment is paid is

$$\$\left[50\,000(1.002^{n+1}) - 501\,000(1.002^{n} - 1)\right].$$ [4]

(ii) Find the number of instalments required for Sam to settle all the amount owed. [2]

(iii) How much does he pay on his last instalment? [2]

(iv) If Sam wishes to settle all the amount owed after paying 19 instalments, what is the minimum amount (to the nearest dollar) he should pay each month? [2]