2023 Term 4 Timed Practice (Structured Remedial Session 1)

<u>Question 1. [N13/9740/P2/Q6]</u>

The continuous random variable Y has the distribution $N(\mu, \sigma^2)$. It is known that $P(Y < 2a) = 0.95$ and $P(Y < a) = 0.25$. Express μ in the form ka, where k is a constant to be determined. [4]

$Y \sim N(\mu, \sigma^2)$

$P(Y < 2a) = 0.95 \Rightarrow P\left(Z < \dfrac{2a-\mu}{\sigma}\right) = 0.95$

From GC (applying invNorm), $\dfrac{2a-\mu}{\sigma} = 1.644\,85$

$\Rightarrow \dfrac{2a-\mu}{1.644\,85} = \sigma \quad -(1)$ [M1]

$P(Y < a) = 0.25 \Rightarrow P\left(Z < \dfrac{a-\mu}{\sigma}\right) = 0.25$

From GC (applying invNorm), $\dfrac{a-\mu}{\sigma} = -0.674\,490$

$\Rightarrow \dfrac{a-\mu}{-0.674\,490} = \sigma \quad -(2)$ [M1]

Equating (1) and (2),

$\dfrac{2a-\mu}{1.644\,85} = \dfrac{a-\mu}{-0.674\,490}$ [M1]

$2a - \mu = -2.4387a + 2.4387\mu$

$3.4387\mu = 4.4387a$

$\mu = \dfrac{4.4387}{3.4387}a$

$= 1.29a$ (to 3 s.f.) [A1]

Question 2. [N16/9740/P2/Q6(modified)]

The Company Secretary obtains a suitable sample of 80 employees in order to carry out a hypothesis test of the Managing Director's belief that the mean age of the employees now is less than 37 years. You are given that the population variance of the ages is 140 years^2.

(i) Write down appropriate hypotheses to test the Managing Director's belief, defining any symbols you use. You are given that the result of the test, using a 5% significance level, is that the Managing Director's belief should be accepted. Determine the set of possible values of the mean age of the sample of employees. [4]

) Let X be the random variable associated to the age of a randomly selected employee from the company, with population mean and variance of X denoted by μ and σ^2 respectively. We test

H_0: $\mu = 37$ against

H_1: $\mu < 37$ at the 5% significance level. [B1]

Using the suitable sample of 80 employees, we denote the distribution sample mean

$$\overline{X} = \frac{X_1 + \ldots + X_{80}}{80}.$$

Since the sample size $n = 80$ is sufficiently large, by the Central Limit Theorem,

$$\overline{X} \sim N\left(37, \frac{140}{80}\right) \text{ approximately,}$$

$$\text{i.e. } Z = \frac{\overline{X} - 37}{\sqrt{\frac{140}{80}}} \sim N(0, 1). \quad \text{[M1]}$$

Denote the mean of the sample of 80 employees as $\bar{x}$. In order to reject H_0 at the 5% significance level, we need to set

$$z = \frac{\bar{x} - 37}{\sqrt{\frac{140}{80}}} < -1.644\,8536$$

$$\bar{x} < 34.824 \quad \text{[M1]}$$

Thus, the set of possible values of the mean age of the sample of employees is

$\{\bar{x} \in \mathbb{R}: 0 < \bar{x} < 34.8\}$ (to 3 s.f.). [A1]

(ii) You are given that the mean age of the sample of employees is 35.2 years, and that the result of a test at α% significance level is that the Managing Directo's belief should not be accepted. Find the set of possible values of α. [3]

With $\bar{x} = 35.2$, in order **not** to reject H_0 at the $\alpha\%$ significance level,

p-value $= 0.086\ 808 > \dfrac{\alpha}{100}$ [B1]

Thus, $\alpha < 8.6808$. [M1]

The set of possible values of α is given by $\{\alpha \in \mathbb{R}: 0 < \alpha < 8.68\}$. [A1]

(Rounded to 3 s.f. and adhering to the above constraint)