Full Name	Class Index No	Class

Anglo-Chinese School (Barker Road)

PRELIMINARY EXAMINATION 2024
SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)

ADDITIONAL MATHEMATICS
4049
PAPER 1

2 HOURS 15 MINUTES

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

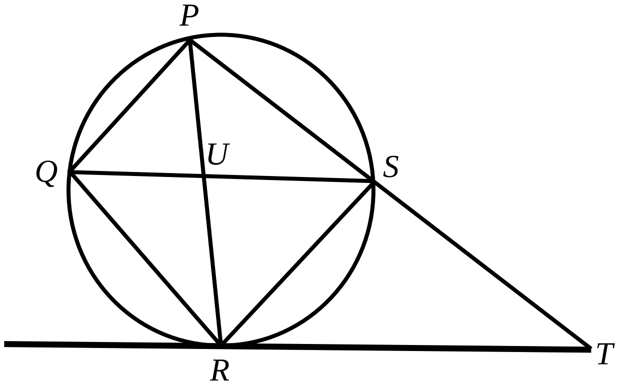
$$\Delta = \frac{1}{2} bc \sin A$$

1 The line $x - y = 3$ intersects the curve $x^2 - 3xy + y^2 + 19 = 0$ at the points P and Q. Find the coordinates of P and of Q. [5]

2 Solve the equation $2^{x+1}+3(2^{-x})=7$. [5]

3 Express the equation $\log_2 x + \log_4 (x-1) = 3$ as a cubic equation in x. [4]

4



The diagonals of a cyclic quadrilateral *PQRS* intersect at *U*. The tangent to the circle at *R* meets *PS* produced at *T*. If *QR* = *RS*, prove that

(a) *QS* is parallel to *RT*, [3]

(b) triangles *QUR* and *RST* are similar. [3]

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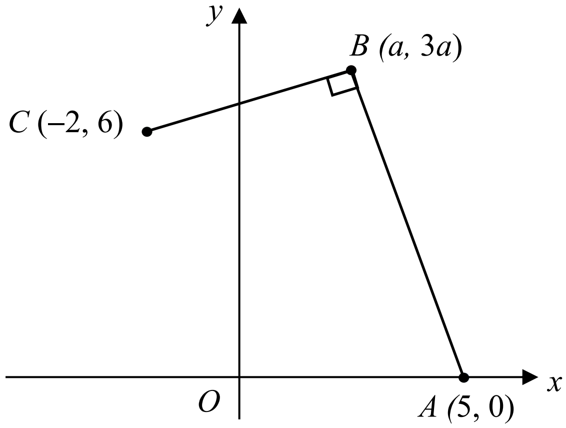
5



The diagram shows points $A(5, 0)$, $B(a, 3a)$ and $C(-2, 6)$ such that the line AB is perpendicular to the line BC.

(a) Show that $a = 2.5$. [3]

5 **(b)** Find the coordinates of the midpoint of *AC*. Hence find the coordinates of *D* such that *ABCD* is a parallelogram. [3]

(c) The area of triangle *AEC* is 5 units^2. Find the value of x given that the point *E* is $(x, 2x)$, where $x > 1$. [2]

6 **(a)** Express $\dfrac{13x-6}{x^2(2x-3)}$ in partial fractions. [5]

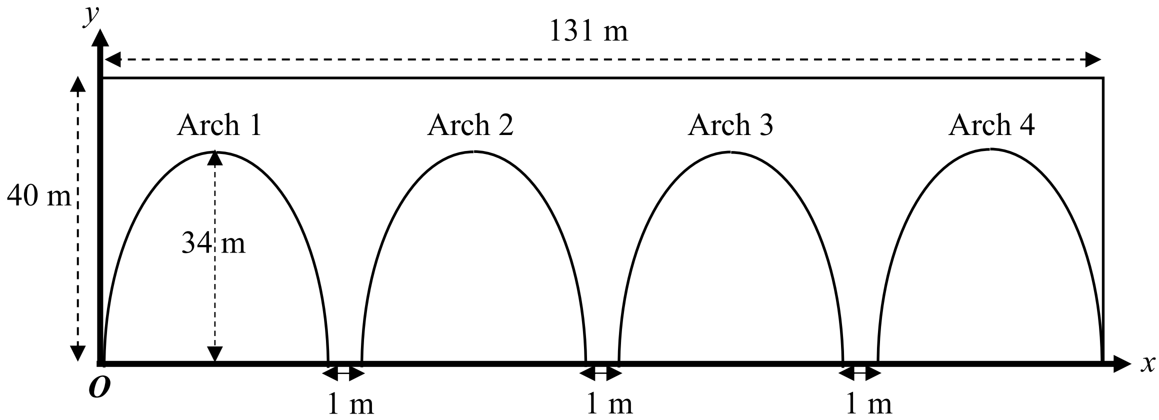
6 **(b)** Hence $\int \frac{13x-6}{x^2(2x-3)}\,\mathrm{d}x$. [3]

7 **(a)** **(i)** Write down and simplify, in ascending powers of x, the first four terms of the expansion of $(1-x)^{12}$. [2]

(ii) Hence find the value of p given that the coefficient of x^3 in the expansion of $(2x^2+17x+p)(1-x)^{12}$ is 6598. [3]

7 **(b)** In the binomial expansion of $\left(x+\dfrac{k}{x}\right)^5$, where k is a positive constant, the coefficients of x^3 and x are the same. Find the value of k. [4]

8



A civil engineer is designing a bridge which is 131 metres long, 40 metres high and is to have four identical parabolic arches along its length. Each arch is 34 metres high and there is one metre between bases of each adjacent pair of arches as shown in the diagram. A set of axes is placed with the origin at the left- hand end of the base of the first arch.

(a) Find the x-intercepts of Arch 1. [2]

(b) The equation representing Arch 1 can be written in the form $y = a(x-p)(x-q)$. Show that $a = -\frac{17}{128}$. [2]

8 **(c)** Explain, with working, if the point (50, 30) lies on Arch 2. [3]

9 **(a)** Differentiate $x\sin x+\cos x$ with respect to x. [2]

(b) Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1}{3}x\sin^3 x\right)=\dfrac{1}{3}\sin^3 x-x\cos^3 x+x\cos x$. [3]

9 **(c)** Using the results found in **part (a)** and **part (b)**, find $\int\left(\sin^3 x - 3x\cos^3 x\right)\mathrm{d}x$. [4]

10 **(a)** Prove the identity $\dfrac{\cos x}{\operatorname{cosec} x - 1} + \dfrac{\cos x}{\operatorname{cosec} x + 1} = 2\tan x$. [5]

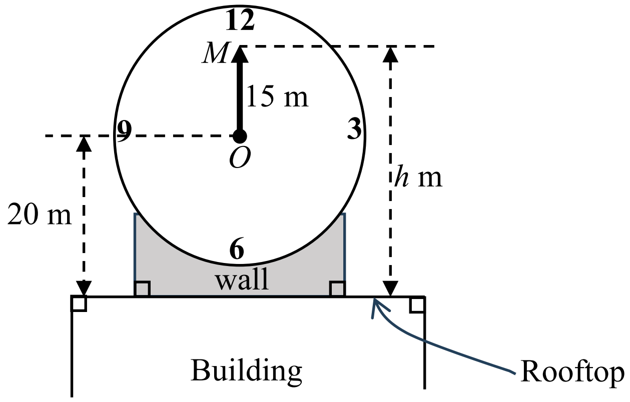
10 **(b)** Solve the equation $\tan x + 2\sec^2 x = 5$ for $0 \le x \le 2\pi$. [5]

11 **(a)** **(i)** Find the range of values of x for which $\ln(x^2 - 3)$ is defined. Leave your answer in surd form. [2]

(ii) Given that $y = \ln\left[e^x(x^2 - 3)\right]$, show that $\frac{\mathrm{d}y}{\mathrm{d}x}$ can be expressed in the form of $\frac{(x+a)(x+b)}{x^2 - 3}$. [3]

11 **(b)** It is given that $y = x^3 + px^2 + qx + 10$ where p and q are integers. The only values of x for which y is a decreasing function of x are those values for which $3 < x < 7$. Find the value of p and of q. [4]

12



A clock is set on the vertical wall on the roof of a building.
The distance from the centre of the clock, O, to the tip, M, of the minute hand is 15 m.
The height, h m, of M above the rooftop is given by $h = a\cos kt + b$, where t is the time in minutes past the hour. O is 20 m above the rooftop. The rooftop is taken to be horizontal.

(a) Write down the value of a and explain why b = 20. [2]

(b) Explain why the value of k is $\dfrac{\pi}{30}$. [1]

12 **(c)** Sketch the graph of h for $0 \leq t \leq 45$. [3]

(d) Find the two timings in the first hour for which the height of M is 10 m. [4]

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