FOR TEACHERS ONLY

PEIRCE SECONDARY SCHOOL 2021
Preliminary Examination
for Secondary 4E/4AO/5A
ADDITIONAL MATHEMATICS Paper 2
Mark Scheme

Question	Answer	Marks	Partial Marks	Guidance
1(i)	$(x-1)^3+64$	2		
	$=(x-1)^3+(4)^3$			
	$=[(x-1)+4]\left[(x-1)^2-(x-1)(4)+(4)^2\right]$		M1	For attempt to apply $a^3+b^3=(a+b)\left(a^2-ab+b^2\right)$. Allow for numerical errors and algebraic slips
	$=[x+3]\left[x^2-2x+1-4x+4+16\right]$			
	$=(x+3)\left(x^2-6x+21\right)$		A1	Ignore extra brackets.
	OR			
	$x^2-6x+21$ $x+3\,\overline{)\,x^3-3x^2+3x+63}$ $\underline{x^3+3x^2}$ $-6x^2+3x$ $\underline{-6x^2-18x}$ $21x+63$ $\underline{21x+63}$ 0	2	M1	Correct dividend: $x^3-3x^2+3x+63$ Correct divisor: $x-3$ For attempt to apply long division
	$(x+3)\left(x^2-6x+21\right)$		A1	Ignore extra brackets.

1(ii)	$(x-1)^3+64=16(x+3)$			0 marks for attempt to solve $x^3-3x^2-13x+15=0$
	$(x+3)(x^2-6x+21)=16(x+3)$			
	$(x+3)(x^2-6x+21)-16(x+3)=0$			
	$(x+3)(x^2-6x+21-16)=0$		**M1**	For attempt to apply factorisation by taking out common factor $x-3$. Allow for algebraic slips and numerical errors but the other factor must be quadratic factor.
	$(x+3)(x^2-6x+5)=0$			
	$(x+3)(x-5)(x-1)=0$		**M1**	For attempt to apply factorisation. Step must be shown explicitly but allow for errors in factors
	$\therefore\ x=-3$, x = 5 or x = 1		**A1**	For **<u>all 3</u>** correct solutions.
2(i)	$\dfrac{7x^2-2}{x^2(2x-1)}=\dfrac{A}{x^2}+\dfrac{B}{x}+\dfrac{C}{2x-1}$	**5**	**M1**	Attempt for partial fractions.
	$7x^2-2=A(2x-1)+Bx(2x-1)+C(x^2)$		**M1**	For attempt to remove denominator. Allow for numeric slips and algebraic slips
	At $x=\dfrac{1}{2}$,			
	$-\dfrac{1}{4}=C\left(\dfrac{1}{4}\right)$		**M1**	Correct choice of value for substitution
	$C=-1$			
	At x = 0,			
	$-2=-A$		**M1**	Correct choice of value for substitution
	$A=2$			
	At $x=1$,			
	$5=2(1)+B(1)-1(1)$			
	$5=B+1$			
	$B=4$			
	$\therefore\ \dfrac{2}{x^2}+\dfrac{4}{x}-\dfrac{1}{2x-1}$		**A1**	Accept $\dfrac{2}{x^2}+\dfrac{4}{x}+\dfrac{-1}{2x-1}$ or o.e.

						Ignore redundant brackets by students.
	ii		$\int \frac{7x^2-2}{x^2(2x-1)}\,dx$			
			$= \int \frac{2}{x^2}\,dx + \int \frac{4}{x}\,dx - \int \frac{1}{2x-1}\,dx$		**M1**	Replace with their partial fractions. Correct application of addition/subtraction rule.
			$= 2\int x^{-2}\,dx + 4\int \frac{1}{x}\,dx - \int \frac{1}{2x-1}\,dx$			
			$= 2\bullet\frac{x^{-1}}{-1} + 4\ln x - \frac{1}{2}\ln(2x-1) + c$		**M1** **M1**	Attempt to apply power rule for 1st term Attempt to apply reciprocal rule for 2nd and 3rd term. No penalty if *c* is missing. Allow for numerical and sign errors
			$= -\frac{2}{x} + 4\ln x - \frac{1}{2}\ln(2x-1) + c$			
			$= \left[-\frac{2}{x} + 4\ln x - \frac{1}{2}\ln(2x-1)\right]_1^2$			
			= 1.2232 − (−2)			
			= 3.22		**A1**	Ignore 'units^2' if given.
			<u>OR</u>		**[4]**	
			$\int_1^2 \frac{7x^2-2}{x^2(2x-1)}\,dx$			
			$= \int_1^2 \frac{2}{x^2}\,dx + \int_1^2 \frac{4}{x}\,dx - \int_1^2 \frac{1}{2x-1}\,dx$		**M1**	Replace with their partial fractions. Correct application of addition/subtraction rule.
			$= 2\bullet\left[\frac{x^{-1}}{-1}\right]_1^2 + 4[\ln x]_1^2 - \frac{1}{2}[\ln(2x-1)]_1^2$		**M1** **M1**	Attempt to apply power rule for 1st term Attempt to apply reciprocal rule for 2nd and 3rd term. Allow for numerical and sign errors No penalty if *c* is present and later omitted.
			$= 2\bullet\frac{1}{2} + 4\bullet\ln 2 - \frac{1}{2}\bullet\ln 3$			
			= 3.22		**A1**	o.e. single expression such as $\ln\frac{16e\sqrt{3}}{3}$

3(i)	$x > \frac{2}{3}$, $x \neq 1$ **OR** $x > 0.667$, $x \neq 1$ **OR** $\frac{2}{3} < x < 1$, $x > 1$ **OR** $0.667 < x < 1$, $x > 1$	**1**	**B1**	Accept 'and' in place of comma. Reject 'or' in place of comma.
3(ii)	$4\log_y 3 - \log_2 \frac{1}{16} = 3\log_3 y$	**5**		
	$\frac{4}{\log_3 y} - (-4) = 3\log_3 y$		**M1** **B1**	Attempt to apply change of base Allow for numerical errors and algebraic slips Correct evaluation to get −4
	Let $u = \log_3 y$,			
	$\frac{4}{u} + 4 = 3u$			
	$4 + 4u = 3u^2$			
	$3u^2 - 4u - 4 = 0$		**M1**	Simplify to any quadratic equation
	$(3u+2)(u-2) = 0$			This steip might be implied.
	$u = -\frac{2}{3}$ or $u - 2$		**M1**	Their u values Allow for numerical errors
	But $u = \log_3 y$,			
	$-\frac{2}{3} = \log_3 y$ and $2 = \log_3 y$			
	$y = 3^{-\frac{2}{3}}$ $y = 9$			
	$y = 0.481$			
	$\therefore$ $y = 0.481$ or 9		**A1**	Not awarded if any value is rejected.

3(iii)	$2\log_8 k = \log_2 \sqrt{z}$	**4**		
	$2\log_8 k = \log_2 z^{\frac{1}{2}}$		**M1**	Correct conversion from radical form to index form
	$2 \bullet \dfrac{\log_2 k}{\log_2 8} = \log_2 z^{\frac{1}{2}}$		**M1**	Attempt to apply change of base Allow for numerical errors and algebraic slips
	$\dfrac{2}{3}\log_2 k = \log_2 z^{\frac{1}{2}}$			
	$\log_2 k^{\frac{2}{3}} = \log_2 z^{\frac{1}{2}}$		**M1**	Single logarithm on each side of equation
	$k^{\frac{2}{3}} = z^{\frac{1}{2}}$			
	$z = k^{\frac{4}{3}}$ OR $z = \sqrt[3]{k^4}$		**A1**	Accept $k^{1\frac{1}{3}}$ and $k^{1.33}$. Reject $(k^4)^{\frac{1}{3}}$ and $\left(k^{\frac{1}{3}}\right)^4$.
4(a)	$\cos\left(A+\dfrac{\pi}{3}\right) = 4\sin\left(A+\dfrac{\pi}{2}\right)$	**4**		
	$\cos A\cos\dfrac{\pi}{3} - \sin A\sin\dfrac{\pi}{3} = 4\left(\sin A\cos\dfrac{\pi}{2} + \cos A\sin\dfrac{\pi}{2}\right)$		**M1**	Attempt to apply addition rule for cosine and for sine Allow for errors in sign
	$\dfrac{1}{2}\cos A - \dfrac{\sqrt{3}}{2}\sin A = 4(0+\cos A)$		**M1**	Attempt to evaluate trigo function of special angles Allow for numerical errors
	$-\dfrac{\sqrt{3}}{2}\sin A = 3\dfrac{1}{2}\cos A$			
	$\dfrac{\sin A}{\cos A} = -\dfrac{3\frac{1}{2}}{\frac{\sqrt{3}}{2}}$		**M1**	Attempt to obtain tan *A* eventually. Allow for numerical errors
	$\tan A = -\dfrac{7}{\sqrt{3}}$		**A1**	Need not rationalize surd in denominator
	$\tan A = -\dfrac{7\sqrt{3}}{3}$			

4(b)(i)	$\cos 105°$	3		
	$= \cos(60° + 45°)$			
	$= \cos 60° \cos 45° - \sin 60° \sin 45°$		M1	Attempt to apply addition rule for cosine Allow for errors in sign
	$= \frac{1}{2} \bullet \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \bullet \frac{\sqrt{2}}{2}$		M1	Attempt to evaluate trigo function of special angles Allow for numerical errors
	$= \frac{\sqrt{2} - \sqrt{6}}{4}$		A1	
4(b)(ii)	$\sec 105°$			
	$= \frac{1}{\cos 105°}$	3		
	$= \frac{4}{\sqrt{2} - \sqrt{6}}$		M1	Substitute result from (b)(i)
	$= \frac{4}{\sqrt{2} - \sqrt{6}} \bullet \frac{\sqrt{2} + \sqrt{6}}{\sqrt{2} + \sqrt{6}}$		M1	Multiplying numerator and denominator by $\sqrt{2} + \sqrt{6}$
	$= \frac{4\left(\sqrt{2} + \sqrt{6}\right)}{\left(\sqrt{2}\right)^2 - \left(\sqrt{6}\right)^2}$			
	$= \frac{4\left(\sqrt{2} + \sqrt{6}\right)}{-4}$			
	$= -\sqrt{2} - \sqrt{6}$		A1	Accept $-\left(\sqrt{2} + \sqrt{6}\right)$ o.e.

5(i)	$2\cos^2 x + 5\sin x\cos x$	**3**		
	$= 2\cos^2 x - 1 + 1 + \frac{5}{2}(2\sin x\cos x)$			
	$= \cos 2x + 1 + \frac{5}{2}\sin 2x$		**M1** **M1**	Uses $\cos 2x = 2\cos^2 x - 1$ Uses $\sin 2x = 2\sin x\cos x$
	$= \frac{5}{2}\sin 2x + \cos 2x + 1$		**A1**	Must be in required order – sin, cos and 1
	$\therefore\ p = 2\frac{1}{2},\ q = 1$			
5(ii)	$\cos x\,(2\cos x + 5\sin x) = 1$	**5**		
	$2\cos^2 x + 5\sin x\cos x = 1$			
	$\frac{5}{2}\sin 2x + \cos 2x + 1 = 1$		**M1**	Simplify to get LHS = part(i)
	$\frac{5}{2}\sin 2x = -\cos 2x$			
	$\tan 2x = -\frac{2}{5}$		**M1**	Simple trigo equation Allow for numerical errors and algebraic slips
	basic $\angle$, $\alpha = 21.801°$		**M1**	Acute angle based on their trigo equation
	$2x = 180-\alpha, 360-\alpha, 180-\alpha+360, 360-\alpha+360$			
	$2x = 158.199, 338.199, 518.199, 698.199$			
	$= 79.1°, 169.1°, 259.1°, 349.1°$		**A1** **A1√**	A1√ for angles in second revolution

6(i)	$\frac{dV}{dt} = 96\pi$ cm^3/s	**4**		
	$h = 3r$			
	At $r = 4$, $\frac{dr}{dt} = ?$			
	$V = \frac{1}{3}\pi r^2 h$			
	$= \frac{1}{3}\pi r^2 (3r)$		**M1**	Replace h with $3r$ Allow for errors in formula for volume
	$V = \pi r^3$			
	$\frac{dV}{dr} = 3\pi r^2$		**M1**	Correct differentiation based on their V.
	$\frac{dV}{dt} = \frac{dV}{dr} \bullet \frac{dr}{dt}$			
	$96\pi = 3\pi r^2 \bullet \frac{dr}{dt}$		**M1**	Correct substitution for Chain Rule
	$\frac{dr}{dt} = \frac{32}{r^2}$			
	After 4 seconds,			
	$V = 96\pi \bullet 4$			
	$V = 384\pi$			
	At $V = 384\pi$,			
	$384\pi = \pi r^3$			
	$r = \sqrt[3]{384}$ OR 7.2684			
	At $r = \sqrt[3]{384}$,			

	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{32}{\left(\sqrt[3]{384}\right)^2}$			
	$= 0.60570$			
	$= 0.606$		**A1**	Ignore wrong or no units
6(ii)	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{32}{r^2}$	**2**		
	$= 32r^{\ 2}$			
	$\frac{\mathrm{d}^2r}{\mathrm{d}t^2} = 32 \bullet -2r^{-3}$		**M1**	To show rate, $\frac{dr}{dt}$, is increasing Use of power rule on their $\frac{dr}{dt}$
	$\frac{\mathrm{d}^2r}{\mathrm{d}t^2} = -\frac{64}{r^3}$			Allow for numerical errors or algebraic slips
	As $r > 0$,			
	$\frac{d^2r}{dt^2} < 0$			
	⇒ the rate $\frac{dr}{dt}$ is a **decreasing function**			
	⇒ the rate $\frac{dr}{dt}$ **will decrease** as t increases.		**B1√**	Argues correctly based on sign of their $\frac{d^2r}{dt^2}$.
7(a)	$y = (k-8)x^2 - 6x + k$	**5**		
	For max. pt., $k - 8 < 0$		**M1**	
	$k < 8$			1st inequality to use later
	For real distinct roots,			
	$b^2 - 4ac > 0$			
	$(-6)^2 - 4(k-8)(k) > 0$		**B1**	
	$36 - 4k^2 + 32k > 0$			
	$4k^2 - 32k - 36 < 0$			

	$k^2-8k-9<0$		M1	Simplify to quadratic inequality
	$(k-9)(k+1)<0$			
	$-1<k<9$		A1√	Allow for solving $(k-9)(k+1)=0$ to arrive at equality, 2nd inequality to use later
	$\therefore\ -1<k<8$		A1√	Combine correctly based on their inequalities
7(b)(i)	$y=p^2+2-px+x^2$	3		
	b^2-4ac			
	$=(-p)^2-4(1)(p^2+2)$		M1	For correct substitution, ignore inequality that follows if any
	$=-3p^2-8$			
	Since $p^2\geq 0$,			
	$-3p^2\leq 0$			
	$-3p^2-8\leq -8$			
	$-3p^2-8<0$			
	Since $b^2-4ac<0$,		M1	Need to show preceding steps beginning from $p^2\geq 0$ or explanation (−ve) x (+ve) = (−ve) (−ve) − (+ve) = (−ve) Accept $-3p^2-8<0$ Accept $-3p^2-8$ is always negative Will not award begin with $-3p^2-8<0$ without leading from $p^2\geq 0$. Will not accept trial and error.
	$\therefore$ the curve has no real roots.		A1	Reject 'no roots', 'no solution'. Must be preceded by above step.
7(b)(ii)	$y=p^2+2-px+x^2$	3		
	$y=5$			
	$p^2+2-px+x^2=5$		M1	Substitute one eqn. into another
	$x^2-px+p^2-3=0$			

	For real equal roots,			
	$b^2 - 4ac = 0$			
	$(-p)^2 - 4(1)(p^2 - 3) = 0$		M1	For correct substitution Allow for variation other than = 0
	$p^2 - 4p^2 + 12 = 0$			
	$p^2 = 4$			
	$p = \pm 2$		A1	Accept $p = 2$ or $p = -2$
8(i)	$m_{AD} = \frac{k-0}{7-3}$	3	M1	Realises calculation of gradient
	$= \frac{k}{4}$			
	$m_{DC} = \frac{10-k}{13-7}$			
	$= \frac{10-k}{6}$			
	Since $m_{AD} \perp m_{DC}$,			
	$\frac{k}{4} \bullet \frac{10-k}{6} = -1$		M1	Realises $m_1 \bullet m_2 = -1$
	$k^2 - 10k - 24 = 0$			
	$(k+2)(k-12) = 0$			
	$k = -2$ or $k = 12$			
	(rejected)			
	$\therefore k = 12$		A1	Answer given. Check preceding steps. Will not accept if $k = 12$ substituted to arrive at -1.
8(ii)	$m_{BC} = \frac{12-0}{7-3}$	2	M1	Realises calculation of gradient
	$= 3$			
	$y - 10 = 3(x - 13)$			
	$y - 10 = 3x - 39$			

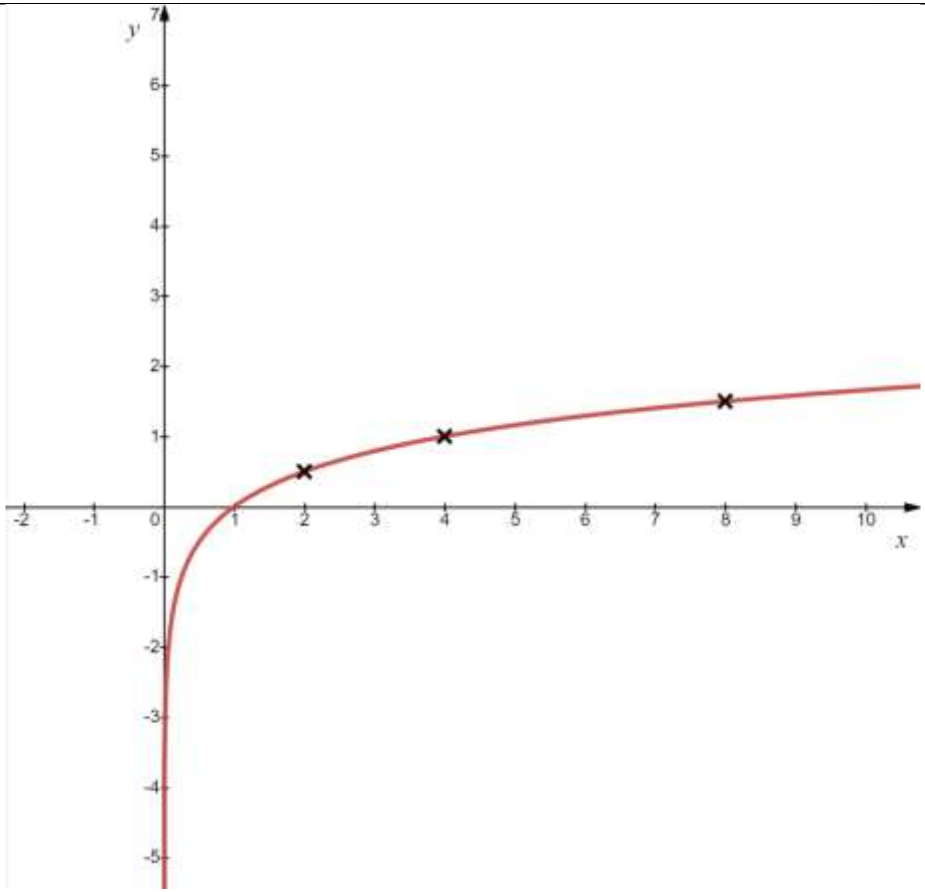
	$y = 3x - 29$		A1	Accept $3x - y - 29 = 0$ or equivalent
8(iii)	$2y - x + 3 = 0$ ____①	3		
	$y = 3x - 29$ ____②			
	Sub. ② into ①:			
	$2(3x - 29) - x + 3 = 0$		M1	Substitute their part (ii) into given eqn. Allow for their incorrect part (ii) eqn.
	$6x - 58 - x + 3 = 0$			
	$x = 11$		M1	Solve to get their x
	Sub. x = 11 into ②:			
	$y = 4$			
	$\therefore$ $B(11, 4)$		A1	
8(iv)	Area = $\frac{1}{2}\begin{vmatrix} 3 & 11 & 13 & 7 & 3 \\ 0 & 4 & 10 & 12 & 0 \end{vmatrix}$	2	M1	Ensure $\frac{1}{2}$ with 5 columns of coordinates Allow for their incorrect B
	$= \frac{1}{2}\lvert 278 - 158 \rvert$			
	= 60 units^2		A1	
8(v)	x-coordinate = 3 + (13-7) = 9	2		
	y−coordinate = 0 − (12−10) = −2			
	$\therefore$ $E(9, -2)$		B1 B1	Correct x-coordinate Correct y-coordinate No penalty for absence of coordinate form (9, −2)
9(i)	(table below)	3	B3	See graph on last page −1 mark if point off by more than 1 tiny square

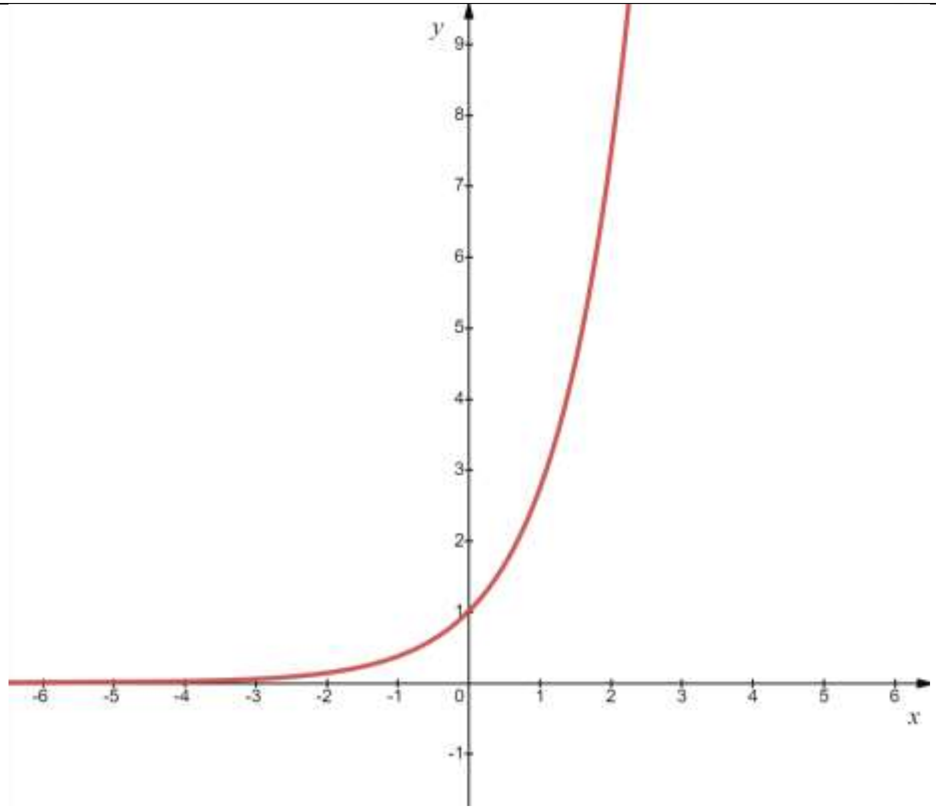
$X = x\sqrt{x}$	1	2.83	5.20	8	11.18
$Y = \frac{y}{\sqrt{x}}$	24.0	31.3	40.8	52.0	64.7

9(ii)	$y = ax^2 + b\sqrt{x}$	3		
	$\frac{y}{\sqrt{x}} = ax\sqrt{x} + b$		M1	
	$Y = mX + c$			

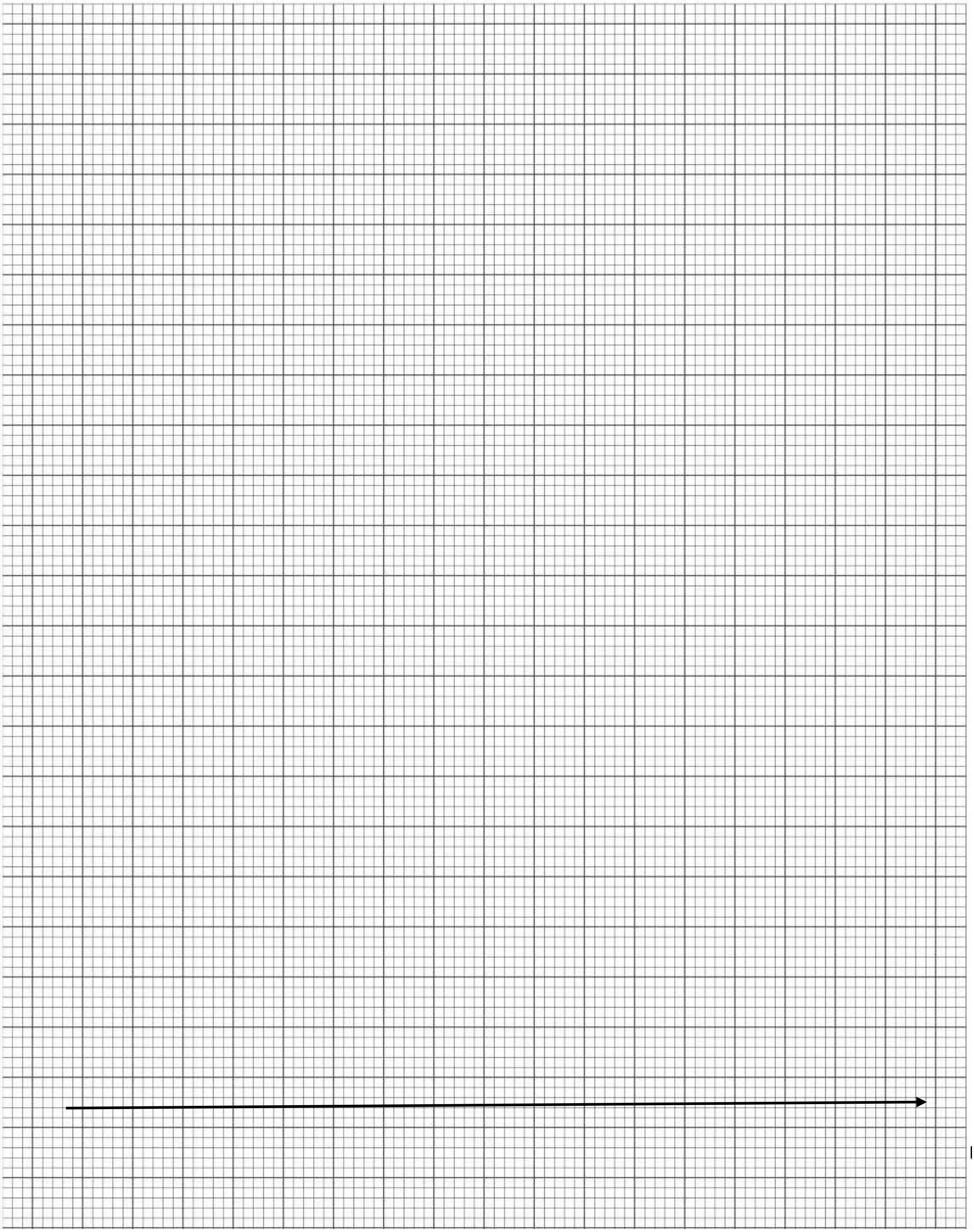
	From the graph, c = 20.0			
	$\therefore\ b = 20.0$		**B1**	Accept values from 19.0 to 21.0 inclusive, no requirement for 1 d.p. or 3 s.f. where appropriate
	$m = \dfrac{68.0 - 20.0}{12 - 0}$			
	$a = 4.00$		**B1**	Accept values from 3.71 to 4.31 inclusive, must be at least 3 s.f. or in fraction
9(iii)	$\dfrac{\sqrt{x}}{y} = \dfrac{1}{50}$	**3**		
	$\dfrac{y}{\sqrt{x}} = 50$		**M1**	Attempt to obtain Y
	$Y - 50$			
	From graph, $X = 7.5$		**M1**	Their X when $Y = 50$ from graph
	$x^{\frac{3}{2}} = 7.5$			
	$x = 3.83$		**A1**	
9(iv)	$y = ax^2 + b\sqrt{x}$	**1**		
	$\dfrac{y}{x^2} = a + \dfrac{b}{x\sqrt{x}}$			
	$\dfrac{y}{x^2} = b \bullet \dfrac{1}{x\sqrt{x}} + a$			
	The horizontal axis is $\dfrac{1}{x\sqrt{x}}$.		**B1**	Accept $\dfrac{1}{\sqrt{x^3}}$ or equivalent
9(v)	a represents the intercept for axis of $\dfrac{y}{x^2}$ OR $\dfrac{y}{x^2}$-intercept OR intercept of the vertical axis	**2**	**B1**	Accept Y-intercept if shown in their graph OR previous working
	b represents the gradient		**B1**	

10(a)(i)	$y = \log_a x$	**3**		
	At (8, 1.5),			
	$1.5 = \log_a 8$			
	$a^{\frac{3}{2}} = 8$			
	$\left(a^{\frac{3}{2}}\right)^{\frac{2}{3}} = 8^{\frac{2}{3}}$			
	$a - 4$		**B1**	
	At a = 4 and (2, b),			
	$b = \log_4 2$			
	$b = \frac{1}{2}$ or 0.5		**B1**	
	At a = 4 and (c, 0),			
	$0 = \log_4 c$			
	$c - 1$		**B1**	

		ii		**2**	**B2**	B1 – correct shape in any quadrant B1 – correct x-intercept For B2, −1 mark if no gap between graph and x−axis.

	b			**2**	**B2**	B1 – correct shape in any quadrant B1 – correct *y*-intercept and with *x*-axis from negative *x* to positive *x*. For B2, −1 mark if no gap between graph and x−axis.

9



Correct scale B1
Correct points B1
Line of best fit B1