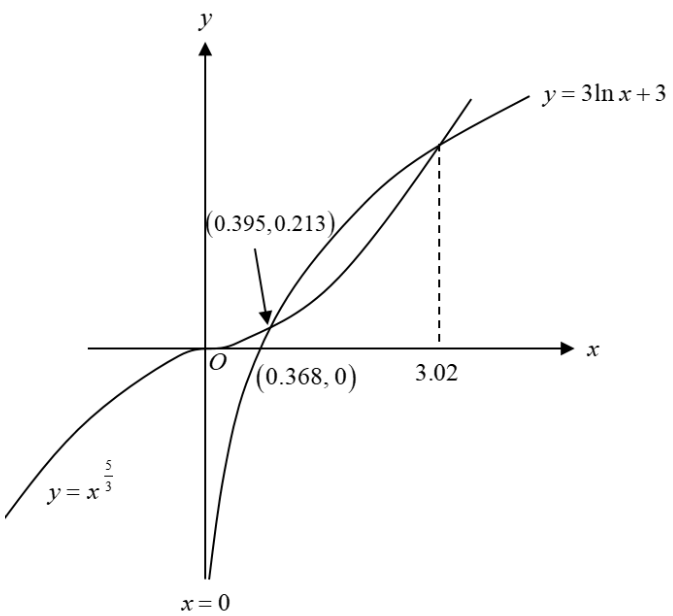
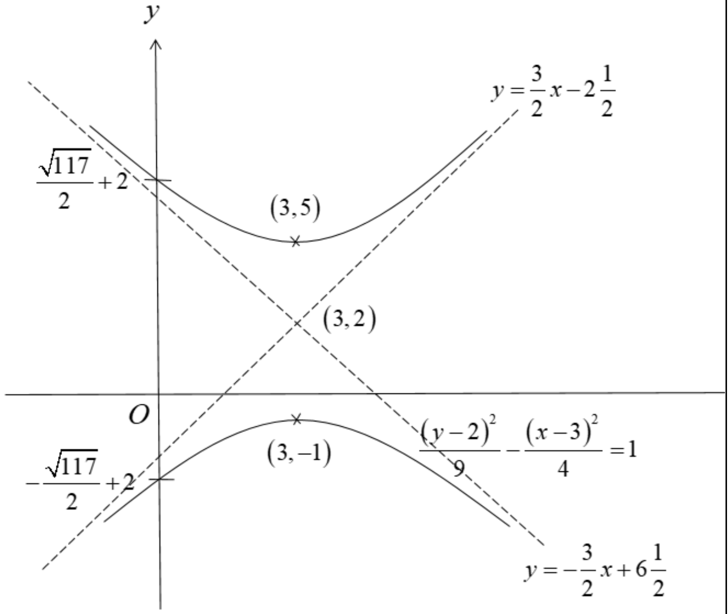
Qn	Solution	Notes
1i		$y = x^{\frac{5}{3}}$ has a stationary point at $(0,0)$ hence gradient of it's graph at the origin should be 0. Graph of $y = 3\ln x + 3$ extends down to infinity as x approaches 0. Equation of asymptote: $x = 0$ and coordinates of x- intercept: $(0.368, 0)$ need to be stated (required by question)
1ii	Graphs of $y = 3\ln x + 3$ and $y = x^{\frac{5}{3}}$ intersect at $x = 0.395$ and $x = 3.02$. $3\ln x + 3 \geq x^{\frac{5}{3}}$ $\therefore 0.395 \leq x \leq 3.02$	
2	$\left(3x^2 - y^2\right)\frac{\mathrm{d}y}{\mathrm{d}x} = 2xy \quad ---(1)$ Differentiating w.r.t. x $\left(3x^2 - y^2\right)\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(6x - 2y\frac{\mathrm{d}y}{\mathrm{d}x}\right)\frac{\mathrm{d}y}{\mathrm{d}x} = 2y + 2x\frac{\mathrm{d}y}{\mathrm{d}x}$ When $x = 0$, $y = 1$ $(0-1)\frac{\mathrm{d}y}{\mathrm{d}x} = 2(0)(1) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 0$ $(0-1)\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + (0 - 2(0))(0) = 2 + 2(0)(0) \Rightarrow \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -2$ $\therefore$ the Maclaurin's series for y is $y = 1 + (0)x + \frac{-2}{2!}x^2 + \ldots$ $= 1 - x^2 + \ldots$	Differentiate (1) immediately using product rule. No need to make $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject before differentiation.

3i	$\int e^{2x}\sin x \, dx$ $= \frac{1}{2}e^{2x}\sin x - \int \frac{1}{2}e^{2x}\cos x \, dx$ $= \frac{1}{2}e^{2x}\sin x - \frac{1}{2}\left[\left(\frac{1}{2}e^{2x}\cos x\right) - \int \frac{1}{2}e^{2x}(-\sin x)\, dx\right]$ $= \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x - \frac{1}{4}\int e^{2x}\sin x \, dx$ $\frac{5}{4}\int e^{2x}\sin x \, dx = \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x + D$ $\int e^{2x}\sin x \, dx = \frac{2}{5}e^{2x}\sin x - \frac{1}{5}e^{2x}\cos x + C$ $= \frac{1}{5}e^{2x}(2\sin x - \cos x) + C$	
3ii	Let $f(x) = e^{2x}(2\sin x - \cos x)$. Observe that $y = e^{2x+2}(2\sin(x+1) - \cos(x+1)) = f(x+1)$ is a translation of $y = f(x)$ by 1 unit in the negative x-direction. Hence, the gradient of the curve $y = f(x+1)$ at $x = \frac{\pi}{2} - 1$ is the gradient of the curve $y = f(x)$ at $x = \frac{\pi}{2}$, which is given by $y = f'\left(\frac{\pi}{2}\right)$. From part (i), $f'(x) = 5e^{2x}\sin x$. Hence, the required gradient is $f'\left(\frac{\pi}{2}\right) = 5e^{\pi}$.	

4i	Asymptotes: $\frac{(y-2)^2}{9}=\frac{(x-3)^2}{4}$ $\frac{y-2}{3}=\pm\frac{x-3}{2}$ $y=\pm\frac{3(x-3)}{2}+2$ $y=\frac{3}{2}x-\frac{5}{2}$ or $y=-\frac{3}{2}x+\frac{13}{2}$  	
4ii	$12y^2-48y+48+ax^2-6ax-3a=0$ $12(y^2-4y)+a(x^2-6x)-3a+48=0$ $12(y^2-4y+4)-48+a(x^2-6x+9)-9a-3a+48=0$ $12(y-2)^2+a(x-3)^2=12a$ $\frac{(y-2)^2}{a}+\frac{(x-3)^2}{12}=1$ which is an ellipse with centre $(3,2)$ and vertices at $(3,2+\sqrt{a})$ and $(3,2-\sqrt{a})$ For curve *C* and *D* to not intersect, $\sqrt{a}<3\Rightarrow a<9$ Since *a* is a positive constant, $\therefore\{a\in\mathbb{R}:0<a<9\}$	Modified mark allocation from 2 to 4.

5i	$\frac{1+x^2}{2-x^2}=\left(1+x^2\right)\left(2-x^2\right)^{-1}$ $=\left(1+x^2\right)2^{-1}\left(1-\frac{x^2}{2}\right)^{-1}$ $=\frac{1}{2}\left(1+x^2\right)\left[1+(-1)\left(-\frac{x^2}{2}\right)+\frac{(-1)(-2)}{2!}\left(-\frac{x^2}{2}\right)^2+\ldots\right]$ $=\frac{1}{2}\left(1+x^2\right)\left(1+\frac{x^2}{2}+\frac{x^4}{4}+\ldots\right)$ $=\frac{1}{2}\left(1+x^2+\frac{x^2}{2}+\frac{x^4}{2}+\frac{x^4}{4}+\ldots\right)$ $=\frac{1}{2}+\frac{3x^2}{4}+\frac{3x^4}{8}+\ldots$	Refer to MF26 and apply the standard series $(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\ldots$
5ii	For expansion to be valid, $\left\|-\frac{x^2}{2}\right\|<1\Rightarrow\left\|\frac{x^2}{2}\right\|<1$ $\left\|x^2\right\|<2$ $x^2<2 \qquad \left(\left\|x^2\right\|=x^2 \text{ since } x^2\geq 0\right)$ $\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)<0$ $\left\{x\in\mathbb{R}:-\sqrt{2}<x<\sqrt{2}\right\}$	Note that the standard series $(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\ldots$ is only valid when $\|x\|<1$. Since the above standard series is applied to $\left(1-\frac{x^2}{2}\right)^{-1}$, hence the expansion is only valid if $\left\|-\frac{x^2}{2}\right\|<1$.
5iii	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1+x^2}{2-x^2}\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}+\frac{3x^2}{4}+\frac{3x^4}{8}+\ldots\right)$ $\frac{\left(2-x^2\right)(2x)-\left(1+x^2\right)(-2x)}{\left(2-x^2\right)^2}=\frac{6x}{4}+\frac{12x^3}{8}+\ldots$ $\frac{4x-2x^3+2x+2x^3}{\left(2-x^2\right)^2}=\frac{6x}{4}+\frac{12x^3}{8}+\ldots$ $6x\left(2-x^2\right)^{-2}=\frac{6x}{4}+\frac{12x^3}{8}+\ldots$ $\left(2-x^2\right)^{-2}=\frac{1}{4}+\frac{x^2}{4}+\ldots$	

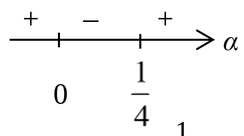
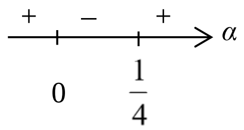
6	$y=\mathrm{g}(x)=\dfrac{ax+b}{2x+c}$ Vertical asym : $x=-\dfrac{c}{2}=\dfrac{3}{2}\Rightarrow c=-3$ Horizontal asym : $y=\dfrac{a}{2}=-2\Rightarrow a=-4$ y-intercept : $y=\dfrac{b}{c}=-\dfrac{4}{3}\Rightarrow b=4$ $\therefore y=\mathrm{g}(x)=\dfrac{-4x+4}{2x-3}$ $y=\mathrm{f}\left(\dfrac{1}{2}x-1\right)$ $\downarrow$ Replace x by $x+2$ $y=\mathrm{f}\left(\dfrac{1}{2}(x+2)-1\right)=\mathrm{f}\left(\dfrac{1}{2}x\right)$ $\downarrow$ Replace x by $2x$ $y=\mathrm{f}\left(\dfrac{1}{2}(2x)\right)=\mathrm{f}(x)$ The graph of $y=\mathrm{f}\left(\dfrac{1}{2}x-1\right)$ is translated 2 units in the negative x-direction and then stretched parallel to the x-axis by factor $\dfrac{1}{2}$ with y-axis invariant. $\mathrm{f}\left(\dfrac{1}{2}x-1\right)=\dfrac{-4x+4}{2x-3}$ $\mathrm{f}\left(\dfrac{1}{2}x\right)=\dfrac{-4(x+2)+4}{2(x+2)-3}$ $=\dfrac{-4x-4}{2x+1}$ $\mathrm{f}(x)=\dfrac{-4(2x)-4}{2(2x)+1}$ $=\dfrac{-8x-4}{4x+1}$	

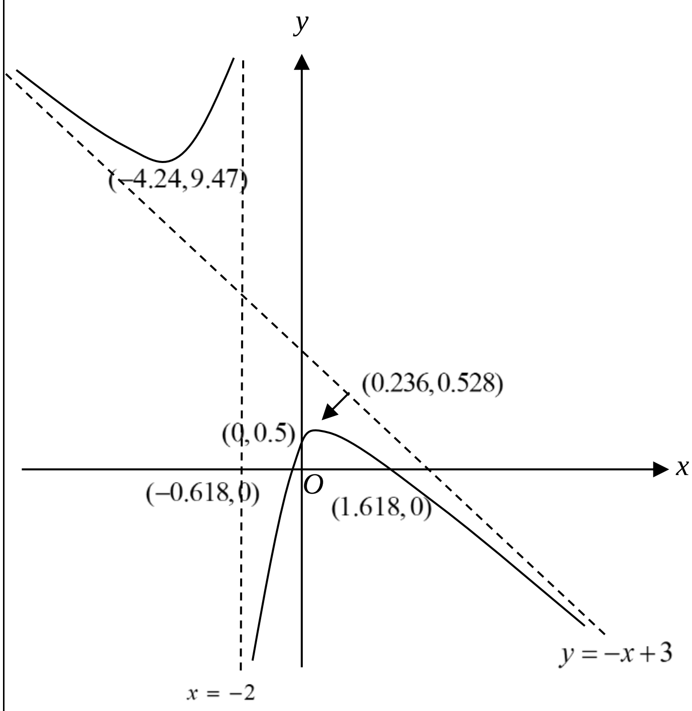
	Alternatively, $y=\mathrm{f}\left(\frac{1}{2}x-1\right)$ ↓ Replace x by $2x$ $y=\mathrm{f}\left(\frac{1}{2}(2x)-1\right)=\mathrm{f}(x-1)$ ↓ Replace x by $x+1$ $y=\mathrm{f}\left((x+1)-1\right)=\mathrm{f}(x)$ The graph of $y=\mathrm{f}\left(\frac{1}{2}x-1\right)$ is stretched parallel to the x-axis by factor $\frac{1}{2}$ with y-axis invariant and then translated 1 unit in the negative x-direction. $\mathrm{f}\left(\frac{1}{2}x-1\right)=\frac{-4x+4}{2x-3}$ $\mathrm{f}(x-1)=\frac{-4(2x)+4}{2(2x)-3}$ $=\frac{-8x+4}{4x-3}$ $\mathrm{f}(x)=\frac{-8(x+1)+4}{4(x+1)-3}$ $=\frac{-8x-4}{4x+1}$	
7a	$\int \sin 2x\cos^6 2x\,\mathrm{d}x=-\frac{1}{2}\int -2\sin 2x\cos^6 2x\,\mathrm{d}x$ $=-\frac{1}{2}\left(\frac{\cos^7 2x}{7}\right)+C$ $=-\frac{\cos^7 2x}{14}+C$	This is of the standard form $\int \mathrm{f}'(x)\left[\mathrm{f}(x)\right]^n\,\mathrm{d}x$ where $\mathrm{f}(x)=\cos 2x$
7bi	$\int \frac{3x}{x^2+2}\,\mathrm{d}x=\frac{3}{2}\int \frac{2x}{x^2+2}\,\mathrm{d}x$ $=\frac{3}{2}\ln\left\|x^2+2\right\|+C$ $=\frac{3}{2}\ln\left(x^2+2\right)+C\ (\because x^2+2>0)$	

7bii	$\dfrac{2}{x-3}+\dfrac{3x+1}{x^2+2}=\dfrac{2\left(x^2+2\right)+(3x+1)(x-3)}{(x-3)\left(x^2+2\right)}$ $=\dfrac{2x^2+4+3x^2-8x-3}{(x-3)\left(x^2+2\right)}$ $=\dfrac{5x^2-8x+1}{(x-3)\left(x^2+2\right)}$	
biii	$\displaystyle\int_0^2 \frac{10x^2-16x+2}{(x-3)\left(x^2+2\right)}\,\mathrm{d}x$ $\displaystyle=2\int_0^2 \frac{5x^2-8x+1}{(x-3)\left(x^2+2\right)}\,\mathrm{d}x$ $\displaystyle=2\int_0^2 \left(\frac{2}{x-3}+\frac{3x+1}{x^2+2}\right)\mathrm{d}x$ $\displaystyle=2\int_0^2 \left(\frac{2}{x-3}+\frac{3x}{x^2+2}+\frac{1}{x^2+2}\right)\mathrm{d}x$ $=2\left[2\ln\lvert x-3\rvert+\dfrac{3}{2}\ln\left(x^2+2\right)+\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{x}{\sqrt{2}}\right)\right]_0^2$ $=2\left[2\ln 1+\dfrac{3}{2}\ln(6)+\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{2}{\sqrt{2}}\right)\right]$ $-2\left[2\ln 3+\dfrac{3}{2}\ln(2)+\dfrac{1}{\sqrt{2}}\tan^{-1}(0)\right]$ $=3\ln 6+\sqrt{2}\tan^{-1}\left(\sqrt{2}\right)-4\ln 3-3\ln 2$ $=\sqrt{2}\tan^{-1}\left(\sqrt{2}\right)-\ln 3$	

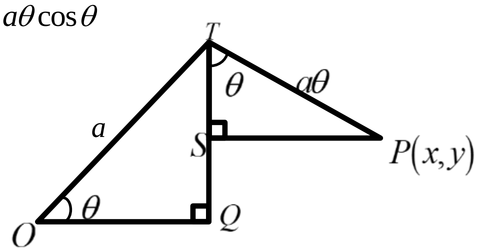
7c	$x=\frac{1}{3}\sin^2\theta \quad \Rightarrow \quad \frac{dx}{d\theta}=\frac{2}{3}\sin\theta\cos\theta$ When $x = 0$, $\theta = 0$; when $x = \frac{1}{4}$, $\theta = \frac{\pi}{3}$. $\int_0^{\frac{1}{4}}\sqrt{\frac{x}{1-3x}}\,dx = \int_0^{\frac{\pi}{3}}\sqrt{\frac{\frac{1}{3}\sin^2\theta}{\cos^2\theta}}\left(\frac{2}{3}\sin\theta\cos\theta\,d\theta\right)$ $=\frac{2}{3\sqrt{3}}\int_0^{\frac{\pi}{3}}\sin^2\theta\,d\theta$ $=\frac{2}{3\sqrt{3}}\int_0^{\frac{\pi}{3}}\frac{1-\cos 2\theta}{2}\,d\theta$ $=\frac{1}{3\sqrt{3}}\left[\theta-\frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{3}}$ $=\frac{1}{3\sqrt{3}}\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right]$	
8ai	$\lvert z^*\rvert=\left\lvert\frac{(-1-i)^3}{1-i\sqrt{3}}\right\rvert=\frac{\lvert -1-i\rvert^3}{\lvert 1-i\sqrt{3}\rvert}=\frac{2\sqrt{2}}{2}=\sqrt{2}$ $\arg\left(z^*\right)=\arg\left(\frac{(-1-i)^3}{1-\sqrt{3}i}\right)$ $=3\arg(-1-i)-\arg\left(1-\sqrt{3}i\right)$ $=3\left(-\frac{3}{4}\pi\right)-\left(-\frac{1}{3}\pi\right)$ $=-\frac{23}{12}\pi\equiv\frac{1}{12}\pi$ $\left\lvert\frac{1}{z}\right\rvert=\frac{1}{\lvert z\rvert}=\frac{1}{\lvert z^*\rvert}=\frac{1}{\sqrt{2}}$ $\arg\left(\frac{1}{z}\right)=-\arg(z)=\arg\left(z^*\right)=-\frac{23}{12}\pi$ or $\frac{1}{12}\pi$	

8aii	$\frac{1}{z^4}=\left(\frac{1}{z}\right)^4=\left(\frac{1}{\sqrt{2}}\mathrm{e}^{-\frac{23}{12}\pi\mathrm{i}}\right)^4=\frac{1}{4}\mathrm{e}^{-\frac{23}{3}\pi\mathrm{i}}=\frac{1}{4}\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}$ $\mathrm{e}^{2a+\mathrm{i}b}=\frac{1}{z^4}\Rightarrow\mathrm{e}^{2a}\cdot\mathrm{e}^{\mathrm{i}b}=\frac{1}{4}\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}$ Therefore we have $\mathrm{e}^{2a}=\frac{1}{4}\Rightarrow 2a=\ln\frac{1}{4}\Rightarrow a=\ln\frac{1}{2}$ or $-\ln 2$ $\mathrm{e}^{\mathrm{i}b}=\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}\Rightarrow b=\frac{1}{3}\pi$	
8b	$\mathrm{i}u-v=3\Rightarrow v=\mathrm{i}u-3$ Then substituting $w=\mathrm{i}z-3$ into the other equation, $u^*+(1-\mathrm{i})(\mathrm{i}u-3)=7+4\mathrm{i}$ $u^*+\mathrm{i}u-3-\mathrm{i}^2u+3\mathrm{i}=7+4\mathrm{i}$ $u^*+\mathrm{i}u+u=10+\mathrm{i}$ $2a+\mathrm{i}(a+\mathrm{i}b)=10+\mathrm{i}$ $2a-b+\mathrm{i}a=10+\mathrm{i}$ Comparing the real and imaginary parts, we get $a=1$ and $2a-b=10\Rightarrow 2-b=10\Rightarrow b=-8$. Therefore $u=1-8\mathrm{i}$ and $v=\mathrm{i}(1-8\mathrm{i})-3=5+\mathrm{i}$	

9	$y=\dfrac{\alpha x^2+x+1}{x+2}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{(x+2)(2\alpha x+1)-(\alpha x^2+x+1)}{(x+2)^2}=\dfrac{\alpha x^2+4\alpha x+1}{(x+2)^2}$ For C to have 2 stationary points, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$ has 2 real roots. For $\alpha x^2+4\alpha x+1=0$ to have 2 real roots, Discrimant > 0 $(4\alpha)^2-4\alpha>0$ $4\alpha(4\alpha-1)>0$  $\alpha<0$ or $\alpha>\dfrac{1}{4}$ $\therefore k=\dfrac{1}{4}$ **<u>Alternatively,</u>** $y=\dfrac{\alpha x^2+x+1}{x+2}=\alpha x+1-2\alpha+\dfrac{4\alpha-1}{x+2}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\alpha-\dfrac{4\alpha-1}{(x+2)^2}$ For C to have 2 stationary points, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$ has 2 real roots. $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow\alpha-\dfrac{4\alpha-1}{(x+2)^2}=0$ $\Rightarrow(x+2)^2=\dfrac{4\alpha-1}{\alpha}$ for equation to have 2 real roots, $\dfrac{4\alpha-1}{\alpha}>0$  $\alpha<0$ or $\alpha>\dfrac{1}{4}$ $\therefore k=\dfrac{1}{4}$	It is a show question. Clear steps on solving quadratic inequality (i.e. number line or equivalent) must be shown clearly.

		Take note to label SIA, i.e. **S**tationary points, **I**ntercepts, **A**symptotes Draw a bigger graph as the maximum point and y-intercept are very close to each other There is NO NEED to find exact coordinates. Use the GC to determine the coordinates of stationary points and intercepts.
10 ai	No. of revolutions in n^{th} minute, $u_n = S_n - S_{n-1}$ $= 54n(29-n) - 54(n-1)(29-n+1)$ $= 1620 - 108n$ $u_n - u_{n-1} = 1620 - 108n - [1620 - 108(n-1)]$ $= -108$ Since $u_n - u_{n-1} = -108$ is a constant (independent of n), the number of revolutions made in each minute follows an arithmetic progression.	**Be *efficient* in algebraic manipulation.** Do not leave your expression for u_n as $u_n = 54n(29-n) - 54(n-1)(29-n+1)$ Use $u_n = 620 - 108n$ to deduce u_{n-1} , it is not efficient to use $u_{n-1} = S_{n-1} - S_{n-2}$
10 aii	For $u_n = 1620 - 108n \leq 0$, $n \geq 15$. Total number of revolutions, $S_{15} = 54(15)(29-15) = 11340$ Distance travelled $= 11340 \times \pi \times 61$ cm $= 21732165$ cm $= 21.7$ km (to 3 s.f.) (shown)	It is a show question. Need to evaluate the answer to at least 5sf before presenting the final answer, i.e. 21.7km.

10b i	$v_1 = (486+20)\left(\frac{2}{3}\right)$ $v_2 = \left[(486+20)\left(\frac{2}{3}\right)+20\right]\left(\frac{2}{3}\right)$ $= 486\left(\frac{2}{3}\right)^2 + 20\left(\frac{2}{3}\right)^2 + 20\left(\frac{2}{3}\right)$ $\vdots$ $v_n = 486\left(\frac{2}{3}\right)^n + 20\left(\frac{2}{3}\right)^n + 20\left(\frac{2}{3}\right)^{n-1} + \ldots + 20\left(\frac{2}{3}\right)$ $= 486\left(\frac{2}{3}\right)^n + 20\left[\frac{\frac{2}{3}\left(1-\left(\frac{2}{3}\right)^n\right)}{1-\left(\frac{2}{3}\right)}\right]$ $= 486\left(\frac{2}{3}\right)^n + 40\left(1-\left(\frac{2}{3}\right)^n\right)$ $= 446\left(\frac{2}{3}\right)^n + 40$ (shown)	Be clear on the <u>number of terms</u> in the GP. $20\left(\frac{2}{3}\right)^n + 20\left(\frac{2}{3}\right)^{n-1} + \ldots + 20\left(\frac{2}{3}\right)$ has n terms $20\left(\frac{2}{3}\right)^{n-1} + 20\left(\frac{2}{3}\right)^{n-2} + \ldots + 20\left(\frac{2}{3}\right)$ has $(n-1)$ terms
10b ii	Since $\left(\frac{2}{3}\right)^n > 0$ for all $n > 0$, $v_n = 446\left(\frac{2}{3}\right)^n + 40 > 40$. Thus, the wheel always rotates at a rate of more than 40 rpm.	Read the question carefully. You need to show the wheel "always rotates at a rate of more than 40rpm", means "**for all values of *n***", not "$n \to \infty$"
10b iii	$446\left(\frac{2}{3}\right)^m + 40 < 45$ $\left(\frac{2}{3}\right)^m < \frac{5}{446}$ $m > \ln\frac{5}{446} \div \ln\frac{2}{3}$ $m > 11.1$ (to 3 s.f.) Least $m = 12$ **<u>Alternative method</u>** $446\left(\frac{2}{3}\right)^m + 40 < 45$ <table><tr><th>m</th><th>$446\left(\frac{2}{3}\right)^m + 40$</th></tr><tr><td>11</td><td>45.156 > 45</td></tr><tr><td>12</td><td>43.437 < 45</td></tr><tr><td>13</td><td>42.292 < 45</td></tr></table>From the GC, least m is 12.	

11	$OQ = a\cos\theta$; $TQ = a\sin\theta$ $TP = a\theta$ (arc length of unit circle) $SP = a\theta\sin\theta$; $TS = a\theta\cos\theta$  $x = OQ + SP$ $= a\cos\theta + a\theta\sin\theta$ (shown) $y = TQ - TS$ $= a\sin\theta - a\theta\cos\theta$ (shown)	
11i	$\frac{dx}{d\theta} = -a\sin\theta + a\sin\theta + a\theta\cos\theta = a\theta\cos\theta$ $\frac{dy}{d\theta} = a\cos\theta - a\cos\theta + a\theta\sin\theta = a\theta\sin\theta$ $\frac{dy}{dx} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$ When $\theta = \frac{\pi}{3}$ $x = a\left(\cos\frac{\pi}{3} + \frac{\pi}{3}\sin\frac{\pi}{3}\right) = a\left(\frac{1}{2} + \frac{\pi\sqrt{3}}{6}\right)$ $y = a\left(\sin\frac{\pi}{3} - \frac{\pi}{3}\cos\frac{\pi}{3}\right) = a\left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right)$ Gradient of normal $= -\frac{1}{\tan\frac{\pi}{3}} = -\frac{1}{\sqrt{3}}$ Equation of normal at W is $y - a\left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}\left(x - \frac{a}{2} - \frac{\pi a\sqrt{3}}{6}\right)$ $\sqrt{3}y - \sqrt{3}a\left(\frac{\sqrt{3}}{2}\right) + \frac{\pi\sqrt{3}a}{6} = -x + \frac{a}{2} + \frac{\pi\sqrt{3}a}{6}$ $\sqrt{3}y = \frac{a}{2} + \frac{3a}{2} - x$ $\sqrt{3}y = 2a - x$ (shown)	Use <u>product rule</u> to find $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ Note: $\frac{d}{d\theta}(a\theta\sin\theta) = a\sin\theta + a\theta\cos\theta$ Be clear on the variables and constants. In this context, the variables are x, y, θ . a is a constant.

11ii	At $\theta = \frac{\pi}{3}$, $\frac{\mathrm{d}x}{\mathrm{d}t} = 0.3$ $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t} = \left(\tan\frac{\pi}{3}\right)(0.3) = \frac{3\sqrt{3}}{10}$ $z = xy$ -----(1) Differentiate (1) w.r.t. t $\frac{\mathrm{d}z}{\mathrm{d}t} = x\frac{\mathrm{d}y}{\mathrm{d}t} + y\frac{\mathrm{d}x}{\mathrm{d}t}$ $= a\left(\frac{1}{2} + \frac{\pi\sqrt{3}}{6}\right)\left(\frac{3\sqrt{3}}{10}\right) + a\left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right)(0.3)$ $= 0.834a$ (3sf) **<u>Alternatively,</u>** $\frac{\mathrm{d}z}{\mathrm{d}x} = y + x\frac{\mathrm{d}y}{\mathrm{d}x}$ $= a\left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right) + \left(\sqrt{3}\right)a\left(\frac{1}{2} + \frac{\pi\sqrt{3}}{6}\right)$ $= 2.7792a$ $\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\mathrm{d}z}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t} = (2.7792a)(0.3) = 0.834a$ (3sf)	Be clear on the variables and constants. In this context, the variables are x, y, θ . ***a* is a constant.** $\frac{\mathrm{d}z}{\mathrm{d}a}, \frac{\mathrm{d}y}{\mathrm{d}a}, \frac{\mathrm{d}a}{\mathrm{d}t}$ have no meaning $\sqrt{3}y = 2a - x$ is the equation of the <u>normal</u> at $\theta = \frac{\pi}{3}$, and should **NOT** be used in this part.