Year 4 Mathematics 2
Supplementary Worksheet
Integration

Name : _____ **( )** **Class :** _____ **Date :** _____

1 If $\int_1^3 \frac{2x+1}{x}\,\mathrm{d}x = 5p$, find the value of p.

[*2008 ACS (Barker Road) AMaths P2*]

2 Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left[(2x-3)\sqrt{4-x}\right]$ can be written in the form $\frac{a-bx}{2\sqrt{4-x}}$ where a and b are integers. Hence evaluate $\int_0^3 \frac{57-18x}{\sqrt{4-x}}\,\mathrm{d}x$.

[*2008 ACS (Barker Road) AMaths P2*]

3 Evaluate $\int_0^{\frac{\pi}{3}} \sin x(\sin x+1)\,dx$.

[*2008 ACS (Barker Road) AMaths P2*]

4(i) Express $\frac{x-16}{2x^2-9x-5}$ in partial fractions.

(ii) Hence evaluate $\int_6^8 \frac{x-16}{2x^2-9x-5}\,\mathrm{d}x$.

[*2008 ACS (I) AMaths P1*]

5 Evaluate $\int_0^2 \frac{1+\mathrm{e}^{2x}}{\mathrm{e}^x}\,\mathrm{d}x$.

[*2008 ACS (I) AMaths P1*]

6 A curve has the equation $y = x\ln 2x$, where $x > 0$.

(i) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(ii) Hence, find $\int_1^2 3\ln 2x\,\mathrm{d}x$.

(iii) Find the range of values of x for which the function $y = x\ln 2x$ is increasing. Leave your answer in terms of e.

(iv) Find the rate of change of x given that y increases at the rate of 4 units/second when x is $\frac{1}{2}\mathrm{e}$ units.

[*2008 ACS (I) AMaths P2*]

7(a) Evaluate

(i) $\int_3^4 \frac{3}{2x-5}\,\mathrm{d}x$, **(ii)** $\int_0^1 \frac{9}{2\sqrt{3x+1}}\,\mathrm{d}x$,

(iii) $\int_0^{\frac{\pi}{2}} 4\sin^2 2x\,\mathrm{d}x$.

(b) The gradient of a curve at the point (x , y) on the curve is given by $3-\frac{k}{x^2}$, where k is a constant. If the tangent to the curve at the point (2 , 8) cuts the y-axis at (0 , 4), find

(i) the value of k,

(ii) the equation of the curve.

[*2008 ACS (I) AMaths P2*]

8 Show that $\frac{2\sin 2x - \sin 4x}{2\sin 2x + \sin 4x} = \tan^2 x$. Hence evaluate $\int_0^{\frac{\pi}{4}} \frac{2\sin 2x - \sin 4x}{2\sin 2x + \sin 4x}\,\mathrm{d}x$, leaving your answer in terms of π.

[*2008 ACS (I) AMaths P2*]

9(a) Express $\frac{2x^2+x-6}{x^2(x-2)}$ in partial fractions.

(b) Hence, find the exact value of $\int_3^5 \frac{2x^2+x-6}{x^2(x-2)}\,\mathrm{d}x$.

[*2008 Anderson Sec AMaths P1*]

10(a) Express $\cos 4x \cos x$ in the form $\frac{\cos A + \cos B}{2}$ and hence find $\int \cos 4x \cos x\,\mathrm{d}x$.

(b) Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x}{1-3x}\right) = \frac{1}{(1-3x)^2}$. Hence evaluate $\int_2^6 \left(\frac{2}{1-3x}\right)^2 \mathrm{d}x$.

[*2008 Anderson Sec AMaths P2*]

11 Show that the gradient function for the curve $y = \frac{\tan x}{\cos x}$ is $\frac{1+\sin^2 x}{\cos^3 x}$.

Hence deduce that the curve has no stationary points for $0 < x < \frac{\pi}{2}$.

[*2008 Anglican High AMaths P1*]

12(i) Express $\frac{5x^2+13x-6}{(2x+1)(x^2-4)}$ in partial fractions.

(ii) Hence evaluate $\int_3^4 \frac{5x^2+13x-6}{(2x+1)(x^2-4)}\,\mathrm{d}x$.

[*2008 Anglican High AMaths P2*]

13 The curve for which $\frac{dy}{dx} = \frac{k}{(2-x)^3}$ where $x > 2$ and k is a constant, is such that the tangent at (3 , 6) passes through the origin. Find

(i) the value of k,

(ii) the equation of the curve.

[*2008 Balestier Hill Sec AMaths P1*]

14 A curve has the equation $y = \frac{2\cos x}{1-\sin x}$.

(i) Show that $\frac{dy}{dx}$ can be written in the form of $\frac{k}{1-\sin x}$ where k is a constant.

State the value of k.

(ii) Hence, evaluate $\int_0^{\frac{\pi}{6}} \frac{1}{1-\sin x}\, dx$.

[*2008 Balestier Hill Sec AMaths P2*]

15(a) Express $\frac{x(x-1)}{(x+1)^2}$ in the form $A + \frac{B}{x+1} + \frac{C}{(x+1)^2}$, where A, B and C are constants.

(b) Evaluate $\int_0^{\frac{1}{4}} \frac{4}{3(1-2x)}\, dx$.

[*2008 Balestier Hill Sec AMaths P2*]

16 A curve is such that $\frac{dy}{dx} = \frac{8}{(2x-1)^3}$. The tangent to the curve, at a certain point, cuts the curve at the point (1, 2). Find the equation of the curve.

[*2008 Catholic High AMaths P1*]

17 Differentiate $2x\cos x$ with respect to x. Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{3}} x \sin x\, dx$.

[*2008 Catholic High AMaths P2*]

18 Express $\frac{2x^2 - 7}{(x+1)(x^2 - x - 6)}$ in partial fractions.

Hence, or otherwise, find $\int \frac{2x^2 - 7}{(x+1)(x^2 - x - 6)}\, dx$.

[*2008 Catholic High AMaths P2*]

19 Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin^2 x - 2\sin 2x)\, dx$.

[*2008 Cedar Girls' Sec AMaths P1*]

20(a) Express $\dfrac{9-4x}{(2x+3)(x-1)^2}$ in partial fractions. Hence evaluate $\int_2^3 \dfrac{9-4x}{(2x+3)(x-1)^2}\,\mathrm{d}x$.

(b) The gradient function of a curve $y = \mathrm{f}(x)$ is given by $\dfrac{\mathrm{d}y}{\mathrm{d}x} = m + \dfrac{n}{x^2}$. The curve passes through the point $A(2, 5)$. The tangent to the curve at point A is parallel to the line $4x - y = 3$ and the curve has a turning point at $x = -\frac{2}{3}$. Find

(i) the value of m and of n,

(ii) the equation of the curve.

[*2008 Cedar Girls' Sec AMaths P2*]

21(a) Find $\int \dfrac{1}{\cos^2 \frac{1}{4}x}\,\mathrm{d}x$.

(b) Express $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\ln \cot 3x\right)$ in the form $\dfrac{k}{\sin px}$, where k and p are constants.

[*2008 CHIJ St Nicholas Girls' AMaths P1*]

22(a) Express $\dfrac{3x^2 - 18x - 8}{x^3 - 4x}$ in partial fractions.

(b) Evaluate $\int_4^9 \left(\dfrac{3x+5}{\sqrt{x}} + \dfrac{3}{5x}\right)\mathrm{d}x$, giving your answer correct to 2 decimal places.

[*2008 CHIJ Toa Payoh Sec AMaths P2*]

23 Differentiate $x^2 \ln x$ with respect to x.

Hence, evaluate $\int_2^3 x \ln x\,\mathrm{d}x$, giving your answer correct to two decimal places.

[*2008 CHIJ Toa Payoh Sec AMaths P1*]

24(a) Express $\dfrac{2x^2 + 16x}{(1-3x)(2x+1)^2}$ in partial fractions.

Hence, find $\int \dfrac{2x^2 + 16x}{(1-3x)(2x+1)^2}\,\mathrm{d}x$.

(b) Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}(x+1)\sqrt{2x-1} = \dfrac{3x}{\sqrt{2x-1}}$.

Hence, evaluate $\int_5^{13} \dfrac{2x}{3\sqrt{2x-1}}\,\mathrm{d}x$.

[*2008 CHIJ St Nicholas Girls' AMaths P2*]

25(a) Find

(i) $\int \frac{24}{(3x-2)^5}\,\mathrm{d}x$,

(ii) $\int 3x(2x-1)^2\,\mathrm{d}x$.

(b) **(i)** Given $y = 5(3x-2)\sqrt{x+1}$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5(9x+4)}{2\sqrt{x+1}}$.

(ii) Hence, find $\int \frac{9x+4}{\sqrt{x+1}}\,\mathrm{d}x$.

[*2008 Clementi Woods Sec AMaths P1*]

26(i) Express $\frac{1}{x(x^2-1)}$ in partial fractions.

(ii) Hence, find $\int \frac{1}{x(x^2-1)}\,\mathrm{d}x$, simplifying your answer.

[*2008 Clementi Woods Sec AMaths P2*]

27(i) Show that $\frac{\mathrm{d}}{\mathrm{d}x}[\ln(\sec x + \tan x)] = \sec x$.

(ii) Express $\frac{\mathrm{d}}{\mathrm{d}x}(\sec x \tan x)$ in terms of $\sec x$.

(iii) Hence, evaluate $\int_0^{\frac{\pi}{3}} \sec^3 x\,\mathrm{d}x$, giving your answer in surd form.

[*2008 Clementi Woods Sec AMaths P2*]

28 Given that $y = \frac{2}{(1+\sin\theta)^2}$, find $\frac{\mathrm{d}y}{\mathrm{d}\theta}$.

Hence, or otherwise, find the value of k given that $\int_0^k \frac{\cos\theta}{(1+\sin\theta)^3}\,\mathrm{d}\theta = \frac{5}{18}$ and $k < 2$.

[*2008 Commonwealth Sec AMaths P2 (modified)*]

29 Express $y = \frac{6x^3+4x^2-3x}{3x+2}$ in partial fractions and hence find $\int y\,\mathrm{d}x$.

[*2008 Commonwealth Sec AMaths P2 (modified)*]

30(i) Evaluate $\int_0^{\frac{\pi}{4}} \left(\cos^2 x - \sin^2 x\right)\mathrm{d}x$.

(ii) Evaluate $\int_0^2 \left(2 - \frac{4}{3x+2}\right)\mathrm{d}x$.

[*2008 Crescent Girls' AMaths P1 (modified)*]

31(i) Express $\dfrac{x-5}{x^2-4x+4}$ in partial fractions.

(ii) Hence, evaluate $\int_4^6 \dfrac{x-5}{x^2-4x+4}\,\mathrm{d}x$.

[*2008 Crescent Girls' AMaths P2*]

32(i) Express $\dfrac{x^2+4x-7}{x^2+x-12}$ in partial fractions.

(ii) Hence, evaluate $\int_4^5 \dfrac{x^2+4x-7}{x^2+x-12}\,\mathrm{d}x$.

[*2008 Holy Innocents High AMaths P2*]

33 Evaluate $\int_0^{\frac{\pi}{6}} (2\cos x + \sin 2x)\,\mathrm{d}x$.

[*2008 Holy Innocents High AMaths P1*]

34(a) Given that $\int_1^3 \mathrm{f}(x)\,\mathrm{d}x = \int_3^6 \mathrm{f}(x)\,\mathrm{d}x = 7$, find the value of A where

$$A = \int_1^6 \left[2\,\mathrm{f}(x) + x\right]\mathrm{d}x.$$

(b) Differentiate $\dfrac{x^2}{2x-1}$ with respect to x, and hence evaluate $\int_1^2 \dfrac{x(x-1)}{(2x-1)^2}\,\mathrm{d}x$.

[*2008 Maris Stella High AMaths P2 (modified)*]

35 Evaluate

(i) $\int_0^1 \dfrac{1}{\sqrt{1+3x}}\,\mathrm{d}x$,

(ii) $\int_0^{\frac{\pi}{3}} (\cos 3x - \sin x)\,\mathrm{d}x$.

[*2008 Ngee Ann Sec AMaths P1*]

36 Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}(2x\sin^2 x) = 1 - \cos 2x + 2x\sin 2x$.

Hence, evaluate $\int_{\frac{\pi}{2}}^{\pi} x\sin 2x\,\mathrm{d}x$, leaving your answer in terms of π.

[*2008 Ngee Ann Sec AMaths P2*]

37(i) Express $\dfrac{2x^2+7x+4}{x^2+3x+2}$ in partial fractions.

(ii) Hence, evaluate $\int_1^2 \dfrac{2x^2+7x+4}{2x^2+6x+4}\,\mathrm{d}x$.

[*2008 Singapore Chinese Girls' AMaths P2*]

38 Given that $y=\sqrt{5-4\mathrm{e}^{x}}$, show that $y\dfrac{\mathrm{d}y}{\mathrm{d}x}+2\mathrm{e}^{x}=0$.

Hence, or otherwise, evaluate $\displaystyle\int_{\ln\frac{1}{2}}^{0}\frac{\mathrm{e}^{x}}{\sqrt{5-4\mathrm{e}^{x}}}\,\mathrm{d}x$.

[*2008 Singapore Chinese Girls' AMaths P2*]

39 Given that $y=\dfrac{(\sin x-\cos x)}{\mathrm{e}^{x}}$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2\cos x}{\mathrm{e}^{x}}$.

Hence, show that $\displaystyle\int_{0}^{\pi}\frac{\cos x}{\mathrm{e}^{x}}\,\mathrm{d}x=\frac{1}{2}\left(\frac{1}{\mathrm{e}^{\pi}}+1\right)$.

[*2008 Temasek Sec AMaths P2*]

40 Evaluate $\displaystyle\int_{0}^{\pi}\left[\mathrm{e}^{3x-1}+\sec^{2}\left(2x+1\right)\right]\mathrm{d}x$, correct your answer to 2 decimal places.

[*2008 Temasek Sec AMaths P1*]

41 Differentiate $\left(x+5\right)\ln\left(x+5\right)$. Hence, evaluate $\displaystyle\int_{0}^{3}\frac{\ln(x+5)}{5}\,\mathrm{d}x$, correct to three significant figures.

[*2008 Unity Sec AMaths P1*]

42 Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x\sqrt{4+3x^{2}}\right)=\dfrac{4+6x^{2}}{\sqrt{4+3x^{2}}}$. Hence, evaluate $\displaystyle\int_{0}^{2}\frac{2+3x^{2}}{\sqrt{4+3x^{2}}}\,\mathrm{d}x$.

[*2008 Victoria School AMaths P2 (modified)*]

43 Express $\dfrac{4x-13}{(x+2)(2x-3)}$ in partial fractions.

Hence, or otherwise, evaluate $\displaystyle\int_{2}^{3}\frac{4x-13}{(x+2)(2x-3)}\,\mathrm{d}x$.

[*2008 Zhonghua Sec AMaths P1 (modified)*]

44(a) Differentiate $\ln\left(\cos 2x\right)$ with respect to x. Hence, or otherwise, evaluate $\displaystyle\int_{0}^{\frac{\pi}{6}}\tan 2x\,\mathrm{d}x$.

(b) Given that $\displaystyle\int_{1}^{6}\mathrm{f}\left(x\right)\mathrm{d}x=12$ and $\displaystyle\int_{4}^{6}\mathrm{f}\left(x\right)\mathrm{d}x=5$, evaluate $\displaystyle\int_{1}^{4}\left[3-\tfrac{3}{2}\mathrm{f}\left(x\right)\right]\mathrm{d}x$.

[*2008 Zhonghua Sec AMaths P2 (modified)*]

Answers

1 $p = 1.02$

2 $\dfrac{19-6x}{2\sqrt{4-x}}$; 54

3 1.79

4(i) $\dfrac{x-16}{2x^2-9x-5} = \dfrac{3}{2x+1} - \dfrac{1}{x-5}$ **(ii)** –0.696

5 7.25

6(i) $1 + \ln 2x$ **(ii)** 3.24 **(iii)** $x > \dfrac{1}{2\mathrm{e}}$ **(iv)** 2 units/s

7(a)(i) 1.65 **(ii)** 3 **(iii)** π

(b)(i) $k = 4$ **(ii)** $y = 3x + \dfrac{4}{x}$

8 $1 - \dfrac{\pi}{4}$

9(a) $\dfrac{1}{x} + \dfrac{3}{x^2} + \dfrac{1}{x-2}$ **(b)** $\ln 5 + \dfrac{2}{5}$

10(a) $\dfrac{\cos 5x + \cos 3x}{2}$; $\dfrac{1}{2}\left(\dfrac{\sin 5x}{5} + \dfrac{\sin 3x}{3}\right) + c$ **(b)** $\dfrac{16}{85}$

12(i) $\dfrac{3}{2x+1} - \dfrac{1}{x+2} + \dfrac{2}{x-2}$ **(ii)** 1.58

13(i) –2 **(ii)** $y = -\dfrac{1}{(2-x)^2} + 7$

14(i) $k = 2$ **(ii)** 0.732

15(a) $1 - \dfrac{3}{x+1} + \dfrac{2}{(x+1)^2}$ **(b)** 0.462

16 $y = -\dfrac{2}{(2x-1)^2} + 4$

17 $2(\cos x - x\sin x)$; 0.342

18 $\dfrac{5}{4(x+1)} + \dfrac{1}{5(x+2)} + \dfrac{11}{20(x-3)}$; $\dfrac{5}{4}\ln|x+1| + \dfrac{1}{5}\ln|x+2| + \dfrac{11}{20}\ln|x-3| + c$

19 $\dfrac{\pi}{8} - \dfrac{3}{4}$ or -0.357

20(a) $\dfrac{12}{5(2x+3)} - \dfrac{6}{5(x-1)} + \dfrac{1}{(x-1)^2}$; –0.0302 **(b)(i)** $m = 4\frac{1}{2}$; $n = -2$ **(ii)** $y = \dfrac{9}{2}x + \dfrac{2}{x} - 5$

21(a) $4\tan\frac{1}{4}x + c$ **(b)** $\dfrac{-6}{\sin 6x}$

22(a) $\dfrac{2}{x} + \dfrac{5}{x+2} - \dfrac{4}{x-2}$ **(b)** 48.49

23 $x + 2x\ln x$; 2.31

24(a) $\frac{2}{1-3x}+\frac{1}{2x+1}-\frac{3}{(2x+1)^2}$; $-\frac{2}{3}\ln|1-3x|+\frac{1}{2}\ln|2x+1|+\frac{3}{2(2x+1)}+c$ **(b)** $11\frac{5}{9}$

25(a)(i) $-\frac{2}{(3x-2)^4}+c$ **(ii)** $3x^4-4x^3+\frac{3}{2}x^2+c$ **(b)(ii)** $2(3x-2)\sqrt{x+1}+c$

26(i) $-\frac{1}{x}+\frac{1}{2(x+1)}+\frac{1}{2(x-1)}$ **(ii)** $\frac{1}{2}\ln|x+1|+\frac{1}{2}\ln|x-1|-\ln|x|+c$

27(ii) $2\sec^3 x-\sec x$ **(iii)** $\sqrt{3}+\frac{1}{2}\ln\left(2+\sqrt{3}\right)$

28 $\frac{-4\cos\theta}{(1+\sin\theta)^3}$; 0.524

29 $2x^2-1+\frac{2}{3x+2}$; $\frac{2x^3}{3}-x+\frac{2}{3}\ln|3x+2|+c$

30(i) 0.5 **(ii)** 2.15

31(i) $\frac{1}{(x-2)}-\frac{3}{(x-2)^2}$ **(ii)** $\ln 2-\frac{3}{4}$ or –0.0569 (to 3 s.f.)

32(i) $1+\frac{2}{x-3}+\frac{1}{x-4}$ **(ii)** 2.50

33 1.25

34(a) $39\frac{1}{2}$ **(b)** $\frac{2x(x-1)}{(2x-1)^2}$; $\frac{1}{6}$

35(i) $\frac{2}{3}$ **(ii)** $-\frac{1}{2}$

36 $-\frac{3\pi}{4}$

37(i) $2-\frac{1}{x+1}+\frac{2}{x+2}$ **(ii)** 1.08

38 0.366

40 1519.42

41 1 + ln(*x* + 5) ; 1.12

42 4

43 $\frac{3}{x+2}-\frac{2}{2x-3}$; –0.429

44(a) 0.347 **(b)** –1.5