Chapter 4 FORCES

Content

- Types of forces
- Equilibrium of forces
- Turning effects of forces
- Centre of gravity

Learning Outcomes

Candidates should be able to:

(a) recall and apply Hooke's law ($F = kx$, where k is the force constant) to new situations or to solve related problems.

(b) describe the forces on mass, charge and current-carrying conductor in gravitational, electric and magnetic fields, as appropriate.
(*To be covered in later topics.*)

(c) show qualitative understanding of normal contact forces, frictional forces and viscous forces including air resistance. (No treatment of the coefficients of friction and viscosity is required).

(d) show an understanding that the weight of a body may be taken as acting at a single point known as its centre of gravity.

(e) define and apply the moment of a force and the torque of a couple.

(f) show an understanding that a couple is a pair of forces which tends to produce rotation only.

(g) apply the principle of moments to new situations or to solve related problems.

(h) show an understanding that, when there is no resultant force and no resultant torque, a system is in equilibrium.

(i) use a vector triangle to represent forces in equilibrium.

(j) derive, from the definitions of pressure and density, the equation $p = \rho gh$. (*Not in H1*)

(k) solve problems using the equation $p = \rho gh$. (*Not in H1*)

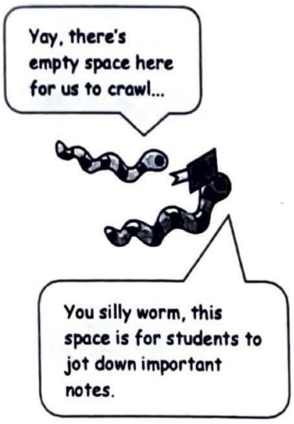
(l) show an understanding of the origin of the force of upthrust acting on a body in a fluid. (*Not in H1*)

(m) state that an upthrust is equal in magnitude and opposite in direction to the weight of the fluid displaced by a submerged or floating object. (*Not in H1*)

(n) calculate the upthrust in terms of the weight of the displaced fluid. (*Not in H1*)

(o) recall and apply the principle that, for an object floating in equilibrium, the upthrust is equal in magnitude and opposite in direction to the weight of the object to new situations or to solve related problems. (*Not in H1*)

4.1 Introduction



To date, physicists recognize four fundamental forces in nature:

1. The gravitational force
2. The electromagnetic force
3. The strong nuclear force } not in the syllabus
4. The weak (nuclear) force } not in the syllabus

All other forces we know can be derived from these four fundamental forces. Other than the gravitational force, most forces such as pushes, pulls and other contact forces like the normal force and friction, can be considered due to the electromagnetic force acting at the atomic level.

In the H2 Physics syllabus, you will learn that a mass experiences a gravitational force in a gravitational field, a charge experiences an electric force in an electric field and a current carrying conductor experiences a magnetic force in a magnetic field. These forces will be covered in later topics.

4.2 Types of Forces

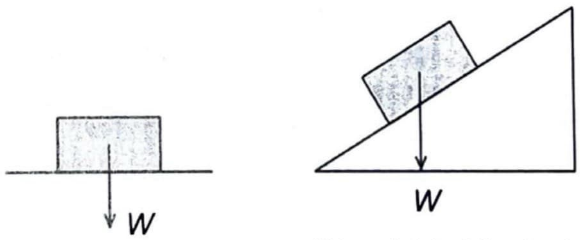
Weight

For a body near the surface of the Earth, its weight W, is defined as the force experienced by a body of mass m in a gravitational field and can be expressed as

$$W = mg$$

where g is the acceleration of free fall.

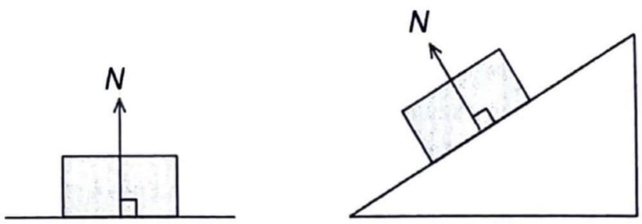
This force acts vertically downwards towards the centre of the Earth through a single point on the body known as its centre of gravity (c.g.).



W: weight of the body

Normal contact force

The normal contact force N is the force exerted on a body when it is in contact with a surface. The normal contact force is always perpendicular to the contact surface and points away from that surface.



N: normal contact force on the body

Friction

Further explanation of static, sliding/kinetic friction can be found in *Appendix A*.

Friction is the force that acts to oppose the relative motion or tendency of relative motion between two surfaces in contact. It acts parallel to the two surfaces in contact.

Advantages: Enables a person to walk and hold objects, rotating wheels to move without slipping etc.

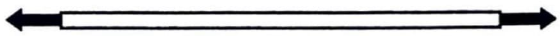
Disadvantages: Causes objects to wear out and energy wasted in the form of heat.

Examples of friction:

- Static friction.
- Sliding / kinetic friction.
- Rolling friction - resistance produced when a rolling body moves over a surface.
- Viscous forces or fluid friction - the friction between moving fluids or between fluids and solids.

Tension and Compression

When an object such as a bar (or rod or wire) is pulled at its ends, we say a force ***F*** is pulling on the bar, and the magnitude of the force is called the tension in the bar, often denoted by *T*.

A bar in tension: *F*  *F*

When the object is pushed on both ends, we say that the object is in compression.

A bar in compression: *F*  *F*

We often deal with springs, strings or ropes that are in tension i.e. experiencing a pulling force.

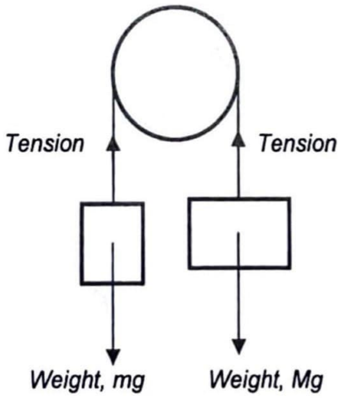
NOTE!

In this syllabus, we always assume that the springs, strings or ropes are massless. We simply regard them as a medium in which forces are transmitted.

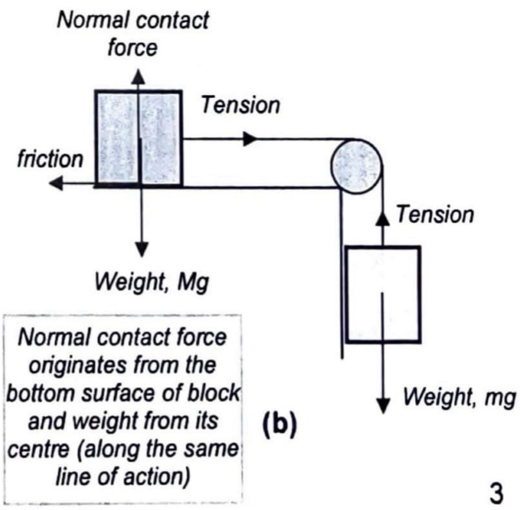
Whether or not the masses are accelerating, the tension *T*, on both sides of the rope will always be of the same magnitude as long as the rope remains taut.

Examples (Sketch and label the forces acting on each block)

You can use symbols (e.g. *N*, *T*) to label forces provided you have a legend for them.!!



(a)



(b)

Hooke's Law

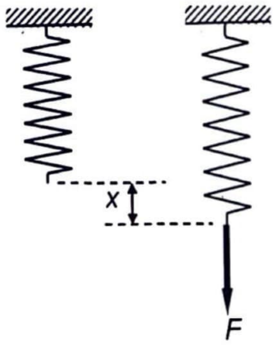
A spring or wire when stretched is found to obey Hooke's Law, up to a limit called its ***limit of proportionality***.

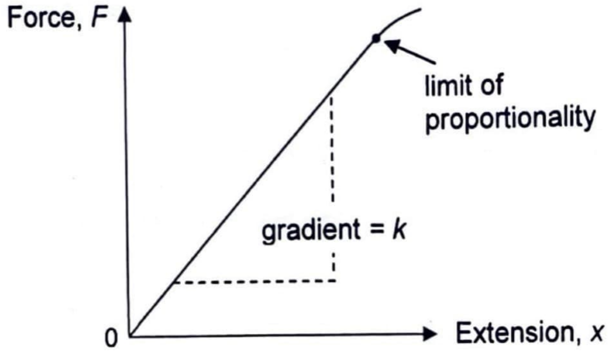
Definition

Hooke's Law states that the extension of a body is proportional to the applied load if the limit of proportionality is not exceeded.

Consider a spring suspended from the ceiling. It has an extension x when a force F is applied to it.

To investigate Hooke's law, a graph of applied force F against the spring extension x, is plotted.





Formula

Equation for Hooke's Law: $F = kx$

where k is the force constant or spring constant which is a measure of the stiffness of the spring, the value of which can be obtained by calculating the gradient of the force–extension (F–x) graph.

Energy stored in a spring

From the F–x graph, we can determine the energy stored in the spring when it is stretched.

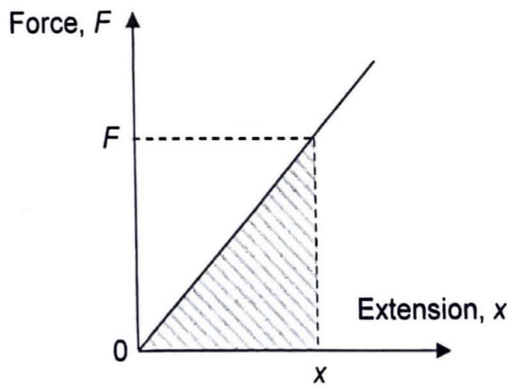
Energy stored in the spring

= Work done W in stretching spring

= Area under F–x graph

$= \frac{1}{2}Fx$

Since $F = kx$,



Formula

Energy stored in a spring $= \frac{1}{2}kx^2$

Example 1

A load of 50 N is suspended from a spring with a force constant of 1000 N m^{-1}. Calculate,

(a) the extension of the spring and
(b) the energy stored in the spring.

(a) Using Hooke's Law, $F = kx$.

$$50 = 1000x$$

$$x = 0.050 \text{ m}$$

(b) Energy stored in spring, $E = \frac{1}{2}kx^2$.

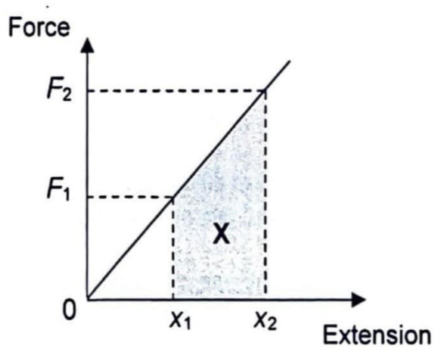
$$= \frac{1}{2}(1000)\left(5.0\times10^{-2}\right)^2$$

$$= 1.3 \text{ J}$$

Example 2

A load F_1 produces an extension of x_1 on a spring with force constant k while a larger load F_2 produces an extension of x_2.

Determine the work required to change the extension of the spring from x_1 to x_2.



Solution

Work done to change the extension of the spring

= Area of trapezium, X

Formula

= ½ $(F_1 + F_2)$ $(x_2 - x_1)$ *or*

avg. force   increase in length

= ½ $(F_2)(x_2)$ – ½ $(F_1)(x_1)$ *or*

= ½ $k(x_2)^2$ – ½ $k(x_1)^2$

This amount of energy is also the additional energy stored in the spring due to the increase in its extension.

Common student errors in this example:

NOTE!!

DO NOT express work done W as: $\frac{1}{2}k\left(\Delta x\right)^2 = \frac{1}{2}k\left(x_2 - x_1\right)^2$ ✖

$$W = \frac{1}{2}k(x_2)^2 - \frac{1}{2}k(x_1)^2$$

$$= \frac{1}{2}k(x_2^{\,2} - x_1^{\,2}) \neq \frac{1}{2}k(x_2 - x_1)^2$$

Fluid Statics

A fluid is any substance that can flow. Hence the term fluid is used for both liquids and gases.

Fluid statics is the study of fluids at rest and we begin by introducing three terms: *density, pressure* and *upthrust.*

Definition

The ***density*** of a substance is defined as its mass per unit volume.

If the mass m of a substance has volume V, its density ρ is given by

Formula

$$\rho = \frac{m}{V}$$

Density depends on factors such as temperature and pressure.

For liquids, density varies very little over wide ranges in temperature and pressure. For gases, density is very sensitive to changes in temperature and pressure.

Definition

Pressure is defined as the force per unit area, where the force is acting at right angles to the area.

Derivation of $p = \rho gh$

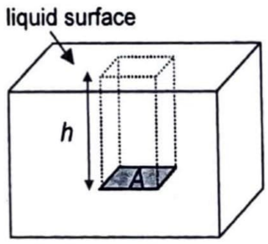
The link between pressure p and density ρ comes about when we deal with liquids or with fluids in general.

Consider a point at a depth h below the surface of a liquid. The pressure due to the liquid at that point is due to the weight of the column of liquid above it.

For a plane surface of area A at a depth h, the weight of the liquid column above it is given by,

$$W = m_{column}g$$
$$= (\rho A h)g$$

Hence, pressure $p = \frac{F}{A} = \frac{W}{A} = h\rho g$



Formula

In general, $p = \frac{F}{A}$

Pressure in a fluid (hydrostatic pressure) is given by:

$$p = \rho g h$$

Taking into account the atmospheric pressure p_{atm}, the total pressure P is

$$P = p_{atm} + \rho g h$$

<u>where</u>

F : is the normal force acting on the object

A: is the surface area of the object

h: is the <u>vertical</u> depth below the fluid surface

ρ: is the density of the fluid

g: is the acceleration of free fall.

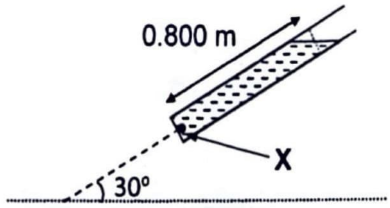
** $p = \rho g h$ is valid for fluids whose density is constant and does not change with depth, i.e. for an incompressible fluid.

- Pressure is a scalar quantity.
- Units of pressure: N m^{-2} or Pascal (Pa)
  atmosphere (atm) where 1 atm = 1.013×10^5 Pa.
- It is an empirical fact that a fluid can exert a pressure in any direction. At any point in a fluid at rest, the pressure is the same in all directions at a given depth given by $p = h\rho g$.

Example 3

A long narrow tube is filled with water of density 1000 kg m^{-3} to a depth of 0.800 m. The tube is then inclined at an angle of 30° to the horizontal as shown below.
If the atmospheric pressure is 100 kPa, what is the pressure at point X, inside the tube?

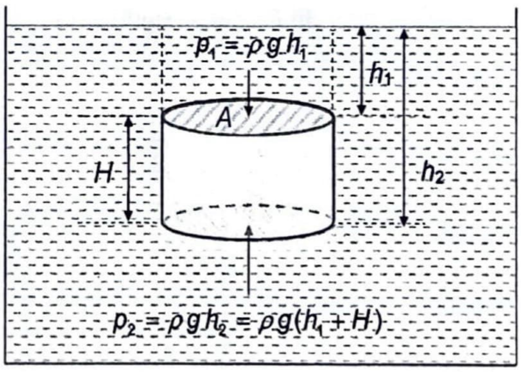
Solution

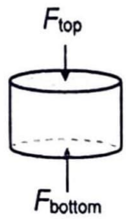


$$P_X = p_{atm} + \rho g h$$
$$= (100 \times 10^3) + (1000)(9.81)(0.800 \sin 30°)$$
$$= 104 \text{ kPa}$$

Upthrust

Consider a cylindrical object of cross–sectional area A submerged in a fluid of density ρ. The fluid exerts a pressure on every part of the object's surface which is in contact with the fluid. Let the top and bottom of the object be a distance h_1 and h_2 below the surface of the fluid respectively.





The downward force on the top of the object due to the fluid is

$$F_{top} = p_1 A = \rho g h_1 A$$

The upward force on the bottom of the object due to the fluid is

$$F_{bottom} = p_2 A = \rho g (h_1 + H) A$$

Since the magnitude of the upward force > downward force (i.e. $F_{bottom} > F_{top}$),

Net force of the fluid on the cylindrical object
$= F_{bottom} - F_{top}$
$= \rho g A H \quad \uparrow$
$= \rho g V$ ($\because$ volume of object, $V = AH$)
$= mg \quad (\rho = \frac{m}{V}$ where m is the mass of fluid displaced)

This **net** upward force is called **upthrust**.

Definition

Upthrust is the vertical upward force exerted on a body by a fluid when it is fully or partially submerged in the fluid due to the difference in fluid pressure.

It is equal in magnitude and opposite in direction to the weight of the fluid displaced by a submerged or floating object.

Archimedes Principle states that the upthrust on a body in a fluid is equal to the weight of the fluid displaced by the body.

Formula

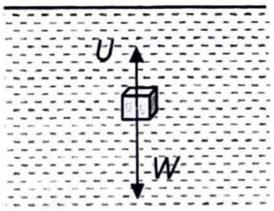
∴ Upthrust = Weight of fluid displaced (Archimedes Principle)

NOTE!!

Do not confuse upthrust with thrust. **Thrust** can be described as a force that is exerted on an object by the expulsion or acceleration of mass in one direction.

Case 1: A Sinking Object

Consider an object of volume V_0 and density ρ_0 that is fully submerged in a fluid of density ρ_f.



W: Weight of object = $\rho_0 V_0 g$

U: Upthrust on the object = $\rho_f V_0 g$

(note that both forces originate from the same point at the centre of the object)

If the object is denser than the fluid, i.e. $W > U$, the object will experience a net downward force given by

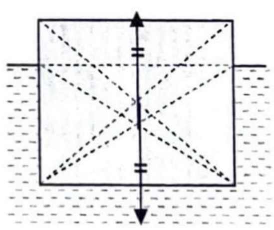
$$F_{net} = W - U$$

The object accelerates downwards i.e. it sinks.

Case 2: A Floating Object

Consider an object, of volume V_0 and density ρ_0 partially submerged in a fluid of density ρ_f. The object displaces a volume V_f of the fluid i.e. volume of the object beneath the fluid surface.

Upthrust, U



Weight, W

Note: for the object in equilibrium, the length of the arrows U and W must be the same. W originates from CG, and U originates from centre of buoyancy (i.e. centroid of fluid displaced).

Weight of the object, $W = \rho_o V_o g$

Upthrust on the object U = Weight of fluid displaced $= \rho_f V_f g$

Since the object is floating in equilibrium,

$$F_{net} = U - W = 0$$
$$\Rightarrow U = W.$$

Hence, weight of fluid displaced = weight of object.

Principle of Flotation

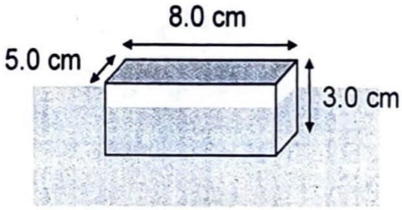
For an object floating in equilibrium, the upthrust is equal in magnitude and opposite in direction to the weight of the object.

NOTE!!

This implies that a floating object displaces its own weight of the fluid in which it floats.

Example 4

An aluminium block with dimensions 8.0 cm × 5.0 cm × 3.0 cm, is floating in mercury as shown.

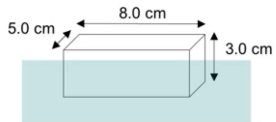


Determine,
1. the upthrust on the aluminium block,
2. the fraction of the aluminium block which is beneath the mercury surface.

Given: density of aluminium = 2700 kg m^{-3} and density of mercury = 13600 kg m^{-3}.

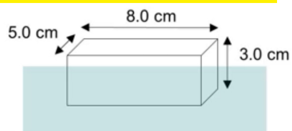
Solution

1. Since the block floats on mercury,



$$U = W$$
$$= V_{block} \rho_{al} g$$
$$= (0.050 \times 0.080 \times 0.030) \times 2700 \times 9.81$$
$$= 3.2 \text{ N}$$

2. $U = W$



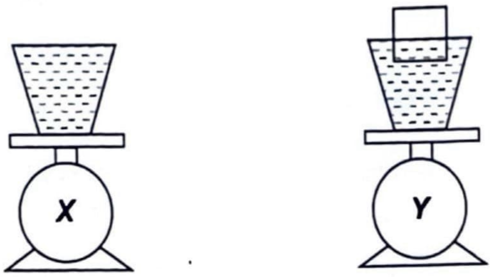
$$V_{submerged} \rho_{Hg} \not{g} = V_{block} \rho_{al} \not{g}$$

$$V_{submerged} / V_{block} = \rho_{al} / \rho_{Hg}$$

Fraction of block submerged $= V_{submerged} / V_{block}$
$= \rho_{al} / \rho_{Hg} = 2700 / 13600$
$= 0.20$

Example 5

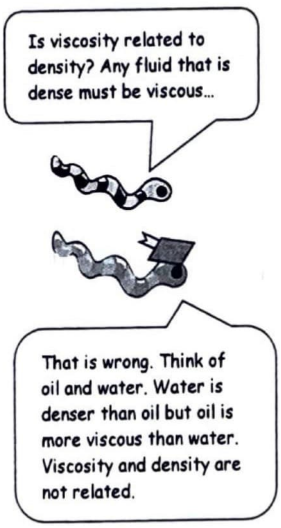
When a beaker of water filled to the brim rests on a balance, the weight indicated is X. A solid object is now placed into the beaker such that it displaces some water and floats on the remaining water. The spilt water is removed from the balance. The weight indicated is now Y. Which of the following is true?



A. $X > Y$
B. $X < Y$
C. $X = Y$
D. Unable to conclude anything.

C

Viscous Force



Viscous force is associated with the resistance that the object experiences due to a fluid when moving relative to each other. It can be understood by considering collisions of particles at the microscopic scale. When a body moves in a fluid, it collides continuously with the molecules of the fluid.

During each collision, the body imparts momentum to the molecule, exerting a force on it. By Newton's third law, the molecule exerts an equal and opposite force on the body. This force will tend to slow the body's motion. The viscous force is the combined force exerted by all the molecules on the body. It acts on any body moving through a fluid.

Air resistance is one commonly experienced viscous force as objects move through air.

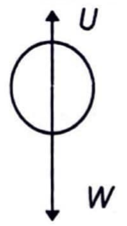
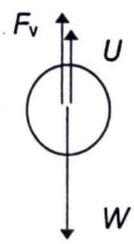
Viscous force depends on:
- the shape of the body,
- the speed of the body, and
- the viscosity of the fluid.

At low speeds, viscous force $F_v \propto v$ and at high speeds $F_v \propto v^2$.

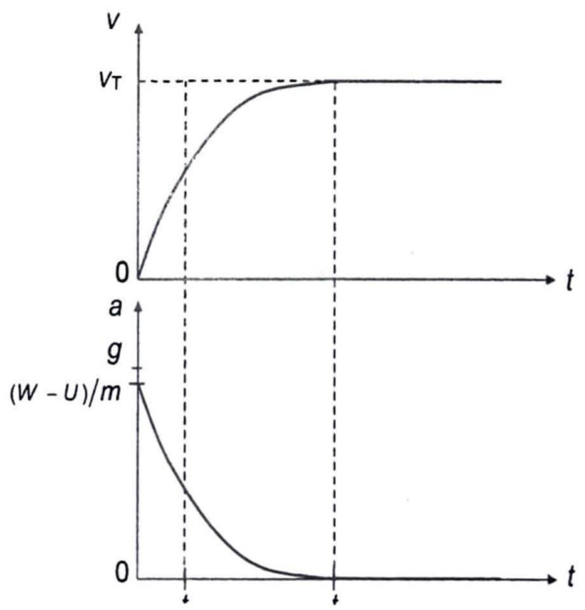
Object falling from rest in a fluid

Consider a sphere of mass *m* falling from rest in a fluid.

Taking downwards as positive:

t / s	*FBD*	*a* / ms^{-2}	*v* / ms^{-1}	Description
0		+ ve	0	• At $t = 0$ s, $v = 0$ m s^{-1} • ∴ no viscous force at this instant since $F_v \propto v$ $F_{net} = W - U$ $\Rightarrow ma = mg - m'g$ $\Rightarrow a < g$ (*m*': mass of fluid displaced)
t_1 (> 0)		+ ve (but ↓)	+ ve (↑ at a ↓ rate)	• sphere <u>accelerates downwards</u> • *v* increases hence • viscous force, F_v increases $F_{net} = W - U - F_v$ • sphere's velocity continues to increase • *a* decreases since F_{net} is smaller.
t_2 (> t_1)		0	+ ve (terminal velocity)	• *v* of the sphere increases further • $\Rightarrow F_v$ increases • F_{net} and *a* become smaller until $a = 0$. $F_{net} = 0 \Rightarrow W = U + F_v$ The sphere reaches a constant velocity called the **terminal velocity** v_T.

The $v - t$ and $a - t$ graphs are as follows:



Summary of Forces

Force	Direction
Weight	Acts vertically downwards towards the centre of Earth through an object's centre of gravity (c.g.).
Normal contact	Always perpendicular to the surfaces in contact.
Friction	Always opposite to the tendency or direction of motion, and parallel to the two surfaces in contact.
Tension	Always directed away from the object affected along string or rod.
Spring under tension	Always away from the object affected along the spring.
Upthrust	**Net upward force** exerted by a fluid on a body submerged fully or partially in the fluid.
Viscous force	Always opposite to the direction of relative motion of the object moving in a fluid.

4.3 Equilibrium of Forces

For a body to be in equilibrium, it must be in both translational and rotational equilibrium.

These are the two conditions necessary for a body to be in **equilibrium**,

1. **The resultant force on the body is zero i.e.** $\sum F = 0$.
2. **The resultant moment on the body about any axis is zero i.e.** $\sum \tau = 0$.

1st Condition: Translational Equilibrium

For a body to be in **translational equilibrium**, the resultant force (vector sum of the forces) acting on it must be zero.

Consider a particle that is subject to two forces, F_1 and F_2.

- The particle will move in the direction of the resultant of F_1 and F_2.
- The resultant force on the particle is zero only if F_1 and F_2 are equal in magnitude and opposite in direction.

Now consider 3 coplanar forces, F_1, F_2 and F_3 acting on a particle A as shown in Fig. 4(a).

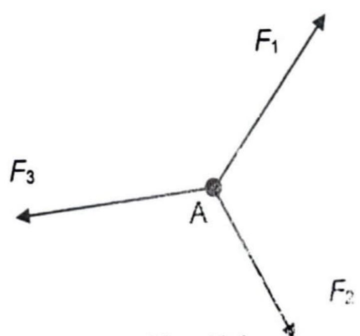


Fig. 4(a)

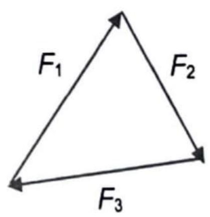


Fig. 4(b)

1st Condition: Translational Equilibrium (cont'd)

If the resultant force is zero, the vector sum of the three forces must be zero. The vector diagram of the three forces will be a closed triangle in one direction (see Fig. 4(b)).

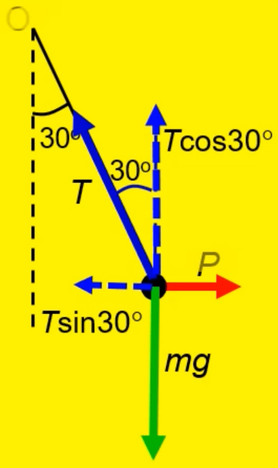
For a particle subject to more than three forces, this same principle applies. By drawing a polygon of forces, it will form a closed polygon in one direction.

Another method used to solve problems associated with objects in translational equilibrium:

- resolve all forces into two perpendicular components
- equating the sum of components in each direction to zero. i.e. $\sum F_x = 0$ and $\sum F_y = 0$.

Example 6

A 5.0 kg mass hangs from a fixed point O by a light inextensible string. It is pulled aside by a horizontal force *P* and rests in equilibrium with the string inclined at an angle of 30° to the vertical. Determine the magnitude of force *P*.



Resolving forces,

$\sum F_x = 0 \quad \Rightarrow \quad T\sin 30^0 = P \quad \cdots(1)$

(horizontally)

$\sum F_y = 0 \quad \Rightarrow \quad T\cos 30^0 = mg \quad \cdots(2)$

(vertically)

(1)/(2),

$\tan 30^0 = \dfrac{P}{mg}$

$P = mg\tan 30^0$

$= \ldots$

$= 28$ N

Example 7

Calculate the tension in each cord (A, B and C) in the figure below if the weight of the suspended mass is 200 N.

Method 1: Forming Force Triangle

$T_C = W = 200$ N

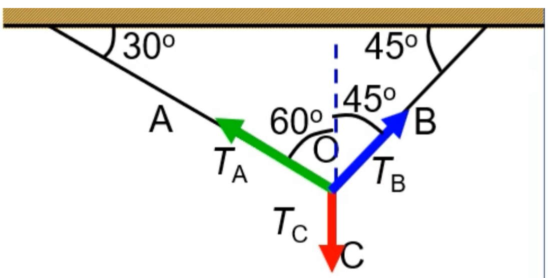
Using sine rule,

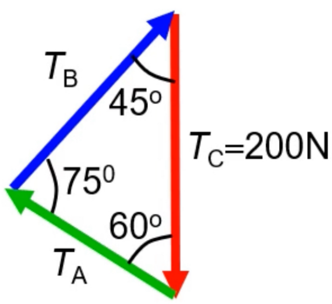
$\dfrac{T_A}{\sin 45^\circ} = \dfrac{200}{\sin 75^\circ}$

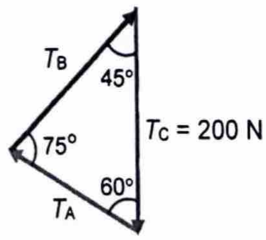
$T_A = 150$ N (2 s.f.)

$\dfrac{T_B}{\sin 60^\circ} = \dfrac{200}{\sin 75^\circ}$

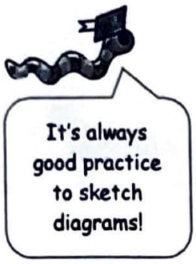
$T_B = 180$ N (2 s.f.)

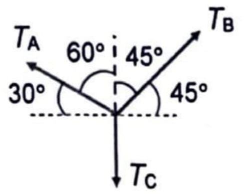






Since the system is in equilibrium, the vector diag. of the 3 tensions will form a closed triangle.





2nd Condition: Rotational Equilibrium

For a body to be in **rotational equilibrium**, the resultant moment on the body about any axis is zero.

Definition

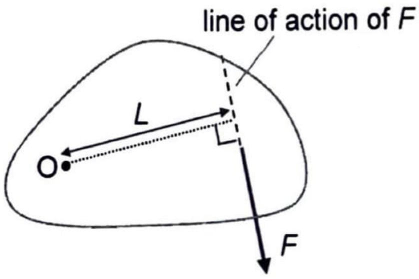
> **Principle of Moments**
> For a body in rotational equilibrium, the sum of all the clockwise moments about any axis must equal the sum of all the anticlockwise moments about the same axis.

Turning effect of a force
The turning effect of a force is called the moment of the force.

Definition

> The **moment** of a force about an axis is defined as the product of the force and the perpendicular distance of the line of action of the force from the axis.

Consider a force F acting on a rigid body in the plane of the paper so as to cause it to turn about an axis perpendicular to the paper, with pivot at O on the same plane as shown in Fig. 4(c).



Moment τ of the force = $L \times F$
Units of moment: N m

Fig. 4(c)

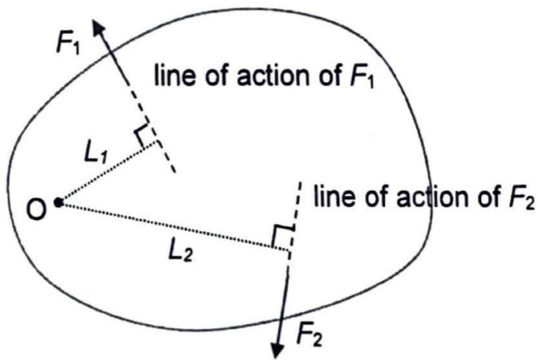


Fig. 4(d)

In Fig. 4(d), a rigid body is acted on by forces F_1 and F_2 lying in the plane of the paper.

F_1 would produce a counter-clockwise rotation, whilst F_2 a clockwise rotation about O.

If we take clockwise moment as positive, anticlockwise moment would be negative.

The net moment of the two forces about O is therefore given by

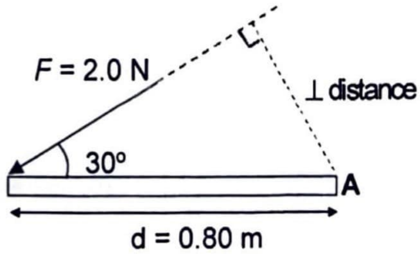
$$\tau = F_2L_2 - F_1L_1$$

Example 8

A force F is applied to a rod. Calculate the moment of F about the end A of the rod.

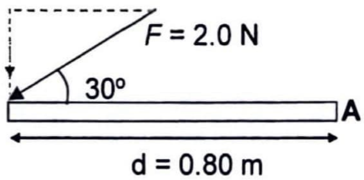
Solution

$$\begin{aligned}\tau &= F \times \text{perpendicular distance} \\ &= (2.0)(0.80\sin 30^\circ) \\ &= 0.80 \text{ N (anticlockwise)}\end{aligned}$$



or

$$\begin{aligned}\tau &= \text{perpendicular component of } F \times \text{d} \\ &= (2.0\sin 30^\circ)(0.80) \\ &= 0.80 \text{ N (anticlockwise)}\end{aligned}$$



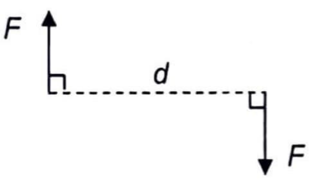
Torque of a Couple

Definition

A **couple** consists of a pair of **equal** and **opposite** forces whose lines of action do not coincide.

Definition

Torque of a couple is the product of the magnitude of one of the forces and the perpendicular distance between the forces.



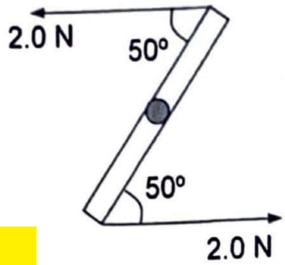
Torque of couple = $\boxed{Fd}$

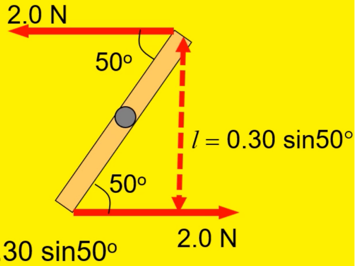
A body acted on by a couple has zero resultant force hence body is in <u>translational equilibrium</u>. However, the couple will produce a turning effect hence the body is <u>**NOT** in rotational equilibrium</u>.

Overall, we say that the body is <u>not in equilibrium</u>.

Example 9

A ruler of length 0.30 m is pivoted at its centre. Equal and opposite forces of magnitude 2.0 N are applied to the ends of the ruler, creating a couple as shown. Determine the magnitude of the torque produced by the couple on the ruler when it is in the position shown.
(N99/I/5)





Torque of the couple $= F\,l$

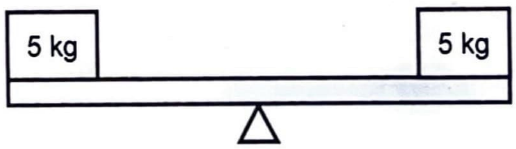
$= 2.0 \times 0.30 \sin 50^\circ$

$= 0.46$ N m (anti-clockwise)

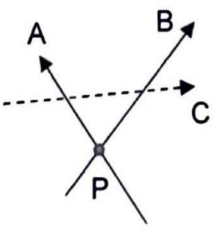
NOTE!

When only three coplanar forces act on a body in equilibrium, their lines of action must either
(i) all be parallel or
(ii) all meet at a point.

Scenario 1 (parallel forces)



Scenario 2 (non-parallel forces)



Since the object is in rotational equilibrium, when taking moments about any axis, net moment should always be zero.

Why is this so?

When three coplanar forces act on an object, the lines of action of any two forces (e.g. A and B, non-parallel) must certainly meet at a point (e.g. P).

If the line of action of the third force C does not meet at the same point P, force C will cause a moment about point P and the object cannot be in rotational equilibrium.

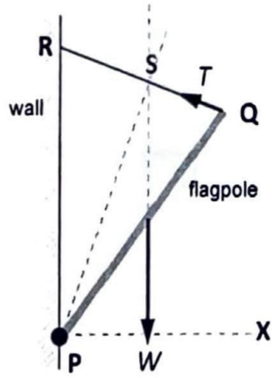
Hence, for object to be in equilibrium, the lines of action of all three forces must meet at a point.

Example 10

The diagram shows a heavy flagpole PQ hinged at a vertical wall at end P and held by a wire connected between end Q and a point R on the wall. The weight of the flagpole is *W* and the tension in the wire is *T*.

What is the direction of the force *F* exerted by the wall on the flagpole?

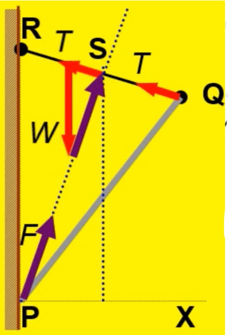
(A) PQ (B) PS (C) PX (D) QP (E) SP



(B)

Solution

- Since the lines of action of *T* and *W* meet at S, the line of action of *F* must be along PS.
- By the closed triangle of forces , *F* should be directed from P to S.

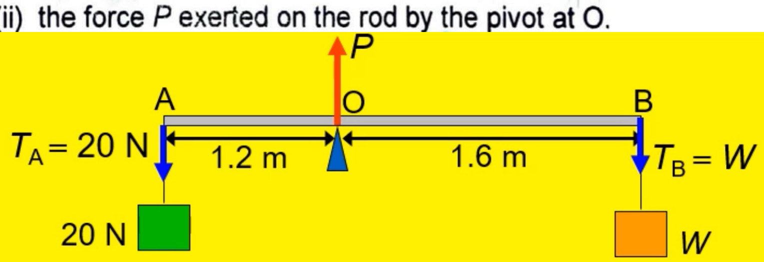


Example 11

A rigid rod of negligible weight is pivoted at point O and carries a block of weight 20 N at end A. Find

(i) the weight *W* of a second block that must be attached at end B if the rod is to be in equilibrium, and

(ii) the force *P* exerted on the rod by the pivot at O.



Using the Principle of Moments, taking moments abt pivot O,

Σ clockwise moment = Σ anti-clockwise moment

$T_B \times 1.6 = 20 \times 1.2$

$\therefore\ T_B = W = 15$ N

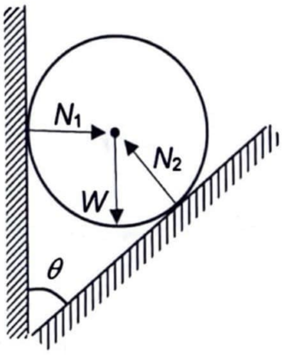
Since resultant force must be zero,

$\therefore P = 20 + 15 = 35$ N.

Example 12

A sphere of weight W is resting against two smooth walls which incline at an angle θ to each other.

Draw the forces acting on it and express the forces in terms of W and θ.



Solution

Resolving forces vertically,

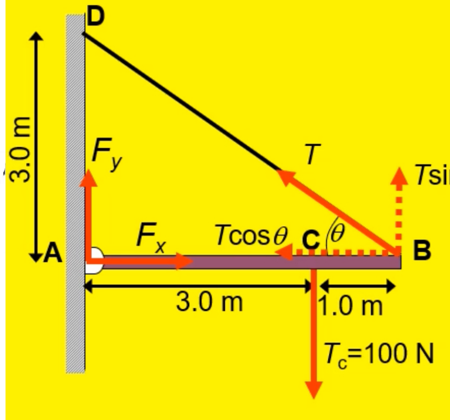
$$N_2 \sin\theta = W \Rightarrow N_2 = \frac{W}{\sin\theta}$$

Similarly, resolving forces horizontally,

$$N_1 = N_2 \cos\theta = \frac{W}{\sin\theta}\cos\theta = \frac{W}{\tan\theta}$$

Example 13

Determine the tension in the cable BD and the horizontal and vertical components of the force exerted on the strut AB at pin A. Assume that the strut is light and has negligible mass.



$$L_{BD} = \sqrt{3.0^2 + 4.0^2} = 5.0 \text{ m}$$

$$\therefore \sin\theta = \frac{3.0}{5.0} = 0.60 \text{ and}$$

$$\cos\theta = \frac{4.0}{5.0} = 0.80$$

Taking moments abt A,

anti-clockwise moment = clockwise moment.

$$(T\sin\theta)L_{AB} = T_C L_{AC}$$

$$T = \frac{T_C L_{AC}}{L_{AB}\sin\theta} = \frac{100 \times 3.0}{4.0 \times 0.60}$$

$$\therefore T = 125 \text{ N}$$

Resolving forces horizontally $\sum F_X = 0$,

$$F_X + (-T\cos\theta) = 0$$

$$F_x = T\cos\theta$$
$$= 125 \times 0.80$$
$$= 100 \text{ N (rightward)}$$

Resolving forces vertically $\sum F_Y = 0$,

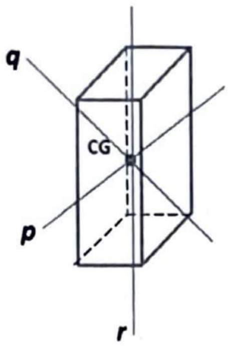
$$F_y + T\sin\theta = T_C$$

$$F_y = T_C - T\sin\theta$$
$$= 100 - (125 \times 0.60)$$
$$\therefore F_y = 25 \text{ N (upward)}$$

4.4 Centre of Gravity

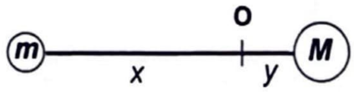
Definition

The **centre of gravity** of a body is the point at which its whole weight (or the resultant of the distributed gravitational attraction on the body) appears to act.



If the body were to be pivoted about its centre of gravity (c.g.), there would be no moment due to the weight of the body about any axis (e.g. *p, q, r*) passing through the c.g..

Consider the two point masses, *m and M*, joined by a light rod, placed in a uniform gravitational field.

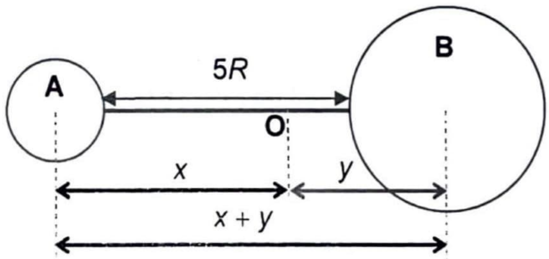


The c.g. O of the system is at a point such that

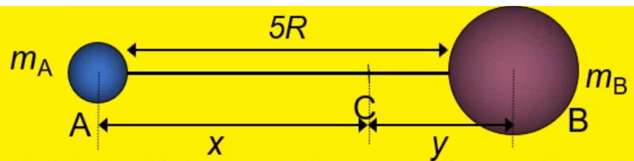
$$\text{Torque about C due to } m = \text{Torque about C due to } M$$
$$mgx = Mgy$$
$$mx = My$$

Example 14

Two spheres A and B of the same material and of radius *R* and 2*R* respectively are joined together by a light rod of length 5*R*. Assuming that the system of spheres is in a uniform gravitational field, determine the distance of the c.g. from the centre of sphere B.



Solution



Let *x* and *y* be the distance of the c.g. to the centres of A and B respectively.

$x + y = 8R$ .... (1)

Taking moments about the c.g.,

$m_A\, g\, x = m_B\, g\, y$

$\left(\frac{4}{3}\pi R^3 \rho\right) g\, x = \left(\frac{4}{3}\pi (2R)^3 \rho\right) g\, y$

$x = 8y$ .... (2)

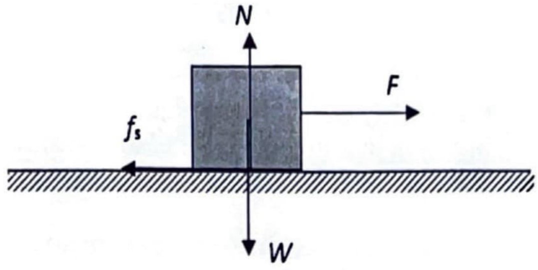
Solving eqn (1) and (2), $y = 0.89$ R.

Centre of gravity of system is 0.89*R* from the centre of sphere B.

Appendix A Types of Friction (not in Syllabus)

Static Friction

Consider a block resting on a rough surface. Apply a small horizontal force F to the right.



If the block remains at rest, static friction $f_s = F$ (Newton's 1st Law).

If F is increased and the block remains at rest, f_s increases.

Eventually when F becomes large enough, the block begins to slide. Thus there exists a maximum possible static friction force $f_{s,max}$ which we call the *limiting frictional force*.

Experimentally, $f_{s,max}$ has the following properties:

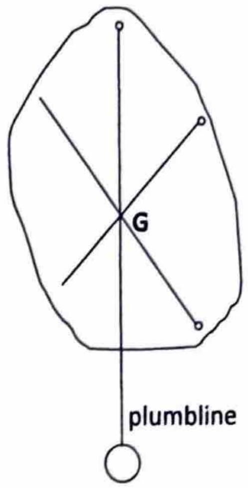
1. $f_{s,max}$ is independent of the surface area that is in contact.
2. For a given pair of surfaces, $f_{s,max} \propto N$. Hence $f_{s,max} = \mu_s N$ where μ_s is called the coefficient of static friction.
3. μ_s depends on the nature of the two surfaces.

Kinetic / Sliding Friction

The force necessary to keep an object sliding at constant velocity is smaller than that required to start it moving. Hence the sliding or kinetic friction force f_k is less than $f_{s,max}$. It is independent of the contact area and it satisfies $f_k = \mu_k N$ where μ_k is called the coefficient of kinetic friction. μ_k is nearly independent of velocity and $\mu_k < \mu_s$.

Appendix B Experimental Determination of Centre of Gravity

A procedure to find the C.G. of a lamina is as follows:



1. Make 3 small holes at well-spaced intervals round the edge of the lamina.

2. A pin is put through one of the holes and held firmly by a clamp and stand so that the lamina can swing freely. The lamina will come to rest with its centre of gravity, G, vertically below the point of support. The vertical line through the support can now be located by means of a plumbline. A plumbline is hung from the pin and the position of the thread marked on the lamina by two small pencil crosses. These crosses are joined by a pencil line.

3. Repeat Step 2 for the other two holes. Trace out the 3 lines and they should all intersect at G.

Tutorial 4 FORCES

Note: Unless stated in the question, take $g = 9.81$ m s^{-2}

Self-Check Questions

S1. Sketch the graph of force against extension for a spring that obeys Hooke's Law. How may the force constant be found from the graph? How may the energy stored in the spring be deduced from the graph?

S2. Define *density* and *pressure*.

S3. State what is *upthrust*?

S4. State *Archimedes' Principle* and the *Principle of Flotation*.

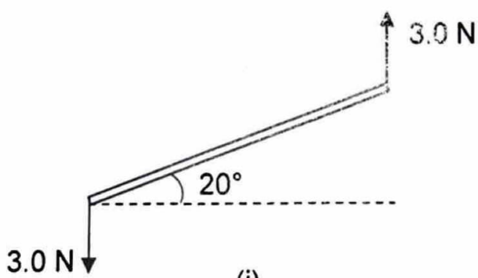
S5. What is meant by the centre of gravity of a body?

S6. Define (i) the *moment of a force*,
(ii) the *torque of a couple*.

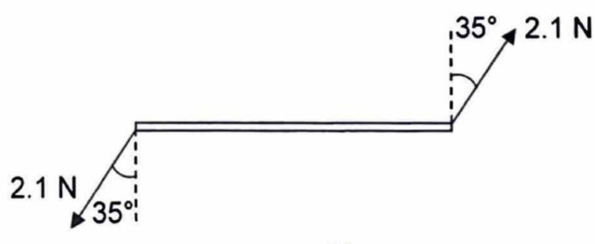
S7. State the conditions for equilibrium of a body which is acted upon by several forces.

Self-Practice Questions

SP1. Calculate the torque acting on the half metre rule in Fig (i) and (ii).

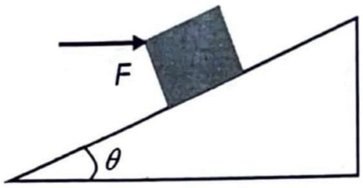


(i)



(ii)

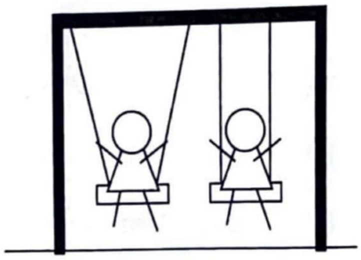
SP2. A block of mass $m = 2.0$ kg is held in equilibrium on a frictionless incline of angle $\theta = 60°$ by the horizontal force F.



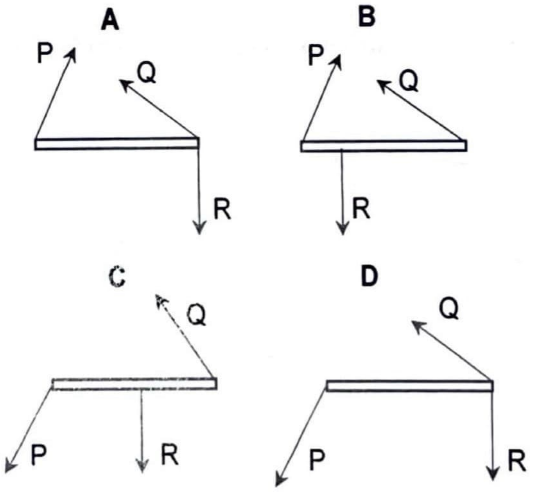
(i) Determine the normal force exerted by the incline on the block.
(ii) Determine the magnitude of F.

SP3. If identical twins of the same mass sit on the swings as shown, which swing is more likely to break?

Explain your answer.

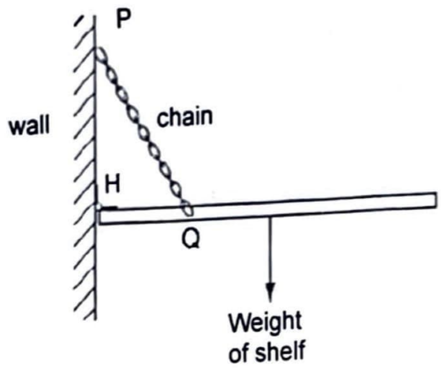


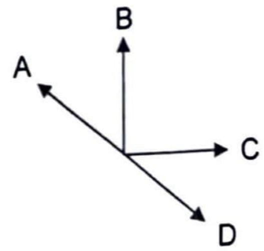
SP4. A light rod is acted upon by 3 forces P, Q and R. Which diagram shows the position and direction of each of the forces when the rod is in equilibrium.



(N05/I/7)

SP5. A hinged shelf is held horizontally against a wall by a chain PQ. The forces acting on the shelf are its weight, the force exerted by the chain and the force exerted by the hinge H. Which arrow could represent the direction of the force the hinge exerts on the shelf?

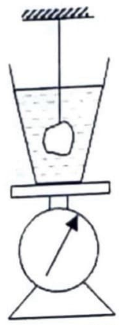




(N06/I/6)

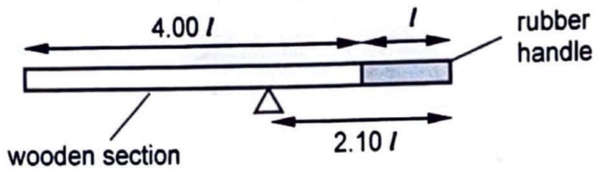
SP6. When a beaker of water rests on a balance, the weight indicated is *X*. A solid object of weight *Y* in air displaces weight *Z* of water when immersed.

What will be the balance reading, in terms of *X*, *Y* and/or *Z* when the object is totally immersed in the water as shown in the diagram?



(N84/II/8)

SP7. A uniform rod has a wooden section and a solid rubber handle as shown.



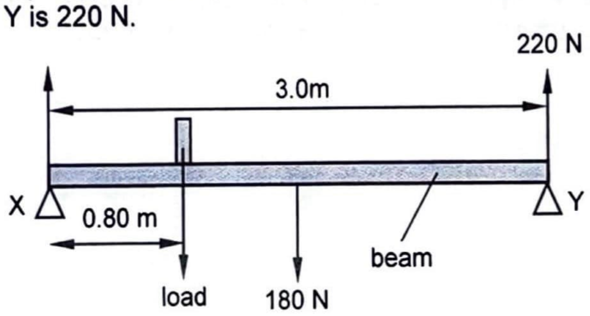
The length of the handle is *l* and the length of the wooden section is 4.00*l*. The rod balances a distance 2.10*l* from the rubber end.

What is the ratio $\frac{\text{density of rubbber}}{\text{density of wood}}$?

A 1.71 **B** 2.25 **C** 2.50 **D** 3.27

(N08/I/8)

SP8. A uniform beam in a roof structure has a weight of 180 N. It is supported in two places X and Y, a distance 3.0 m apart. A load is placed on the beam a distance of 0.80 m from X. The support provided by Y is 220 N.



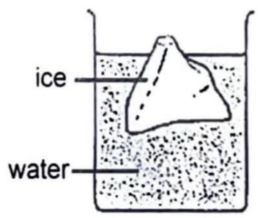
What is the value of the load?

A 270 N **B** 490 N **C** 520 N **D** 830 N

(N10/I/9)

SP9. A lump of ice floats in water as shown.

Which statement is correct?



A The lump of ice floats because the area of its lower surface is larger than the area of its upper surface.

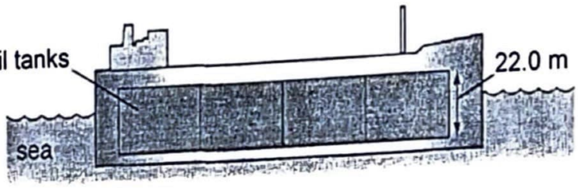
B The pressure difference between the lower and the upper surfaces of the lump of ice give rise to an upthrust equal to its weight.

C The ice has a greater density than the water.

D The mass of water displaced by the ice is equal to the upthrust.

(N07/I/8)

SP10. An oil tanker, with vertical sides, has an external cross-sectional area of 36500 m^2 in the plane of the sea.



The tanker carries oil of density 930 kg m^{-3} in its tanks, which have a constant cross-sectional area of 34000 m^2 and depth 22.0 m. Sea water has density 1030 kg m^{-3}.

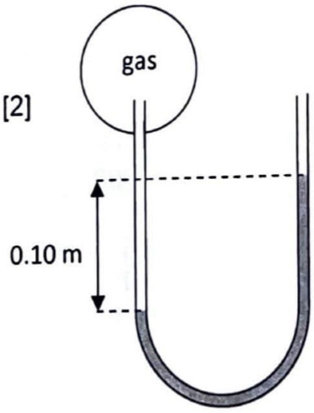
By how much does the tanker rise in the water when it unloads its oil?

A 26.2 m **B** 22.7 m **C** 21.3 m **D** 18.5 m

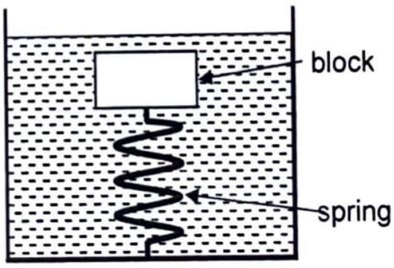
(N09/I/10)

Discussion Questions

D1. Find the pressure of the gas in the cylinder if the atmospheric pressure is 100 kPa, density of liquid is 13.6×10^3 kg m^{-3}. [2]



D2. One end of a light spring of force constant k = 150 N m^{-1}, is attached to the bottom of a tank filled with water. The other end of the spring was attached to a block of wood. The block-spring system is completely submerged in water and is at static equilibrium with the spring vertical, as shown. If the mass of the block is 2.0 kg and its density is 650 kg m^{-3}, calculate



(a) the extension of the spring [2]

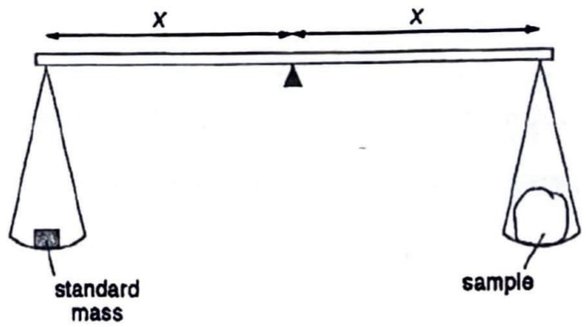
(b) the energy stored in the spring. [2]

[density of water = 1000 kg m^{-3}]

D3. **(i)** An object of mass *m* and density *d* is surrounded by air of density ρ. Show that the resultant force *F* acting downward on the object is given by

$$F = mg\left(1 - \frac{\rho}{d}\right)$$ [2]

(ii) A chemist uses an accurate balance to weigh a sample as show in the figure below.



The chemist ignores the effect of upthrust and records the mass of the sample as 0.17851 kg. The density of the sample is 940.0 kg m^{-3}, the density of the standard mass is 8493 kg m^{-3} and the density of air is 1.29 kg m^{-3}.

Calculate the actual mass of the sample. [4]

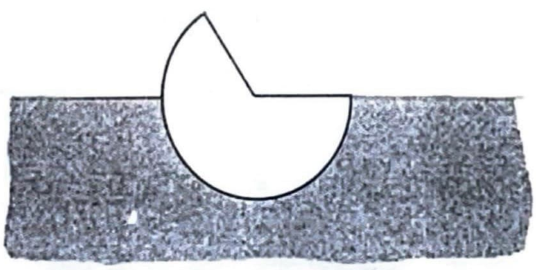
[N04/III/10(part)]

D4. **(a)** A block of ice of mass 150 kg is floating in a fresh-water lake. The density of water is 1000 kg m^{-3}. Determine

(i) the upthrust on the ice,
(ii) the weight of water displaced,
(iii) the volume of water displaced,
(iv) the volume of water produced when all the ice melts. [6]

(b) Explain why, when ice floating in a jug of water melts, there is no change in the water level. [2]

(c) The following figure shows an object that is not in equilibrium, partially submerged in wate



The density of the object is uniform and is less than the density of water.

(i) Make a rough copy of the figure. Draw arrows on your copy to show the weight of the object and the upthrust. Pay particular attention to the relative positions of the lines of action of the two forces.

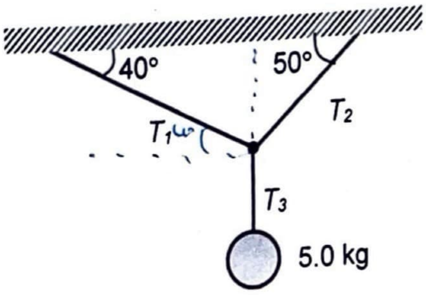
(ii) Describe what will happen to the object and suggest its approximate final position after it comes to equilibrium. [3]

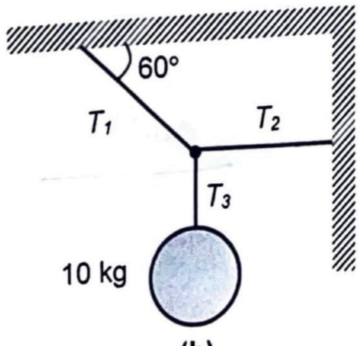
(N02/III/10)

D5. Drag is sometimes referred to as fluid friction. Describe a way in which drag and friction between solids are similar, and a way in which they differ. [2]

(N03/III/12(a))

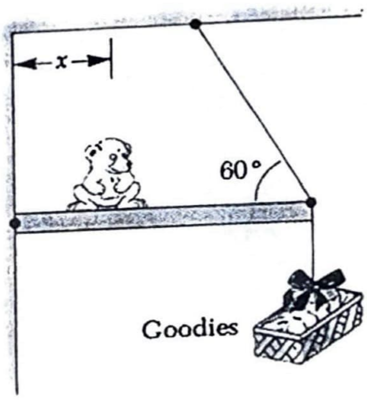
D6. Determine the tension in each cord for the systems shown below.





[6]

D7. A hungry bear weighing 700 N walks out on a uniform beam in an attempt to retrieve some "goodies" hanging at the end of the beam. The beam is 6.0 m long and weighs 200 N. The goodies weigh 80 N.

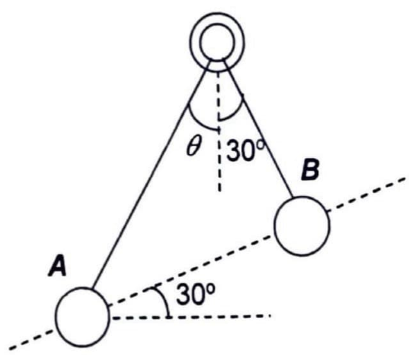


(i) Draw a diagram showing all the forces acting on the beam. [2]

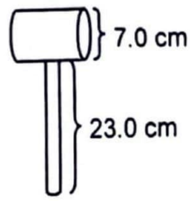
(ii) When the bear is at x = 1.0 m, find the tension in the wire and the contact force at the hinge. [3]

(iii) If the wire can withstand a maximum tension of 900 N, what is the maximum distance x the bear can walk before the wire breaks? [3]

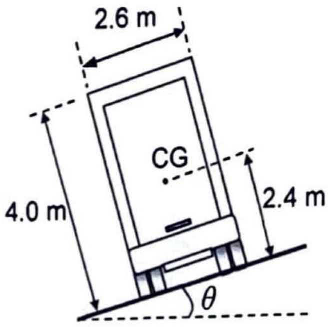
D8. Two spheres, A and B are connected by <u>different</u> strings. They have equal weight and are electrostatically charged such that the repulsive force between them has a magnitude of 3.0 mN and is directed along a line joining the centres of the two spheres. The spheres are in equilibrium as shown. Determine the weight W of each sphere, and hence determine the tension in the cords and the angle θ. [4]



D9. A mallet consists of a uniform cylindrical head of mass 1.80 kg and a diameter of 7.0 cm mounted on a uniform cylindrical handle of mass 0.45 kg and length 23.0 cm, as shown. How far above the bottom of the handle is the centre of gravity of the mallet? [3]

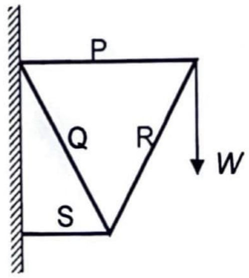


D10. The ~~centre of gravity~~ stability of a loaded truck depends on how the truck is parked. If it is 4.0 m high and 2.6 m wide and its C.G. is 2.4 m above the ground, how steep a slope can the truck be parked on without tipping over? [2]



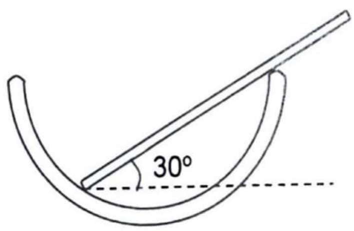
Challenging Questions

C1. To support a load W, four light hinged rods P, Q, R and S are connected as shown and mounted in a vertical plane.
Which rods are in compression and which in tension?



	In compression	*In tension*
A	P	Q,R,S
B	P,Q	R,S
C	Q,R	P,S
D	Q,R,S	P
E	R,S	P,Q

C2. A uniform rod rests in equilibrium at 30° with the horizontal within a fixed smooth hemispherical bowl. Draw in the forces acting on the rod. Find the forces at the points of contact with the bowl in terms of W, the weight of the rod.



Ans: $0.58\,W \left(\text{i.e. } \dfrac{W}{\sqrt{3}}\right)$

C3. Four bricks, each of length l, are put on top of one another (see Fig. C3(a) below) in such a way that part of each extends beyond the one beneath.

(a) Show that the largest equilibrium extensions are

(i) top brick overhanging the one below by $l/2$,

(ii) second brick from top overhanging the one below by $l/4$, and

(iii) third brick from top overhanging the bottom one by $l/6$.

(b) Determine a general formula for the maximum total distance spanned by n bricks if they are to remain stable.

(c) A builder wants to construct a corbelled arch (Fig. C3 (b)) based on the principle of stability discussed in (a) and (b) above. What minimum number of bricks, each 0.30 m long, is needed if the arch is to span 1.0 m.

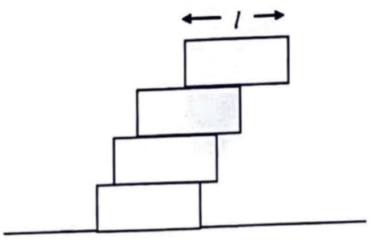


Fig. C3 (a)

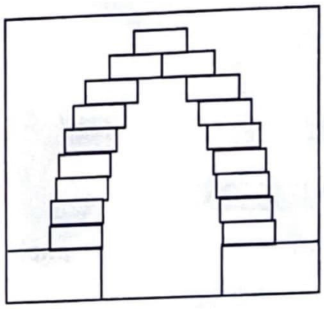


Fig. C3 (b)

Ans: **(b)** $\sum_{i=1}^{n} \frac{l}{2i}$ **(c)** 43

Answers

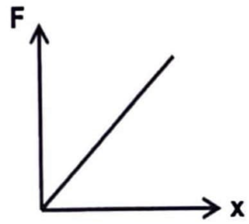
D1. 113 kPa
D2. (a) 0.070 m (b) 0.37 J
D3. (ii) 0.17873 kg
D4. (i) 1470 N, (ii) 1470 N, (iii) 0.15 m^3, (iv) 0.15 m^3
D6. (a) 31.5 N, 37.6 N, 49.1 N
(b) 113 N, 56.7 N, 98.1 N
D7. (ii) 700 N at 70° above the horizontal, (iii) 5.1 m
D8. 6.0 mN, 7.9 mN, 5.2 mN, 19.1°
D9. 23.5 cm
D10. 28°
C1. E
C2. 0.58 W
C3. L/2, L/4, L/6, $\sum_{i=1}^{n} \frac{L}{2i}$

Suggested Solutions

SELF-CHECK QUESTIONS

S1. Force constant = gradient of the graph.

Energy stored = area under graph.



S2. The **density** of a substance is defined as its mass per unit volume.

Pressure is defined as the magnitude of the normal force per unit surface area.

S3. The **force of upthrust** *is* the net vertical upward force exerted on a body by a fluid when it is fully or partially submerged in the fluid due to the difference in fluid pressure.

S4. **Archimedes' principle** states that the buoyant force (upthrust) is equal in magnitude and opposite in direction to the weight of the fluid that is displaced by a submerged or floating object.

The **principle of floatation** states an object floating in equilibrium, the upthrust is equal in magnitude and opposite in direction to the weight of the object.

S5. The **centre of gravity** of a body is the point at which its whole weight (or the resultant of the distributed gravitational attraction on the body) appears to act.

S6. **(i) The moment of a force** about a point is defined as the product of the force and the perpendicular distance of the line of action of the force from the pivot.

(ii) Torque of a couple is the product of one of the forces and the perpendicular distance between the forces.

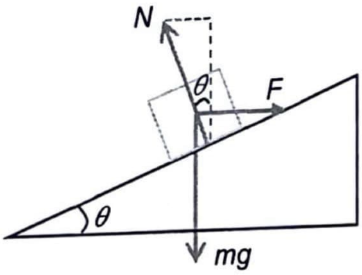
S7. The resultant force on the object is zero, i.e., $\sum F = 0$.
The resultant torque on the object about any axis is zero, i.e., $\sum \tau = 0$.

SELF-PRACTICE QUESTIONS

SP1. Draw dotted lines and extend each line of action of each force (for better visualization). Then draw a perpendicular line between each line of action of force.

(i) $\tau = F \times d = 3.0 \times 0.500 \cos 20° = 1.41$ N m
(ii) $\tau = F \times d = 2.1 \cos 35° \times 0.500 = 0.860$ N m

SP2.



Resolving N into its vertical and horizontal components which balances weight and F respectively.

(i) $N \cos\theta = mg \Rightarrow N = \dfrac{mg}{\cos\theta}$

$$N = \frac{(2.0)(9.81)}{\cos 60°} = 39.2 \text{ N}$$

(ii) $N \sin\theta = F$

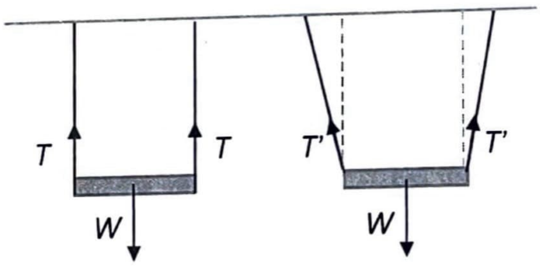
$F = (39.2)(\sin 60°) = 33.9$ N

SP3. The swing on the left is more likely to break.
The ropes of the swing on the left will be taking a larger tension because it has an angle with the vertical. This means that to carry the same weight, the tension in the rope has to be higher such that the sum of the vertical components of the tension in each rope is of the same magnitude as the weight.

$2T = W$ and $2T' \cos\theta = W$

$\Rightarrow T = T' \cos\theta$

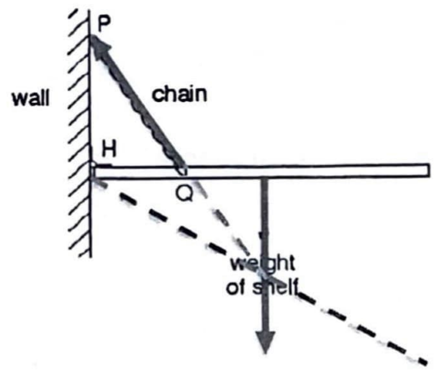
$\Rightarrow T' > T \quad (\because \cos\theta < 1)$



SP4. **B**
Since system is in equilibrium, the lines of action of the three non-parallel forces MUST intersect. Hence only option B and C are possible. Since the vector triangle must be a closed loop, therefore B is the correct option.

SP5. D

If three non-parallel forces cause a system to be in equilibrium, then the three forces MUST intersect. Thus, the force that the hinge exerts on the shelf MUST lie along the blue dotted line as shown. The force that the hinge exerts on the shelf points in the direction of D as its horizontal component (rightward) can then cancel the leftward horizontal component of the tension in the chain. (Vector triangle must be a closed loop.).



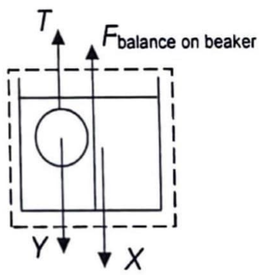
SP6. By Archimedes' principle, upthrust acting on object is the weight Z of displaced water.

Looking at the object as a system in equilibrium,

$Z + T = Y$

$\Rightarrow T = Y - Z$ ..... (1)

Looking at the beaker of water and object as a system in equilibrium,



$T + F_{\text{balance on beaker}} = Y + X$

$F_{\text{balance on beaker}} = Y + X - T$ ..... (2)

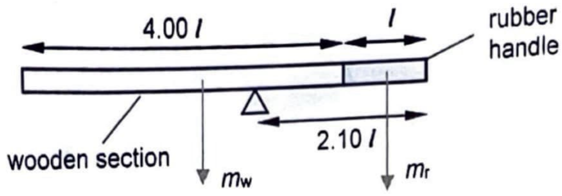
Sub (1) into (2),

$$F_{\text{balance on beaker}} = Y + X - T$$
$$= Y + X - (Y - Z)$$
$$= X + Z$$

By Newton's 3rd Law,

$F_{\text{beaker on balance}} = F_{\text{balance on beaker}} = X + Z$

SP7. B



Taking moments about the pivot:

$$m_w\left(5.00-2.10-\frac{4.00}{2}\right)l=m_r\left(2.10-\frac{1.00}{2}\right)l$$

$$m_w(0.90l)=m_r(1.60l)$$

$$\frac{m_r}{m_w}=\frac{0.90}{1.60}$$

$$\frac{\rho_r}{\rho_w}=\frac{m_r/(Al)}{m_w/\left[A(4.00l)\right]}=4\frac{m_r}{m_w}=4\left(\frac{0.90}{1.60}\right)=2.25$$

SP8. B

Taking moments about X

(Load)(0.80) + (180)(1.5) = (220)(3)

Load = 490 N

SP9. B

Upthrust arises due to the difference in pressure in a fluid.

*Note: option D is incorrect as it should be "the weight of water displaced" instead of "mass".

SP10. D

Since tanker is in equilibrium, the additional upthrust due to oil = weight of oil.

$$V\rho_{sea}g=(mg)_{oil}=V_{oil}\rho_{oil}g$$

$$V=\frac{V_{oil}\rho_{oil}g}{\rho_{sea}g}$$

$$=\frac{(34000\times 22.0)(930)(9.81)}{(1030)(9.81)}$$

$$h=\frac{V}{A}$$

$$=\frac{(34000\times 22.0)(930)(9.81)}{(1030)(9.81)\times 36500}$$

$$=18.5\text{ m}$$