2024 TKGS PRELIM MATH P2 MARK SCHEME

Qn			Solution	Content/Success Criteria	
1	**(a)**		$\dfrac{3-2x}{7} = \dfrac{1}{3}(2x-1)$ $3(3-2x) = 7(2x-1)$ $9-6x = 14x-7$ $20x = 16$ $x = 0.8 / \dfrac{4}{5}$	I can multiply LCM to remove the denominators on both sides of a linear equation involving fractions. I can solve a linear equation.	
				Content	L
				Complexity	L
				Context	L
				Response Strategy	Simple
				Assessment Objective	AO1
	(b)	**(i)**	$\dfrac{25a^2 - 9b^2}{15a - 10ab + 6b^2 - 9b}$ $= \dfrac{(5a)^2 - (3b)^2}{5a(3-2b) + 3b(2b-3)}$ $= \dfrac{(5a-3b)(5a+3b)}{5a(3-2b) - 3b(3-2b)}$ $= \dfrac{(5a-3b)(5a+3b)}{(5a-3b)(3-2b)}$ $= \dfrac{5a+3b}{3-2b}$	I can factorise algebraic expressions by identity and grouping.	
				Content	L
				Complexity	L
				Context	L
				Response Strategy	Simple
				Assessment Objective	AO1
		(ii)	$\dfrac{3c}{4ab} \div \dfrac{c^3}{12a^2b}$ $= \dfrac{3c}{4ab} \times \dfrac{12a^2b}{c^3}$ $= \dfrac{36a^2bc}{4abc^3}$ $= \dfrac{9a}{c^2}$	I can take reciprocal to convert division to multiplication. I can apply laws of indices to simplify algebraic expressions.	
				Content	L
				Complexity	L
				Context	L
				Response Strategy	Routine
				Assessment Objective	AO1
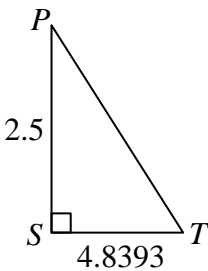
Qn			Solution	Content/Success Criteria	
(c)	**(i)**		$-x^2 + 9x - \dfrac{7}{2} = 0$		
			$x^2 - 9x + \dfrac{7}{2} = 0$	I can complete the square.	
			$\left(x - \dfrac{9}{2}\right)^2 + \dfrac{7}{2} - \left(-\dfrac{9}{2}\right)^2 = 0$	I can solve equation using complete the square method	
			$\left(x - \dfrac{9}{2}\right)^2 - \dfrac{67}{4} = 0$	Content	L
				Complexity	L
			$\left(x - \dfrac{9}{2}\right)^2 = \dfrac{67}{4}$	Context	L
				Response Strategy	Routine
			$x = \pm\sqrt{\dfrac{67}{4}} + \dfrac{9}{2}$	**Assessment Objective**	AO1
			$x = 8.59268$ or 0.407324		
			$x = 8.59$ or 0.41		
	(ii)		$-x^2 + 9x - \dfrac{7}{2} = -(x - 4.5)^2 + 16.75 / \dfrac{67}{4}$	I know that $y = -x^2 + 9x - \dfrac{7}{2}$ has max. value.	
			Since the maximum value of $y = -x^2 + 9x - \dfrac{7}{2}$ is $16.75 / \dfrac{67}{4} < 18$. There is no solution when $y = 18$.	I can find max. value with complete square form of quadratic equation.	
				Content	L
				Complexity	M
				Context	M
				Response Strategy	Routine
				Assessment Objective	AO3
OR			$-x^2 + 9x - \dfrac{7}{2} = 18$		
			$-x^2 + 9x - \dfrac{43}{2} = 0$		
			$\text{Discriminant} = (9)^2 - 4(-1)\left(-\dfrac{43}{2}\right)$		
			$= -5$		
			Since discriminant < 0, $-x^2 + 9x - \dfrac{7}{2} = 18$ has no solution.		
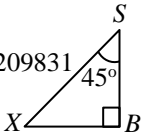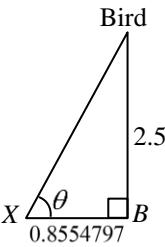
Qn		Solution	Content/Success Criteria	
2	**(a)**	P(man who passed test) $= \dfrac{1 \times 3}{7 \times 3} = \dfrac{3}{21}$ $\therefore$ the number of men who passed the test $= 3$ Let the number of women who failed the test be x P(both women failed the test) $= \dfrac{x}{21} \times \dfrac{x-1}{20} = \dfrac{1}{10}$ $$x^2 - x = \dfrac{420}{10}$$ $$x^2 - x - 42 = 0$$ $$(x-7)(x+6) = 0$$ $$x = 7 \quad \text{or} \quad -6 \text{ (rejected)}$$ Women who passed the test $= 21 - 9 - 7 - 3$ $\qquad\qquad\qquad\qquad\qquad = 2$	I can find the probability of a single event. I can use multiplication of probabilities for simple combined events.	

	Passed	Failed
Men	3	**9**
Women	2	7

Content	M
Complexity	M
Context	M
Response Strategy	Unfamiliar
Assessment Objective	AO2

Qn			Solution	Content/Success Criteria	
2	**(b)**	**(i)**	A set of students who only listen to pop music but not classical music.	I can represent elements of set notation in words. I can find the number of elements in any sets.	
		(ii)	10	Content	L
		(iii)	4	Complexity	M
		(iv)	9	Context	M
				Response Strategy	Simple
				Assessment Objective	AO2

Qn		Solution	Content/Success Criteria	
3	**(a)**	−2	I can find the value by substitution.	
	(b)	See graph below	I can draw graph that passes through all points plotted smoothly.	
			Content	L
			Complexity	L
			Context	L
			Response Strategy	Routine
			Assessment Objective	AO1

$y = 2x - 15 + \dfrac{20}{x}$

$y = x - 2$

$y = 7$

(10, 2.5)

(0, −5)
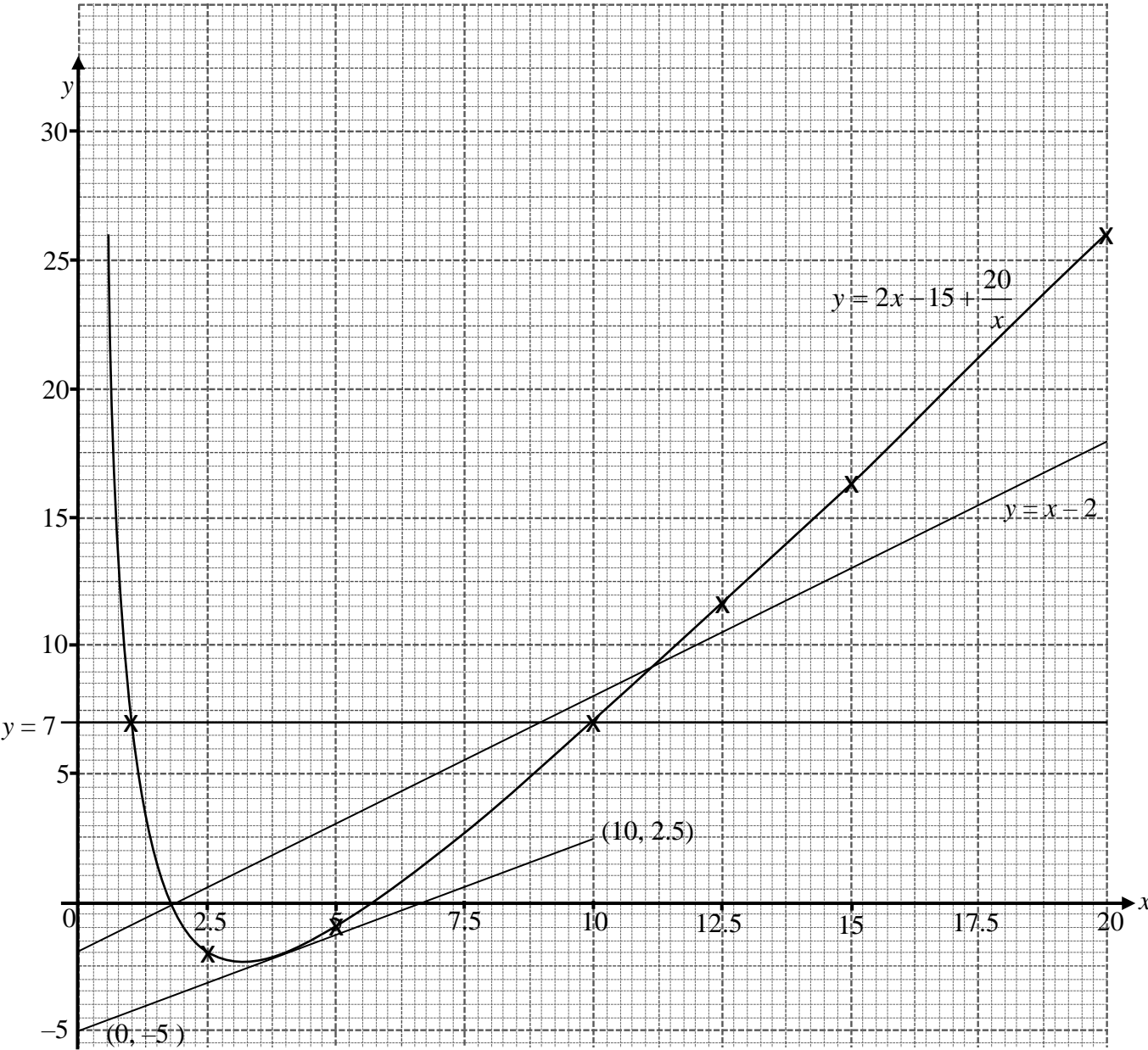
Qn	Solution	Content/ Success Criteria	
(c)	$2x + \dfrac{20}{x} < 22$ $2x + \dfrac{20}{x} - 15 < 22 - 15$ $2x + \dfrac{20}{x} - 15 < 7$ Draw a line of $y = 7$ on the graph $\therefore\ 1 < x < 10$	I can solve inequality using graph.	
		Content	M
		Complexity	M
		Context	L
		Response Strategy	unfamiliar
		Assessment Objective	AO2
(d)	Draw a tangent at $x = 4$ gradient $= \dfrac{2.5 - (-5)}{10 - 0}$ $= 0.75$ (accept ± 0.2)	I know how to find tangent on a graph.	
		Content	L
		Complexity	L
		Context	L
		Response Strategy	Routine
		Assessment Objective	AO1
(e)	$x^2 - 13x + 20 = 0$ $\dfrac{x^2}{x} - \dfrac{13x}{x} + \dfrac{20}{x} = \dfrac{0}{x}$ $x - 13 + \dfrac{20}{x} = 0$ $2x - 15 + \dfrac{20}{x} = x - 2$ Draw $y = x - 2$ on the graph $x = 1.783$ or 11.217 (accept ± 0.15)	I can solve quadratic equation using graph.	
		Content	L
		Complexity	M
		Context	M
		Response Strategy	Routine
		Assessment Objective	AO2

Qn		Solution	Content/Success Criteria	
4	**(a)**	Using sine rule, $$ST = \dfrac{7.5}{\sin 95^\circ} \times \sin 40^\circ \quad \textbf{\color{purple}Essential step}$$ $$= 4.83932$$ $$SX = \dfrac{1}{4}ST$$ $$= 1.209831$$ $$= 1.21 \text{ m}$$	I can use sine rule to find unknown length	
			Content	L
			Complexity	L
			Context	L
			Response Strategy	simple
			Assessment Objective	AO1
	(b)	$\angle RSX = 180^\circ - 40^\circ - 95^\circ$ (angle sum of triangle) $\qquad = 45^\circ$ Using cosine rule, **Essential step** $$XR = \sqrt{1.209831^2 + 7.5^2 - 2(1.209831)(7.5)\cos 45^\circ}$$ $$= 6.69937$$ $$= 6.70 \text{ m}$$	I can apply cosine rule to find unknown length	
			Content	L
			Complexity	L
			Context	L
			Response Strategy	simple
			Assessment Objective	AO1
	OR	Using sine rule, $$TR = \dfrac{7.5 \times \sin 45^\circ}{\sin 95^\circ} \quad \textbf{\color{purple}Essential step}$$ $$= 5.3236 \text{ m}$$ $$XT = \dfrac{3}{4} \times 4.83932$$ $$= 3.62949 \text{ m}$$ Using cosine rule, **Essential step** $$XR = \sqrt{3.62949^2 + 5.3236^2 - 2(3.62949)(5.3236)\cos 95^\circ}$$ $$= 6.69937$$ $$= 6.70 \text{ m}$$		
	(c)	By Pythagoras' Theorem, $$PT = \sqrt{2.5^2 + 4.8393^2}$$ $$= 5.4469$$ $$= 5.45 \text{ m}$$	I can use Pythagoras' Theorem to find unknown length	
			Content	L
			Complexity	L
			Context	L
			Response Strategy	simple
			Assessment Objective	A01

Qn		Solution	Content/Success Criteria	
	(d)	Let the point directly below the bird be *B*. $\angle SBX = 90°$ for *XB* to be the shortest distance $\boxed{XB = \sin 45° \times 1.209831}$ **Essential step** $\quad = 0.8554797$ m $\boxed{\tan \theta = \dfrac{2.5}{0.8554797}}$ **Essential step** $\theta = \tan^{-1}\left(\dfrac{2.5}{0.8554797}\right)$ $\quad = 71.10947$ $\quad = 71.1°\,(1\text{ d.p})$	I know that shortest distance gives largest angle of elevation I can use trigo ratio to find unknown angle	
			Content	L
			Complexity	M
			Context	M
			Response Strategy	taught
			Assessment Objective	A02
OR		$\dfrac{1}{2} \times 7.5 \times h = \dfrac{1}{2} \times 1.20983 \times 7.5 \times \sin 45°$ $\qquad h = 0.85547$ $\theta = \tan^{-1}\left(\dfrac{2.5}{0.8554797}\right)$ $\quad = 71.10947$ $\quad = 71.1°\,(1\text{ d.p})$		

Qn			Solution	Content/Success Criteria	
5	**(a)**	**(i)**	$\angle ABD = \angle ACD$ ($\angle s$ in same segment) $\quad = 30°$	I can apply circle properties to find angle	
		(ii)	$\angle ACB = \angle OBC$ (base $\angle s$ of isosceles Δ) $\quad = 40°$ $\angle BCD = 40° + 30°$ $\quad = 70°$ $\angle BAD = 180° - 70°$ ($\angle s$ in opposite segments) $\quad = 110°$	I can apply circle properties to find angle	
				Content	L
				Complexity	L
				Context	L
				Response Strategy	taught
				Assessment Objective	AO1
		(iii)	Let r be the radius of the circle, $\angle DOB = 140°$ ($\angle$ at centre $= 2\angle$ at circumferene) $\dfrac{140°}{360°} \times 2\pi r = 11$ $r = \dfrac{11}{\frac{7}{18} \times 2\pi}$ $\quad = 4.5$	I can find radius given angle	
				Content	L
				Complexity	L
				Context	M
				Response Strategy	taught
				Assessment Objective	AO2
	OR		Let r be the radius of the circle, $\angle DOB = 140°$ ($\angle$ at centre $= 2\angle$ at circumferene) $r\theta = 11$ $r = \dfrac{11}{140 \times \frac{\pi}{180}}$ $\quad = 4.5$		
	(b)	**(i)**	$\angle BTC = \angle BCD = 70°$ (found in (a(ii)) $\angle OBT = 90°$ (tan $\perp$ radius) $\angle CBT = 90° - 40°$ $\quad = 50°$ $\angle BOC = 180° - (40° \times 2)$ (base $\angle$s of isosceles Δ) $\quad = 100°$ $\angle BDC = \dfrac{100°}{2}$ ($\angle$ at centre $= 2\angle$at circumference) $\quad = 50°$ $\therefore \angle CBT = \angle BDC = 50°$ $\therefore \Delta CTB$ is similar to ΔBCD by AA similary test	I can apply circle properties to find angle to prove similarity	
				Content	L
				Complexity	M
				Context	M
				Response Strategy	taught
				Assessment Objective	AO2

Qn			Solution	Content/Success Criteria	
		(ii)	$\dfrac{CT}{BC} = \dfrac{BC}{DB}$ (corresponding sides of similar triangles) $CT = \dfrac{BC}{DB} \times BC$ $CT = \dfrac{x}{y} \times x$ $= \dfrac{x^2}{y}$	I can use ratios of corresponding sides of similar triangles to solve unknown	
				Content	L
				Complexity	M
				Context	M
				Response Strategy	taught
				Assessment Objective	AO2

Qn			Solution	Content/Success Criteria	
6	**(a)**	**(i)**	median $= 17\,\text{min}$	I know how to read CFC and find median and IQR.	
		(ii)	interquartile $= 20 - 13$ $\quad\quad\quad\quad = 7\,\text{min}$	I know the meaning of percentile.	
		(iii)	number of customers $= \dfrac{35}{100} \times 60$ $\quad\quad\quad\quad\quad\quad\quad = 21$ From the graph, 35th percentile $= 15\,\text{min}$	Content	L
				Complexity	L
				Context	L
				Response Strategy	simple
				Assessment Objective	AO1
	(b)		If 20 % waited <u>more than</u> x minutes, then $100\% - 20\% = 80\%$ waited <u>less than or equal to</u> x minutes $\dfrac{80}{100} \times 60 = 48$ customers From the graph, $x = 21.5\,\text{min}$	I can interpret the 'more than' CFC.	
				Content	L
				Complexity	L
				Context	L
				Response Strategy	taught
				Assessment Objective	AO1
	(c)	**(i)**	median $= \dfrac{15 + 17}{2}$ $\quad\quad\quad\quad = 16\,\text{min}$	I know how to find median and IQR for discrete data.	
		(ii)	interquartile $= 18 - 14$ $\quad\quad\quad\quad = 4\,\text{min}$	Content	L
				Complexity	L
				Context	L
				Response Strategy	taught
				Assessment Objective	AO1
	(d)		The sample sizes are not the same. **OR** Sample size for restaurant B is not large enough.	Content	M
				Complexity	M
				Context	M
				Response Strategy	unfamiliar
				Assessment Objective	AO3

Qn			Solution	Content/Success Criteria	
7	(a)	(i)	$\dfrac{h}{h+16}=\sqrt{\dfrac{5}{125}}$ $\dfrac{h}{h+16}=\sqrt{\dfrac{1}{25}}$	I can use ratio of the areas of similar figures to solve for unknown	
			$h=\dfrac{1}{5}(h+16)$	Content	L
				Complexity	M
			$h=\dfrac{1}{5}h+\dfrac{16}{5}$	Context	M
				Response Strategy	taught
			$\dfrac{4}{5}h=\dfrac{16}{5}$ $h=4\text{ (shown)}$	Assessment Objective	AO2
		(ii)	Vol. of large pyramid $=\dfrac{1}{3}\times(125)\times(16+4)$	I can apply the formula to find volume of pyramid	
			$=833\dfrac{1}{3}\text{ cm}^3$	Content	L
				Complexity	L
			Vol. of small pyramid $=\dfrac{1}{3}\times(5)\times(4)$	Context	L
			$=6\dfrac{2}{3}\text{ cm}^3$	Response Strategy	taught
			Vol. of frustum $=833\dfrac{1}{3}-6\dfrac{2}{3}$ $=826\dfrac{2}{3}$	Assessment Objective	AO1
		(iii)	% removed $=\left(\dfrac{4}{20}\right)^3\times100$	I can apply ratio of volumes of similar pyramid to find percentage	
			$=\dfrac{1}{125}\times100$	Content	L
				Complexity	L
			$=\dfrac{4}{5}\%\ /\ 0.8\%$	Context	M
				Response Strategy	simple
				Assessment Objective	AO1
	(b)		$\dfrac{\text{Volume of cone }A}{\text{Volume of cone }B}=\dfrac{\dfrac{1}{3}\pi(3r)^2h}{\dfrac{1}{3}\pi(2r)^2h}$	I know how to find unknown volume when figures are not similar	
			$\dfrac{\text{Volume of cone }A}{240}=\dfrac{9}{4}$	Content	L
				Complexity	M
			Volume of cone $A=\dfrac{9}{4}\times240$	Context	M
				Response Strategy	taught
			$=540\text{ cm}^3$	Assessment Objective	AO2

Qn			Solution	Content/Success Criteria	
8	**(a)**	**(i)**	Since $\overrightarrow{AB} \,//\, \overrightarrow{CD}$, $$n\begin{pmatrix} -3 \\ 9 \end{pmatrix} = \begin{pmatrix} d \\ 4 \end{pmatrix}$$ $$\begin{pmatrix} -3n \\ 9n \end{pmatrix} = \begin{pmatrix} d \\ 4 \end{pmatrix}$$ $9n = 4$ and $d = -3n$ $n = \dfrac{4}{9}$ and $d = -3\left(\dfrac{4}{9}\right)$ $d = -\dfrac{4}{3}$	I can use scalar multiples to find unknown given parallel vectors.	
				Content	L
				Complexity	M
				Context	M
				Response Strategy	Simple
				Assessment Objective	AO2
		(ii)	$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ $\overrightarrow{AC} = \begin{pmatrix} -3 \\ 9 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ $= \begin{pmatrix} -1 \\ 14 \end{pmatrix}$ $\left|\overrightarrow{AC}\right| = \sqrt{(-1)^2 + (14)^2}$ $= \sqrt{197}$ $= 14.0$ unit	I can find the magnitude of vectors	
				Content	M
				Complexity	L
				Context	L
				Response Strategy	Simple
				Assessment Objective	AO2
	(b)	**(i)**	$\overrightarrow{RM} = \overrightarrow{PM} - \overrightarrow{PR}$ $= \dfrac{1}{2}\mathbf{a} - \mathbf{b}$ $\overrightarrow{PS} = \overrightarrow{PR} + \overrightarrow{RS}$ $= \mathbf{b} + k\left(\dfrac{1}{2}\mathbf{a} - \mathbf{b}\right)$ $= \mathbf{b} + \dfrac{1}{2}k\mathbf{a} - k\mathbf{b}$ $= \dfrac{1}{2}k\mathbf{a} + (1-k)\mathbf{b}$ (there should only be one $\mathbf{a}$ & one $\mathbf{b}$ in the final answer)	I can express a vector in the form of 2 other vectors.	
				Content	M
				Complexity	H
				Context	H
				Response Strategy	Unfamiliar
				Assessment Objective	AO3
		(ii)	$\overrightarrow{QR} = \overrightarrow{PR} - \overrightarrow{PQ}$ $= \mathbf{b} - \mathbf{a}$ $\overrightarrow{PT} = \overrightarrow{PQ} + \overrightarrow{QT}$ $= \mathbf{a} + h(\mathbf{b} - \mathbf{a})$ $= \mathbf{a} + h\mathbf{b} - h\mathbf{a}$ $= (1-h)\mathbf{a} + h\mathbf{b}$ (there should only be one $\mathbf{a}$ & one $\mathbf{b}$ in the final answer)	I can express a vector in the form of 2 other vectors.	
				Content	M
				Complexity	H
				Context	H
				Response Strategy	Unfamiliar
				Assessment Objective	AO3

Qn			Solution	Content/Success Criteria	
		(iii)	Since P, S and T are collinear, $$\dfrac{\frac{1}{2}k}{1-h} = \dfrac{1-k}{h}$$ $$\left(\frac{1}{2}k\right)h = (1-h)(1-k)$$ $$\frac{1}{2}hk = 1-h-k+hk$$ $$hk = 2-2h-2k+2hk$$ $$2h+2k = hk+2$$ $$2(h+k) = hk+2 \text{ (shown)}$$	I know that for collinear point, the ratio of the scalar multiples of 2 parallel lines are equal.	
				Content	M
				Complexity	H
				Context	H
				Response Strategy	Unfamiliar
				Assessment Objective	AO3
		(iv)	$$\dfrac{\left|\overrightarrow{PS}\right|}{\left|\overrightarrow{PT}\right|} = \dfrac{\frac{1}{2}\left(\frac{4}{5}\right)}{1-\frac{1}{3}} = \dfrac{3}{5}$$ $PS \ : \ ST = 3 \ : \ 5-3$ $PS \ : \ ST = 3 \ : \ 2$	I can find the ratio of 2 line segments using vectors.	
				Content	M
				Complexity	L
				Context	L
				Response Strategy	Simple
				Assessment Objective	AO2

Qn		Solution	Content/Success Criteria	
9	**(a)**	Area $= 11 \times 17$ $= 187$ inch^2 $= 187 \times 2.54^2$ cm^2 $= \dfrac{1206.4492}{100^2}$ m^2 $= 0.12064492$ m^2 $= 0.121$ m^2	I can convert inches to metres	
			Content	L
			Complexity	L
			Context	L
			Response Strategy	routine
			Assessment Objective	AO1
	(b)	Total area of 1200 bulletin posters $= 1200 \times 0.12064492$ $= 144.774$ m^2 Since the area > 120 m^2, there is an additional 5% charge. Total cost $= 2 \times 1200 + \dfrac{5}{100} \times 2 \times 1200$ $= \$2520$	I can calculate the cost	
			Content	L
			Complexity	M
			Context	M
			Response Strategy	taught
			Assessment Objective	AO2
	(c)	Let the time taken by the new printer to print 1 bulletin posters be t. The time taken by the old printer to print 1 bulletin posters will be $t+15$. $\dfrac{6 \times 60 \times 60}{t} + \dfrac{6 \times 60 \times 60}{t+15} = 1200$ $\dfrac{21600(t+15) + 21600t}{t(t+15)} = 1200$ $21600(t+15) + 21600t = 1200t(t+15)$ $21600t + 324000 + 21600t = 1200t^2 + 18000t$ $1200t^2 - 25200t - 324000 = 0$ $t^2 - 21t - 270 = 0$ $t = 30$ or -9 (reject) Number of posters printed by the old printer $= \dfrac{21600}{30+15}$ $= 480$ Total area printed by old printer $= 480 \times 0.21064$ m^2 $= 57.91$ m^2 Since the older printer only printed 57.9 m$^2 < 60$ m^2. It will need to be serviced.	I can solve problem using various heuristics	
			Content	M
			Complexity	H
			Context	H
			Response Strategy	unfamiliar
			Assessment Objective	AO2 / AO3
OR		To print an area of 60m^2, printer must print $\dfrac{60}{0.12064492} = 497.33$ posters. Since the older printer only printed 480 posters < 495.86 posters, it will need to be serviced.		