1(i)	Using de Moivre's theorem, $(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$ $\cos 6\theta = \mathrm{Re}\left((\cos\theta + \mathrm{i}\sin\theta)^6\right)$ $= \cos^6\theta - 15\cos^4\theta\sin^2\theta + 15\cos^2\theta\sin^4\theta - \sin^6\theta$ $\sin 6\theta = \mathrm{Im}\left((\cos\theta + \mathrm{i}\sin\theta)^6\right)$ $= 6\cos^5\theta\sin\theta - 20\cos^3\theta\sin^3\theta + 6\cos\theta\sin^5\theta$ $\tan 6\theta = \dfrac{6\cos^5\theta\sin\theta - 20\cos^3\theta\sin^3\theta + 6\cos\theta\sin^5\theta}{\cos^6\theta - 15\cos^4\theta\sin^2\theta + 15\cos^2\theta\sin^4\theta - \sin^6\theta}$ Dividing both numerator annd denominator by $\cos^6\theta$, $\tan 6\theta = \dfrac{6\tan\theta - 20\tan^3\theta + 6\tan^5\theta}{1 - 15\tan^2\theta + 15\tan^4\theta - \tan^6\theta}$ (Shown)	
(ii)	Substituting $\theta = \dfrac{\pi}{5}$, $\tan\left(\dfrac{6\pi}{5}\right) = \dfrac{6\tan\left(\frac{\pi}{5}\right) - 20\tan^3\left(\frac{\pi}{5}\right) + 6\tan^5\left(\frac{\pi}{5}\right)}{1 - 15\tan^2\left(\frac{\pi}{5}\right) + 15\tan^4\left(\frac{\pi}{5}\right) - \tan^6\left(\frac{\pi}{5}\right)}$ As $\tan\left(\dfrac{6\pi}{5}\right) = \tan\left(\dfrac{\pi}{5}\right)$, $\tan\left(\dfrac{\pi}{5}\right) = \dfrac{6\tan\left(\frac{\pi}{5}\right) - 20\tan^3\left(\frac{\pi}{5}\right) + 6\tan^5\left(\frac{\pi}{5}\right)}{1 - 15\tan^2\left(\frac{\pi}{5}\right) + 15\tan^4\left(\frac{\pi}{5}\right) - \tan^6\left(\frac{\pi}{5}\right)}$ Since $\tan\left(\dfrac{\pi}{5}\right) \neq 0$, $1 = \dfrac{6 - 20\tan^2\left(\frac{\pi}{5}\right) + 6\tan^4\left(\frac{\pi}{5}\right)}{1 - 15\tan^2\left(\frac{\pi}{5}\right) + 15\tan^4\left(\frac{\pi}{5}\right) - \tan^6\left(\frac{\pi}{5}\right)}$ Letting $v = \tan^2\left(\dfrac{\pi}{5}\right)$ and rearranging, we have	

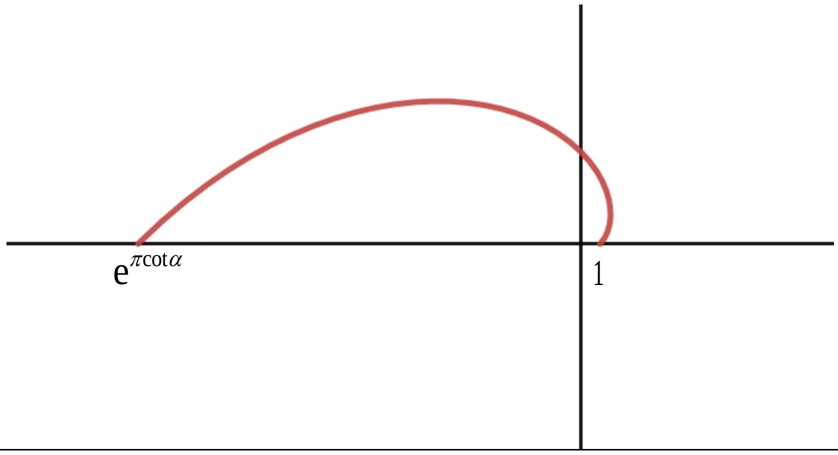
	$v^3-9v^2-5v+5=0$ Let $\mathrm{f}(v)=v^3-9v^2-5v+5$ $\mathrm{f}(-1)=(-1)^3-9(-1)^2-5(-1)+5=0$ $\therefore (v+1)$ is a factor $(v+1)(v^2-10v+5)=0$ $v=-1\left(\text{rej. since }\tan^2\left(\frac{\pi}{5}\right)>0\right)$ or $v=\frac{10\pm\sqrt{80}}{2}$ $=\frac{10\pm\sqrt{16\times5}}{2}$ $=5\pm2\sqrt{5}$ Since $\tan^2\left(\frac{\pi}{5}\right)<\tan^2\left(\frac{\pi}{4}\right)=1$, $\tan^2\left(\frac{\pi}{5}\right)=5-2\sqrt{5}$. (Shown)	

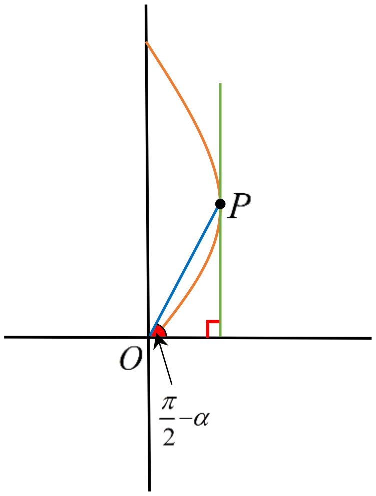
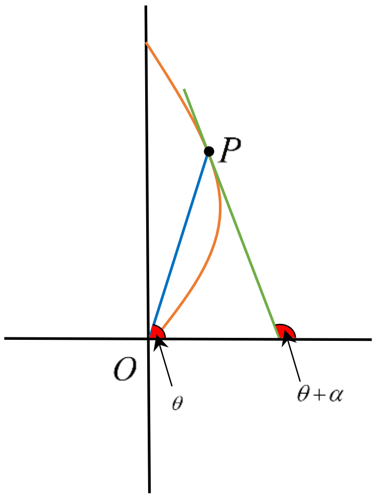
2(a) **[2]**	$\mathbf{A}\begin{pmatrix} r\cos\alpha \\ r\sin\alpha \end{pmatrix}=\begin{pmatrix} r(\cos\theta\cos\alpha-\sin\theta\sin\alpha) \\ r(\cos\alpha\sin\theta+\cos\theta\sin\alpha) \end{pmatrix}=\begin{pmatrix} r\cos(\theta+\alpha) \\ r\sin(\theta+\alpha) \end{pmatrix}$ Hence T_1 gives an anti-clockwise rotation of angle θ about the origin.	
(b) **[1]**	It is a reflection in the *x*-axis.	
(c) **[2]**	**M** is obtained when we let $\theta=\frac{\pi}{6}$ in matrix **A**. Rotating a point about O for 2π radians brings it back to the original position. Since $2\pi=12\times\frac{\pi}{6}$, the smallest positive integer $k=12$.	
(d) **[3]**	$\mathbf{C}=\mathbf{MBM}^{-1}=\begin{pmatrix} \frac{1}{2}\sqrt{3} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2}\sqrt{3} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \frac{1}{2}\sqrt{3} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2}\sqrt{3} \end{pmatrix}^{-1}$,	

	hence **C** has eigenvalues 1 and -1. Hence we have $\mathbf{Cx}_1 = 1\mathbf{x}_1$ and $\mathbf{Cx}_2 = -1\mathbf{x}_2$ for eigenvectors $\mathbf{x}_1$ and $\mathbf{x}_2$, where $\mathbf{x}_1 = \begin{pmatrix} \frac{1}{2}\sqrt{3} \\ \frac{1}{2} \end{pmatrix}$ and $\mathbf{x}_2 = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2}\sqrt{3} \end{pmatrix}$. So the two invariant lines are $y = \frac{1}{\sqrt{3}}x$ and $y = -\sqrt{3}x$, which pass through the origin.	

3(i) **[5]**	Differentiate $\frac{\mathrm{d}b}{\mathrm{d}t} = \beta g - \alpha b$ with respect to *t*, $\frac{\mathrm{d}^2 b}{\mathrm{d}t^2} = \beta \frac{\mathrm{d}g}{\mathrm{d}t} - \alpha \frac{\mathrm{d}b}{\mathrm{d}t}$ $= \beta(1 - \beta g) - \alpha \frac{\mathrm{d}b}{\mathrm{d}t}$ $= \beta\left[1 - \left(\frac{\mathrm{d}b}{\mathrm{d}t} + \alpha b\right)\right] - \alpha \frac{\mathrm{d}b}{\mathrm{d}t}$ $= \beta - \beta\frac{\mathrm{d}b}{\mathrm{d}t} - \alpha\beta b - \alpha \frac{\mathrm{d}b}{\mathrm{d}t}$ Hence $\frac{\mathrm{d}^2 b}{\mathrm{d}t^2} + (\alpha + \beta)\frac{\mathrm{d}b}{\mathrm{d}t} + \alpha\beta b = \beta$ Solving auxiliary equation, $\lambda^2 + (\alpha + \beta)\lambda + \alpha\beta = 0$ $(\lambda + \alpha)(\lambda + \beta) = 0$ $\lambda = -\alpha$ or $\lambda = -\beta$ Hence complementary function is $b = A\mathrm{e}^{-\alpha t} + B\mathrm{e}^{-\beta t}$ Let the particular solution be $b = c$, then $\alpha\beta c = \beta \Rightarrow c = \frac{1}{\alpha}$	

	Solution is $b = Ae^{-\alpha t} + Be^{-\beta t} + \frac{1}{\alpha}$	
(ii) **[2]**	Given that $\alpha = 0.2$ and $\beta = 0.7$, and $b = 7.6$, $\frac{\mathrm{d}b}{\mathrm{d}t} = -1.12$ when $t = 0$. $\begin{cases} 7.6 = A + B + \frac{1}{0.2} \Rightarrow A + B = 2.6 \\ -1.12 = -0.2A - 0.7B \Rightarrow 0.7 = 0.5A \Rightarrow A = 1.4, B = 1.2 \end{cases}$ Hence $b = 1.4e^{-0.2t} + 1.2e^{-0.7t} + 5$	
(iii) **[2]**	Using GC, when $1.4e^{-0.2t} + 1.2e^{-0.7t} + 5 = 5.69$, $t = 4.07$ hours (3s.f.)	

4(i)		
(ii)	$x = e^{\theta\cot\alpha}\cos\theta$ $\frac{\mathrm{d}x}{\mathrm{d}\theta} = e^{\theta\cot\alpha}\left(\cot\alpha\cos\theta - \sin\theta\right) = 0$ $\tan\theta = \cot\alpha$ $\theta = \frac{\pi}{2} - \alpha$	

(iii)	$y=\mathrm{e}^{\theta\cot\alpha}\sin\theta$ $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\mathrm{e}^{\theta\cot\alpha}\left(\cot\alpha\sin\theta+\cos\theta\right)$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{e}^{\theta\cot\alpha}\left(\cot\alpha\sin\theta+\cos\theta\right)}{\mathrm{e}^{\theta\cot\alpha}\left(\cot\alpha\cos\theta-\sin\theta\right)}$ $=\dfrac{\cot\alpha\tan\theta+1}{\cot\alpha-\tan\theta}$ $=\dfrac{\tan\theta+\tan\alpha}{1-\tan\theta\tan\alpha}$ $=\tan\left(\theta+\alpha\right)$  Acute angle between the tangent at *P* and the line *OP* $=\ \pi-\left(\dfrac{\pi}{2}-\alpha\right)-\dfrac{\pi}{2}=\alpha$. OR  Acute angle between the tangent at *P* and the line *OP* $=\ \left(\theta+\alpha\right)-\theta=\alpha$.	
(iv)	Total length of arc	

$$= \int_0^{\pi} \sqrt{r^2 + \left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2}\ \mathrm{d}\theta$$

$$= \int_0^{\pi} \sqrt{\mathrm{e}^{2\theta\cot\alpha} + \mathrm{e}^{2\theta\cot\alpha}\cot^2\alpha}\ \mathrm{d}\theta$$

$$= \int_0^{\pi} \mathrm{e}^{\theta\cot\alpha}\sqrt{1+\cot^2\alpha}\ \mathrm{d}\theta$$

$$= \sqrt{\frac{\tan^2\alpha+1}{\tan^2\alpha}}\left[\frac{\mathrm{e}^{\theta\cot\alpha}}{\cot\alpha}\right]_0^{\pi}$$

$$= \sqrt{\frac{\tan^2\alpha+1}{\tan^2\alpha\cot^2\alpha}}\left[\mathrm{e}^{\theta\cot\alpha}\right]_0^{\pi}$$

$$= \sec\alpha\left[\mathrm{e}^{\pi\cot\alpha}-1\right]$$

OR

Total length of arc

$$= \int_0^{\pi} \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2}\ \mathrm{d}\theta$$

$$= \int_0^{\pi} \sqrt{\mathrm{e}^{2\theta\cot\alpha}\left(\cot\alpha\cos\theta-\sin\theta\right)^2 + \mathrm{e}^{2\theta\cot\alpha}\left(\cot\alpha\sin\theta+\cos\theta\right)^2}\ \mathrm{d}\theta$$

$$= \int_0^{\pi} \mathrm{e}^{\theta\cot\alpha}\sqrt{\cot^2\alpha\left(\cos^2\theta+\sin^2\theta\right)+\cos^2\theta+\sin^2\theta}\ \mathrm{d}\theta$$

$$= \int_0^{\pi} \mathrm{e}^{\theta\cot\alpha}\sqrt{\cot^2\alpha+1}\ \mathrm{d}\theta$$

$$= \sqrt{\frac{\tan^2\alpha+1}{\tan^2\alpha}}\left[\frac{\mathrm{e}^{\theta\cot\alpha}}{\cot\alpha}\right]_0^{\pi}$$

$$= \sqrt{\frac{\tan^2\alpha+1}{\tan^2\alpha\cot^2\alpha}}\left[\mathrm{e}^{\theta\cot\alpha}\right]_0^{\pi}$$

$$= \sec\alpha\left[\mathrm{e}^{\pi\cot\alpha}-1\right]$$

5(a)	Improved Euler with step size $h=\dfrac{\alpha}{2}$, $g(x,y)=\left(x\cos^2 x\right)-2xy$ $x_1=0,\ y_1=0,\ u_2=y_1+hg(x_1,y_1)=0+\dfrac{\alpha}{2}(0)=0$ $y_2=y_1+\dfrac{h}{2}\left[g(x_1,y_1)+g(x_2,u_2)\right]$ $=0+\dfrac{\alpha}{4}\left(0+\left(\dfrac{\alpha}{2}\cos^2\left(\dfrac{\alpha}{2}\right)\right)-2\left(\dfrac{\alpha}{2}\right)(0)\right)$ $y_2=\dfrac{\alpha^2}{8}\cos^2\left(\dfrac{\alpha}{2}\right)$ $u_3=y_2+hg(x_2,y_2)$ $=\dfrac{\alpha^2}{8}\cos^2\left(\dfrac{\alpha}{2}\right)+\dfrac{\alpha}{2}\left(\left(\dfrac{\alpha}{2}\cos^2\left(\dfrac{\alpha}{2}\right)\right)-2\left(\dfrac{\alpha}{2}\right)\left(\dfrac{\alpha^2}{8}\cos^2\left(\dfrac{\alpha}{2}\right)\right)\right)$ $=\dfrac{\alpha^2}{8}\cos^2\left(\dfrac{\alpha}{2}\right)+\dfrac{\alpha^2}{4}\cos^2\left(\dfrac{\alpha}{2}\right)-\dfrac{\alpha^4}{16}\cos^2\left(\dfrac{\alpha}{2}\right)$ $=\left[\dfrac{\alpha^2}{16}\cos^2\left(\dfrac{\alpha}{2}\right)\right]\left(6-\alpha^2\right)$ $y_3=y_2+\dfrac{h}{2}\left[g(x_2,y_2)+g(x_3,u_3)\right]$ $=\dfrac{\alpha^2}{8}\cos^2\left(\dfrac{\alpha}{2}\right)$ $+\dfrac{\alpha}{4}\left[\begin{array}{l}\left(\dfrac{\alpha}{2}\cos^2\left(\dfrac{\alpha}{2}\right)\right)-2\left(\dfrac{\alpha}{2}\right)\left[\dfrac{\alpha^2}{8}\cos^2\left(\dfrac{\alpha}{2}\right)\right]\\+\left(\alpha\cos^2\alpha\right)-2\alpha\left[\dfrac{\alpha^2}{16}\cos^2\left(\dfrac{\alpha}{2}\right)\right]\left(6-\alpha^2\right)\end{array}\right]$ $=\dfrac{\alpha^2}{32}\left[\left(8\cos^2\alpha\right)+\left(\alpha^4-7\alpha^2+8\right)\cos^2\left(\dfrac{\alpha}{2}\right)\right]$	
(b) (i)	$I_n=\int_0^{\alpha}x^n e^{x^2}\,dx$ $=\int_0^{\alpha}x^{n-1}xe^{x^2}\,dx$ $=\left[x^{n-1}\cdot\dfrac{1}{2}e^{x^2}\right]_0^{\alpha}-\int_0^{\alpha}\dfrac{1}{2}e^{x^2}\cdot(n-1)x^{n-2}\,dx$ $=\dfrac{1}{2}\alpha^{n-1}e^{\alpha^2}-\dfrac{1}{2}(n-1)\int_0^{\alpha}x^{n-2}e^{x^2}\,dx$ $\therefore\ I_n=\dfrac{1}{2}\alpha^{n-1}e^{\alpha^2}-\dfrac{1}{2}(n-1)I_{n-2}$ (shown)	

(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=x\left(1-\frac{x^2}{2}\right)^2-2xy$ $\frac{\mathrm{d}y}{\mathrm{d}x}+2xy=x-x^3+\frac{x^5}{4}$ Integrating Factor: $\mathrm{e}^{\int 2x\,\mathrm{d}x}=\mathrm{e}^{x^2}$ $y\mathrm{e}^{x^2}=\int \mathrm{e}^{x^2}\left(x-x^3+\frac{x^5}{4}\right)\mathrm{d}x$ $y(\alpha)\mathrm{e}^{\alpha^2}-y(0)\mathrm{e}^0=\int_0^{\alpha}\left(x\mathrm{e}^{x^2}-x^3\mathrm{e}^{x^2}+\frac{1}{4}x^5\mathrm{e}^{x^2}\right)\mathrm{d}x$ Since $y(0)=0$, $\mathrm{e}^{\alpha^2}y(\alpha)=I_1-I_3+\frac{1}{4}I_5$ $=I_1-I_3+\frac{1}{4}\left(\frac{1}{2}\alpha^4\mathrm{e}^{\alpha^2}-2I_3\right)$ $=I_1-\frac{3}{2}I_3+\frac{1}{8}\alpha^4\mathrm{e}^{\alpha^2}$ $=I_1-\frac{3}{2}\left(\frac{1}{2}\alpha^2\mathrm{e}^{\alpha^2}-I_1\right)+\frac{1}{8}\alpha^4\mathrm{e}^{\alpha^2}$ $=\frac{5}{2}I_1-\frac{3}{4}\alpha^2\mathrm{e}^{\alpha^2}+\frac{1}{8}\alpha^4\mathrm{e}^{\alpha^2}$ Since $I_1=\int_0^{\alpha}x\mathrm{e}^{x^2}\,\mathrm{d}x=\frac{1}{2}\mathrm{e}^{\alpha^2}-\frac{1}{2}$, $\mathrm{e}^{\alpha^2}y(\alpha)=\frac{5}{2}\left(\frac{1}{2}\mathrm{e}^{\alpha^2}-\frac{1}{2}\right)-\frac{3}{4}\alpha^2\mathrm{e}^{\alpha^2}+\frac{1}{8}\alpha^4\mathrm{e}^{\alpha^2}$ $=\frac{5}{4}\mathrm{e}^{\alpha^2}-\frac{5}{4}-\frac{3}{4}\alpha^2\mathrm{e}^{\alpha^2}+\frac{1}{8}\alpha^4\mathrm{e}^{\alpha^2}$ $\therefore\ y(\alpha)=\frac{5}{4}-\frac{5}{4}\mathrm{e}^{-\alpha^2}-\frac{3}{4}\alpha^2+\frac{1}{8}\alpha^4$	
	Substituting $\alpha=0.1$, for part (a), $\mathrm{f}(0.1)=\frac{0.1^2}{32}\left[\left(8\cos^2 0.1\right)+\left(0.1^4-7(0.1)^2+8\right)\cos^2\left(\frac{0.1}{2}\right)\right]$ $=0.00495$, for part (b)(ii), $\mathrm{f}(0.1)=\frac{5}{4}-\frac{5}{4}\mathrm{e}^{-0.1^2}-\frac{3}{4}(0.1)^2+\frac{1}{8}(0.1)^4=0.00495$ Both approximations are comparable. However, to obtain the estimate given by the small angle approximation, it requires the solving of the differential equation, whereas the improved Euler method does not.	

Section B: Probability and Statistics [50 marks]

6	Given $\left(\hat{p}_A - 1.9600\sqrt{\frac{\hat{p}_A\hat{q}_A}{n}}, \hat{p}_A + 1.9600\sqrt{\frac{\hat{p}_A\hat{q}_A}{n}}\right) = (0.229, 0.371)$, $\hat{p}_A = \frac{0.229 + 0.371}{2} = 0.3$ $1.9600\sqrt{\frac{0.3\times 0.7}{n}} = \frac{0.371 - 0.229}{2} = 0.071$ $n = 160.03 \approx 160$ Based on the 2 samples, $\hat{p} = \frac{(160\times 0.3) + 36}{160 + 100} = \frac{84}{260} = \frac{21}{65}$ A symmetric $\alpha\%$ confidence interval for p $= \left(\hat{p} - z\sqrt{\frac{\hat{p}\hat{q}}{n}}, \hat{p} + z\sqrt{\frac{\hat{p}\hat{q}}{n}}\right)$ $2z\sqrt{\frac{\frac{21}{65}\times\frac{44}{65}}{260}} \le 0.1$ $z \le 1.7240$ $\alpha \le 100 \times \mathrm{P}(-1.7240 < Z < 1.7240)$ $\alpha \le 91.529$ Largest possible value of $\alpha = 91.5$ (1 d.p.)	

7	H_0 : The two factors, level of reading activity and literacy skills, are independent. H_1 : The two factors, level of reading activity and literacy skills, are not independent. Perform a χ^2 test on independence. Observed Frequencies (O_{ij})	

		Level of Reading Activity			
		High	Medium	Low	Total
Literacy Skills	Good	25	12	7	44
	Average	35	57	27	119
	Poor	9	13	20	42
	Total	69	82	54	205

Under H_0, Expected Frequencies (E_{ij})

		Level of Reading Activity			
		High	Medium	Low	Total
Literacy Skills	Good	14.81	17.6	11.59	44
	Average	40.05	47.6	31.35	119
	Poor	14.14	16.8	11.06	42
	Total	69	82	54	205

Contributions to χ^2_{cal} $\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\left(O_{ij}-E_{ij}\right)^2}{E_{ij}}\right)$:

		Level of Reading Activity		
		High	Medium	Low
Literacy Skills	Good	7.01167	1.78182	1.81794
	Average	0.63763	1.85630	0.60264
	Poor	1.86640	0.85952	7.21862

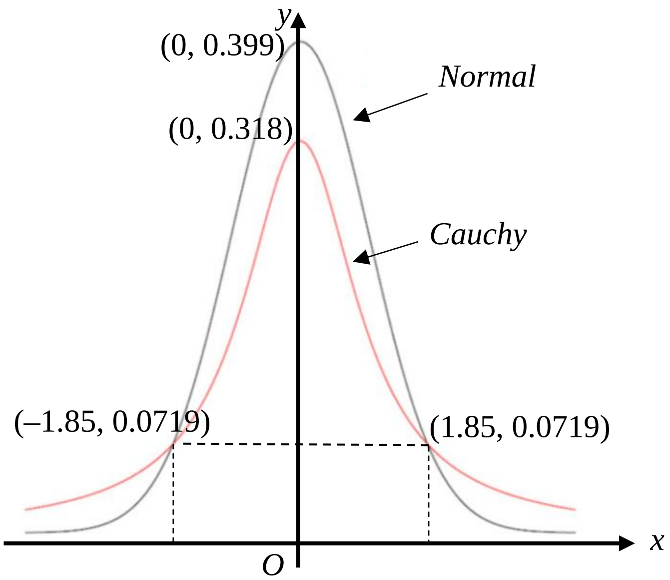
The Degrees of Freedom, $\upsilon = (3-1)\times(3-1) = 4$

Using the GC, χ^2_{cal} = 23.653 and *p*-value = 9.37×10^{-5}

Since *p*-value = 9.37×10^{-5} < 0.0001, we reject H_0 at 0.01% level of significance and conclude that there is sufficient evidence to claim that the two factors, level of reading activity and literacy skills, are not independent.

As the ***p*** value is very small, the test indicates that there is very strong evidence of an association between the 2 factors, with the strongest evidence for association in the Good/High cell and Poor/Low cell, where the observed frequencies are much greater than the expected frequencies under H_0.

For the Good/High cell, as there are more students with good literacy skills and a high level of reading activity than expected, it is not a cause for concern. For the Poor/Low cell, the higher observed value is an issue the academic department should be concerned about as it suggests that the students' poor literacy skills are causing the level of reading activity to be low (and vice versa).

8(a)	Since the line of symmetry of the curve $y=\left(x-\alpha\right)^2+\gamma^2$ lies along $x=\alpha$, median of $X=\alpha$	
(b)	$\mathrm{P}(X>\alpha+\gamma)=\frac{\gamma}{\pi}\int_{\alpha+\gamma}^{\infty}\frac{1}{\left[\left(x-\alpha\right)^2+\gamma^2\right]}\,\mathrm{d}x$ $=\frac{\gamma}{\pi}\left[\frac{1}{\gamma}\tan^{-1}\left(\frac{x-\alpha}{\gamma}\right)\right]_{\alpha+\gamma}^{\infty}$ $=\frac{\gamma}{\pi}\left[\frac{1}{\gamma}\left(\frac{\pi}{2}\right)-\frac{1}{\gamma}\left(\frac{\pi}{4}\right)\right]$ $=0.25$ Since median of $X=\alpha$, by symmetry, IQR = $\left(\alpha+\gamma\right)-\left(\alpha-\gamma\right)=2\gamma$	
(c)		
(d)	$\mathrm{P}(C<c)=\mathrm{P}(\tan U<c)$ $=\mathrm{P}\left(U<\tan^{-1}c\right)$ $=\frac{\tan^{-1}c-\left(-\frac{\pi}{2}\right)}{\left(\frac{\pi}{2}\right)-\left(-\frac{\pi}{2}\right)}=\frac{1}{\pi}\tan^{-1}c+\frac{1}{2}$ Pdf of C: $\mathrm{f}(c)=\frac{\mathrm{d}}{\mathrm{d}c}\left(\frac{1}{\pi}\tan^{-1}c+\frac{1}{2}\right)=\frac{1}{\pi\left(1+c^2\right)}=\frac{1}{\pi\left[1^2+\left(c-0\right)^2\right]},\quad c\in\mathbb{R}$ Therefore C has a Cauchy distribution with $\alpha=0,\gamma=1$.	

9**(i)**	For $r = 0, 1, 2, \ldots$ $\mathrm{P}(X+Y=r)=\sum_{k=0}^{r}\mathrm{P}(X=k)\mathrm{P}(Y=r-k)$ $=\sum_{k=0}^{r}\left[\frac{\mathrm{e}^{-\lambda}\lambda^{k}}{k!}\right]\left[\frac{\mathrm{e}^{-\mu}\mu^{r-k}}{(r-k)!}\right]$ $=\frac{\mathrm{e}^{-\lambda-\mu}}{r!}\sum_{k=0}^{r}\frac{r!}{k!(r-k)!}\lambda^{k}\mu^{r-k}$ $=\frac{\mathrm{e}^{-(\lambda+\mu)}}{r!}\sum_{k=0}^{r}\binom{r}{k}\lambda^{k}\mu^{r-k}$ $=\frac{\mathrm{e}^{-(\lambda+\mu)}}{r!}(\lambda+\mu)^{r}$ $X+Y\sim\mathrm{Po}(\lambda+\mu)$ (Shown).	
(ii)	The conditions are: (i) The EMS calls are made independently. (ii) The EMS calls are made singly. (iii) The average rate of EMS calls made remains a constant.	
(iii)	Let X be the number of EMS calls made in an hour. Then $X\sim\mathrm{Po}\left(\frac{585}{24}\right)$, i.e., $X\sim\mathrm{Po}(24.375)$. $\mathrm{P}(X>30)=1-\mathrm{P}(X\le 30)=0.110\ (3\text{ sf})$.	
(iv)	Let E be the number of EMS calls made in a week. Let F be the number of Fire calls made in a week. Then $E\sim\mathrm{Po}(4095)$, $F\sim\mathrm{Po}(35)$ and $E+F\sim\mathrm{Po}(4130)$. $\mathrm{P}(F=50 \mid E+F=5000)=\frac{\mathrm{P}(E=4950)\mathrm{P}(F=50)}{\mathrm{P}(E+F=5000)}$ $=0.0294\ (3\text{ sf})$. Assume that E and F are independent.	
(v)	The average time interval between EMS calls $=\frac{1}{585}$ days. Thus, the average time interval is $=\frac{1}{585}\times 24\times 60\times 60=148$ seconds (3 sf).	

<table>
<tr><td>10(i
)</td><td>The decision rules for the test do not depend on parameters of the population.
OR
The data is not quantitative.

The decision rules for the test do not depend on requirements concerning the distribution of the population.
OR
The distribution of the parent population from which the samples are drawn is unknown.
OR
It is not known whether the distribution of parent population from which the samples are drawn is normally distributed.</td><td></td></tr>
<tr><td>(ii)</td><td>John should use a sign test and not a Wilcoxon test to carry out his investigation because based on his assumption, it can be determined from the data whether the sugar content of each of the 10 drinks is greater or less than 5 grams per 100 ml but not the magnitude of the difference.</td><td></td></tr>
<tr><td>(iii)</td><td>Let m be the median sugar content (in grams per 100 ml) of the packaged drinks.

To test $H_0 : m = 5$ vs $H_1 : m < 5$.

Perform a 1-tail test at 5% level of significance.

Replace the data with either a plus sign (+) or minus sign (–) depending on whether the sugar content represented by the Nutri-Grade mark is greater or less than 5, we have
<table>
<tr><td>Drink</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td></td><td>–</td><td>–</td><td>–</td><td>+</td><td>+</td><td>–</td><td>–</td><td>–</td><td>+</td><td>–</td></tr>
</table>
Let S_+ be the number of + signs out of 10.
From the table, $s_+ = 3$, $s_– = 7$.
Under H_0, $S_+ \sim \text{B}(10, 0.5)$
$p\text{-value} = \text{P}(S_+ \le s_+) = \text{P}(S_+ \le 3) = 0.172$ (3 sf).
Since p-value = 0.172 > 0.05, John should not reject H_0 and conclude that there is insufficient evidence, at the 5% level of significance, that the median sugar content of packaged drinks sold in his neighbourhood supermarket is less than 5 grams per 100 ml.</td><td></td></tr>
<tr><td>(iv)</td><td>Let X be the sugar content of the packaged drinks and $D = X - 5$.
Let m_D be the population median of D.</td><td></td></tr>
</table>

To test $H_0 : m_D = 0$ vs $H_1 : m_D < 0$.
Perform a 1-tail test at 5% level of significance.

Drink	1	2	3	4	5	6	7	8	9	10
Sugar Content	4.6	2.3	0.7	5.9	10.3	3.7	4.9	4.8	10.2	3.9
d	–0.4	–2.7	–4.3	0.9	5.3	–1.3	–0.1	–0.2	5.2	–1.1
Rank of $\|d\|$	3	7	8	4	10	6	1	2	9	5

The sum of the positive ranks, $P = 4 + 9 + 10 = 23$

The sum of the negative ranks, $Q = \frac{10(11)}{2} - 23 = 32$

So, T = min (P, Q) = 23

From the table, we reject H_0 if $T \leq 10$. Since $T = 23 > 10$, we do not reject H_0 and conclude that at the 5% level of significance, there is insufficient evidence to conclude that the packaged drinks in his neighbourhood supermarket have a lower sugar content as compared to a reference drink containing 5 grams per 100 ml of sugar.

Although the same conclusion is reached for both tests, the Wilcoxon test is usually regarded as a more accurate and reliable test as compared to the sign test because the Wilcoxon test takes into account the magnitude of the sugar content differences and not merely their signs. However, one disadvantage of the Wilcoxon test is that it must make an additional assumption that the distribution of the sugar content differences D must be symmetric about the median.